On Mathematical Thought and the Teaching of Ordinary Differential Equation

Ye Zhao¹, Qian Xu^{2,*}

¹Beijing Institute of Petrochemical Technology, Beijing, China

²Beijing Union University, Beijing, China

*Correspondence Author, xuqian098@163.com

Abstract: *Mathematics plays an important role in human culture. The concept of mathematical thought and culture are meaningful to mathematical teaching. Ordinary differential equation (ODE) is an important part of college mathematics curriculum. Combined with some instance and experience during the teaching processing, this paper elaborates some mathematical thoughts and methods in the teaching and research of ODE, which is beneficial to the all-round development of college students.*

Keywords: Mathematical thought and culture, Ordinary differential equation, Mathematical methods.

1. Introduction

Mathematical science, like other subjects, is the manifestation of human evolution and intellectual development, and is also one of the pursuit and expression of human goodness and beauty. For mathematics teaching, it is not only a course, a subject, but also a mode of thinking and a rational spirit. Mathematics education is not a single dissemination of knowledge and skills, because knowledge and skills in mathematics teaching are far from representing mathematics itself. It's like learning an instrument. Playing a moving piece of music can not represent music itself. Mathematics education should also include culture literacy education, which is the unity of "the connotation of mathematics, rational thinking of mathematics, and aesthetic thought of mathematics", and the display of "emotion, attitude and value".

ODE is one of the compulsory courses of mathematics and applied mathematics majors in colleges and universities in China. It is an important branch of modern mathematics and an important channel of connecting mathematics with practical problems. ODE has a profound and vivid practical background, which originates from production practice and scientific technology, and becomes a powerful tool for modern scientific analysis and problem solving. As a mathematics educator, we should carry out the teaching purpose of "cultivating talents by virtue" and constantly explore and study the thought culture of mathematics, and integrate it into the classroom teaching, so that students can be affected by the mathematics culture.

2. Mathematical Thoughts and Methods in ODE

Many mathematical thoughts and methods are covered in ODE. Teachers should understand them deeply and become the guide of students' learning thoughts and methods.

2.1 The Thoughts of Transformation

The transformation means that the general problem is changed into the special one by a certain transformation, and the complicated one is changed into the simple one [1]. The core

of the thought of transformation is simplification and reduction. In ODE, the idea of reduction is to transform a differential equation which is difficult to solve or directly discuss its properties into an equation which is easy to solve or directly discuss its properties.

For example, the homogeneous equation is transformed into a variable separable equation by variable substitution. In addition, the method of integral factor is used to transform the improper equation into the proper equation, the Bernoulli equation into the first-order linear equation, the Riccati equation into the Bernoulli equation, etc.

2.2 The Idea of Constant Variation

The constant variation method is an important method in the initial integral method for solving ODE. It is a method that the solution of homogeneous linear differential equations is transformed into the solution of non-homogeneous linear differential equations. The theory of linear system of ODE has been perfected. For students, the structure of linear equation solution is also a good review and application of linear space theory.

The solution of $\frac{dy}{dx} = p(x)y$ is obtained by the method of

separating variables, $y = ce^{\int p(x)dx}$. For the corresponding

inhomogeneous equation $\frac{dy}{dx} = p(x)y + q(x)$, it can be written

in differential form $dy = [p(x)y + q(x)]dx$. So we have

$\frac{dy}{y} = \left(p(x) + \frac{q(x)}{y(x)} \right) dx$. And by formal integral of both sides,

$y = \pm e^{\int p(x)dx} \cdot e^{\int \frac{q(x)}{y(x)}dx}$ can be obtained. Because $y(x)$ is

unknown, $\pm e^{\int \frac{q(x)}{y(x)}dx}$ can be donated by $c(x)$. That is, the

formal solution of the equation is $y = c(x)e^{\int p(x)dx}$. The

expression of $c(x)$ can be computed by substituting the formal solution into the original equation.

2.3 The Thought of Mathematical Model

Any theoretical knowledge can not be separated from reality as the basis, and ODE is no exception. In the elementary mathematics, the equation studies the relationship between discrete variables, while the ODE studies the properties of continuous variables. The essence of the equation is to establish the related differential relationship model reflecting physics, biology, engineering and other fields. The method of mathematical model is a classical method of mathematical theory and an effective method to solve practical problems in the field of science and technology. It can organically unify all kinds of problems in different mathematical models, so that problems of different forms can be solved effectively by mathematical methods.

For example, the mathematical model of object cooling [2].

The object is placed in the air. The temperature is measured $u_0 = 150^\circ\text{C}$ at time $t = 0$, and the temperature is measured $u_1 = 100^\circ\text{C}$ after 10 minutes. We investigate the relationship between the temperature u of the object and time t and calculate the temperature of the object after 20 minutes. Here we assume that the air temperature is kept at $u_a = 24^\circ\text{C}$.

Some basic laws of thermodynamics:

- (1) Heat is always conducted from a high temperature object to a low temperature object;
- (2) Newton's law of cooling: within a certain temperature range, the temperature change speed of an object is proportional to the difference between the temperature of the object and the temperature of the medium.

The following differential equations can be derived from known conditions:

$$\frac{du}{dt} = -k(u - u_a)$$

Where k is a positive constant. By separating variables $\frac{d(u - u_a)}{u - u_a} = -kdt$ are available. After integral of two sides, the general solution of the equation $u = u_a + ce^{-kt}$ can be obtained.

By substituting initial conditions $u(0) = 150^\circ\text{C}$, $u(10) = 100^\circ\text{C}$, we obtain $k \approx -0.051$. So $u = 24 + 126e^{-0.051t}$. Substituting $t = 20$, $u(0) = 150^\circ\text{C}$, $u(20) = 69.43^\circ\text{C}$ is available.

Further, it can be seen from the expression of the solution that the temperature of the object tends to the temperature of the environment as time increases.

In the teaching of mathematical model, the teacher can guide the students to use numerical method to further study the change of the model's solution, and let the students consult the data and summarize the application of the model in real life. For example, the cooling model can be used to identify

the dead time of the corpse, so that students can learn in order to practice. Furthermore, students can feel the "application beauty" of mathematical model method in the exploration.

3. Thoughts and Methods in the Teaching and Research of ODE

As a math educator, we don't have to let students remember the concept and theorem in class all their lives, but we can let students cultivate rational thinking and aesthetic thinking of mathematics while learning knowledge. As a famous mathematics educator in Japan once said, "For the mathematical knowledge learned in school, if there is no chance to use it after graduation, one or two years later, it will soon be forgotten. However, no matter what work they do, only the spirit of mathematics, mathematical thinking methods, research methods, reasoning methods and the point of view of the problem, are engraved deeply on their mind. And that will benefit them throughout their lives at anytime and anywhere." Therefore, in order to effectively carry out teaching, being familiar with some mathematics teaching ideas and methods is very helpful.

3.1 Attaching Importance to Concepts—Erudition and Thoughtfulness

The concept is the essential attribute abstracted from the concrete thing, and is the thought form that people form in the mind to reflect the object essential attribute. The concept has two basic characteristics: connotation and extension, and changes with the development of the objective world [3]. The forming process of concept is a profound thinking activity and an important process for the development of human thinking ability. People's understanding of the objective world and the truth is realized in the formation and renewal of a series of concepts. The cognition and learning of ODE should follow such a replacement process, and the cognition and learning of concepts should be respected.

The teaching or learning of the concept of ODE should clarify the connotation of the concept and the extension of the concept. For example, the concept of "order" in ODE: In an ordinary differential equation, the order of the highest derivative of the unknown function is called the order of the equation [4]. In this concept, there are two key messages, one is the "unknown function", and the other is the "highest-order derivative". It is necessary to grasp these two characteristics. This is the connotation of the concept, and the extension of the concept is also an important way to understand the concept itself. For example, a differential equation $\frac{d^2y}{dx^2} = x +$

$\frac{d^3}{dx^3} \sin x$, although a third-order derivative appears in the equation, this derivative is not a derivative of the "unknown function". So it is not a third-order, but a second-order differential equation.

3.2 Induction and Analogy—Careful Thinking

"Mathematics as a part of culture, its most fundamental characteristic is that it expresses a spirit of exploration," said mathematician Minyou Qi. The students of colleges and

universities should explore knowledge actively, and apply it to practice. This can help them to achieve mastery through a comprehensive study of the subject and to form knowledge chain. Induction and analogy provide an effective way.

In the teaching of ODE, teachers can assist students to classify the methods used to solve the equations, organize the related methods together and compare them. For example, the above-mentioned homogeneous equations in section 2.1 can be classified into the methods of solving the separated variables by the transformation of variables. The teacher can let the students make a mind map, thus they can clear the thought of refined content to grasp the knowledge. On the other hand, the teacher should also pay attention to divergent thinking in teaching of ODE. For example, the first-order linear inhomogeneous differential equations mentioned above can be solved from the point of view of constant variation or by the method of integral factor. So that knowledge can be systematized.

4. Conclusion

“Mathematics evokes enthusiasm and suppresses impatience, purifies the soul and eliminates prejudice and error. Bad habit is the root of error, chaos and hypocrisy, and all truths are countered with it. And mathematical truths are better for young people to abandon evil habits.” says Albutnot John. Therefore, the study and education of mathematics not only promotes and improves the level of science and technology, but also plays an important role in cultivating talents, promoting the scientific thinking of human beings, cultivating the sentiment of human beings and promoting the spiritual quality of human beings. As a college teacher, we need to understand the meaning of mathematics education, melt the thinking, philosophy and spirit of mathematics into the dissemination of mathematics knowledge and skills. At the same time, we should strengthen and attach importance to the principle of cultural quality in mathematics learning and education, to better serve the society.

5. Acknowledgement

This work is supported by education and teaching reform foundation of Beijing Institute of Petrochemical Technology (NO.ZDFS GG202107001).

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Author Profile

Ye Zhao received his PhD in Partial Differential Equation from Capital Normal University in 2007. Now she is a lecturer in Beijing Institute of Petrochemical Technology, China.