ESTUDO DO PROBLEMA DE PERDA DE ESTABILIDADE DE ESTRUTURAS CILINDRICAS DE PAREDES FINAS SOB EXPOSIÇÃO TÉRMICA LOCAL INTENSA

RESEARCH OF THE PROBLEM OF LOSS OF STABILITY OF CYLINDRICAL THIN-WALLED STRUCTURES UNDER INTENSE LOCAL TEMPERATURE EXPOSURE

ИССЛЕДОВАНИЕ ЗАДАЧИ ПОТЕРИ УСТОЙЧИВОСТИ ЦИЛИНДРИЧЕСКИХ ТОНКОСТЕННЫХ КОНСТРУКЦИЙ ПРИ ИНТЕНСИВНОМ ЛОКАЛЬНОМ ТЕМПЕРАТУРНОМ ВОЗДЕЙСТВИИ

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RESUMO

Atualmente, a tecnologia de impressão tridimensional está se desenvolvendo rapidamente. Isso se deve à necessidade de criar produtos de formas complexas, cuja produção pelos métodos padrão existentes é muito trabalhosa e desvantajosa e, às vezes, tecnicamente impossível. O objetivo do estudo é estudar o problema da estabilidade de temperatura de uma estrutura cilíndrica de paredes finas sob exposição térmica local não estacionária, simulando o movimento de um ponto de feixe de laser ao longo de uma das extremidades. Para resolver o problema, foi utilizado o método dos elementos finitos. Como as áreas geométricas do cálculo foram parametrizadas, para cada conjunto de parâmetros, sua própria malha de elementos finitos foi usada para todos os tipos de análise. Foi construído um modelo de elemento finito espacial paramétrico de um invólucro cilíndrico preso ao longo da extremidade inferior. O fluxo de calor em movimento local atua no lado oposto, simulando o movimento do feixe de laser durante a formação aditiva do elemento de parede fina. São obtidas as soluções numéricas do problema de condução de calor dinâmico não estacionário, dependências de temperatura espaço-temporais e soluções do problema quase-estático da perda da estabilidade local e completa do estado de equilíbrio do cilindro em vários instantes de tempo devido à ocorrência de tensões compressivas locais durante aquecimento intenso. São obtidas as dependências da potência crítica do fluxo de calor correspondentes à perda de estabilidade na espessura da parede do cilindro e sua altura. Os resultados acima podem ser usados como parâmetros de calibração do processo de criação de estruturas de paredes finas, usando as tecnologias aditivas da classe Laser Melting para produtos que exigem requisitos maiores de precisão de fabricação.

Palavras-chave: *tecnologias aditivas, capacidade de carga, corpos em crescimento, modelagem completa, temperatura de alto gradiente.*

ABSTRACT

Presently, three-dimensional printing technology is developing rapidly. This happens due to the need to create products of complex shape, the production of which by existing standard methods is very difficult and disadvantageous, and sometimes technically impossible. The study aims to investigate the problem of temperature stability of a thin-walled cylindrical structure under unsteady local thermal exposure, simulating the motion of a laser beam spot along one of the ends. To solve the problem, the finite element method was used. Since the geometric areas of calculation were parameterized, then for each set of parameters, its own finite element mesh was used for each type of analysis. A parametric spatial finite element model of cylindrical shell pinched at the bottom end was constructed. The local moving heat flux acts on the opposite side, simulating the movement of a laser beam during the additive formation of a thin-walled element. Numerical solutions of the non-stationary dynamic heat conduction problem were obtained, spatiotemporal temperature dependences were

obtained, and solutions of the quasistatic problem of the loss of both local and complete loss of stability of the equilibrium state of the cylinder at various times due to the occurrence of local compressive stresses during intense heating were obtained. The dependences of critical heat flux power corresponding to the loss of stability on the cylinder wall thickness and its height were obtained. The above results can be used as calibration parameters of the process of creating thin-walled structures using additive technologies of Laser Melting class for products that require increased requirements for manufacturing accuracy.

Keywords: *additive technologies, load-bearing capacity, growing bodies, full-fledged modeling, high-gradient temperature.*

АННОТАЦИЯ

В настоящее время технологии трехмерной печати развивается стремительными темпами. Это связано с необходимостью создавать изделия сложной формы, производство которых существующими стандартными методами весьма трудоемко и невыгодно, а порой и технически неосуществимо. Целью исследования является исследование задачи температурной устойчивости тонкостенной цилиндрической конструкции при нестационарном локальном тепловом воздействии, моделирующим движение пятна лазерного пучка вдоль одного из торцов. Для решения задачи был использован метод конечного элемента. Так как геометрические области расчета были параметризованы, то для каждого набора параметров использовалась своя конечно-элементная сетка для всех типов анализа. Построена параметрическая пространственная конечно-элементная модель цилиндрической оболочки, заземленной по нижнему торцу. На противоположную сторону действует локальный движущийся тепловой поток, моделирующий движение лазерного луча при аддитивном формировании тонкостенного элемента. Получены численные решения нестационарной динамической задачи теплопроводности, получены пространственно-временные зависимости температуры и получены решения квазистатической задачи о потере как местной, так и полной потери устойчивости равновесного состояния цилиндра в различные моменты времени вследствие возникновения локальных сжимающих напряжений при интенсивном нагреве. Получены зависимости критической мощности теплового потока, соответствующие потере устойчивости от толщины стенки цилиндра и его высоты. Приведенные результаты могут применяться в качестве калибровочных параметров процесса создания тонкостенных конструкций методами аддитивных технологий класса Laser Melting для изделий, требующих повышенных требований к точности изготовления.

Ключевые слова: *аддитивные технологии, несущая способность, растущие тела, конечно-элементное моделирование, высокоградиентные температурные поля.*

1. INTRODUCTION:

Presently, three-dimensional printing technology is developing rapidly (You *et al.*, 2017; Jacobs and Lin, 2017; Park *et al.*, 2017; Smith *et al.*, 2018; Tyburec *et al.*, 2019). This happens due to the need to create products of complex shape, the production of which by existing standard methods is very laborious and disadvantageous, and sometimes technically impossible (Kinash *et al.*, 2017; Boitsov *et al.*, 2018; Du *et al.*, 2018). A large number of works have been done on the topic of use of additive technologies, in particular a review (Uriondo *et al.*, 2015; Chen *et al.*, 2017; Etienne and Timiryazev, 2018; Kasatkin *et al.*, 2019; Ostroukh *et al.*, 2019; Teslenko *et al.*, 2019). Additive technologies allow us to create products with mechanical characteristics close to the features of products obtained by traditional methods (Kablov, 2015; Sokolov and Razov, 2018). It is worth noting that, in addition to advantages, additive technologies have a number of disadvantages, residual stresses and

deformations (Buchbinder *et al.*, 2015; Gerasimenko *et al.*, 2017; Leicht *et al.*, 2018; Fedorenko *et al.*, 2019), which reduce load-bearing capacity of products, as well as hardly predicted distortion of the initial shape that occurs during molding (Simagina, 2017; Formalev *et al.*, 2018a; Bahaadini and Saidi, 2018; Kurbatov *et al.*, 2018; Matveenkov *et al.*, 2019; Faizan *et al.*, 2019).

Attempts to develop high-quality mathematical model of growing bodies are associated with problems of adequate comprehensive formulation and large computational resources (Gao *et al.*, 2018; Nahmany *et al.*, 2017; Formalev and Kolesnik, 2019; Kopecki *et al.*, 2019; Cetin and Baykasoğlu, 2019). At the same time, full-fledged modeling based on classical models of mechanics of deformable solids appears impossible or insufficiently accurate (Lomakin *et al.*, 2017; Jin *et al.*, 2017; Rzeszut and Voronoi, 2018; Wen *et al.*, 2018; Xin *et al.*, 2018; Bai *et al.*, 2019). Nonetheless, taking into account high-gradient temperature stresses associated with

heterogeneity of the temperature fields during the synthesis of the product is also possible based on relatively simple models (Khoshdarregi and Altintas, 2018; Li *et al.*, 2018; Li *et al.*, 2019; Gokhale *et al.*, 2019; Sun *et al.*, 2019). In particular, the presence of local areas of compressive stresses can lead to loss of stability of thin-walled products (Ogibalov and Gribanov, 1968; Falta *et al.*, 2018; Iwicki *et al.*, 2019; Basaglia *et al.*, 2019; Ma *et al.*, 2019).

The study aimed to investigate the problem of temperature stability of a thin-walled cylindrical structure under unsteady local thermal exposure, simulating the motion of a laser beam spot along one of the ends.

2. MATERIALS AND METHODS:

The linearized problem of loss of stability of thin-walled cylindrical shell under local edge heating by high-intensity radiation source simulating the action of the laser beam was considered, and the critical power of the source for its different positions on the circuit was calculated (Formalev *et al.*, 2018b). The shell panels in form of circular cylinder in the three-dimensional quasistatic formulation of thermoelasticity problem were considered. External influence follows from solutions of the problem of unsteady heat conduction. Equations of the static equilibrium have the form (Equation 1). Kinematic relationships correspond to geometrically linear theory (Equation 2). The defining equations of linear thermoelastic medium have the form (Equation 3), where the temperature field follows from the solution of the heat conduction problem described below.

Boundary conditions correspond to rigid termination of one of the ends of cylinder; the remaining surfaces are free of bonds (Equation 4). The heat conduction problem can be considered in a non-stationary setting (Equation 5). At boundary conditions corresponding to the absence of heat exchange with the external environment (Equation 6) and in homogeneous initial conditions (Equation 7). At the upper end of the cylinder, the field of heat flux vector with a finite carrier is set, directed along the normal, and decaying according to exponential law. The position of non-zero flux is defined by the point modeling center of the spot of the laser beam (Equation 8) (Rabinskiy and Tushavina, 2019a). The accepted law of distribution of amplitude of heat flux over time is presented in Figure 1. The solution to the heat conduction problem has a general form $T=T(x,y,z,t)$.

The geometric model represents a cylindrical shell with the height and wall thickness specified parametrically (Rabinskiy and Tushavina, 2019b). Tasks can be solved using the finite element method. Since the geometric areas of calculation were parameterized, then for each set of parameters, its own finite element mesh was used for each type of analysis. Volumetric tetrahedral four-node elements formed the grid. The number of nodes for each option was not less than 150.000 (Figure 2).

In the heat conduction problem, initial and boundary conditions (6)-(7) were applied to all surfaces. At the upper limit at point (R, 0, H), heat flow (8) is used. The problem is solved in a dynamic setting. The solution to the problem of heat conduction is a temperature field set table at points corresponding to the nodes of the finite element grid $T=T_{(i,k)}$ (i is the node number, k is the time instant) (Rabinskiy *et al.*, 2019). The linear thermoelasticity problem can be solved numerically in three-dimensional formulation. Lower side surface is rigidly pinched (4). The temperature field is used as a load at a fixed moment of time in accordance with the solution of previous problem.

The obtained solution of thermoelasticity problem corresponds to the subcritical state of the simulated system. The issue of stability of the equilibrium state for the discrete analog of the original problem is considered in bifurcation formulation; that is, it can be reduced to the eigenvalue problem for the operator of homogeneous initial-boundary value problem of bending deformation under strains corresponding to the thermoelasticity problem. Then, the temperature field generated by source (laser) and causing subcritical stress state acts as one-parameter loading of the system. The load parameter will be the coefficient at the amplitude of heat flux, i.e. (Equation 9).

The result of solving the spectral problem can be a set of eigenvalues and eigenvectors of a discrete analogue of the original problem, while the smallest positive value is taken as critical.

3. RESULTS AND DISCUSSION:

The problem was solved for various combinations of wall thickness and cylindrical shell height parameters. The result of integrating the equations of unsteady heat conduction was a solution in the form of isothermal contours similar to concentric circles. At the same time, the density decreased with increasing time (temperature gradients decreased). When calculating linear

thermoelasticity tasks, the obtained stress fields had high gradients at the initial time points, which sharply reduced over time. Since the overall heating of structure increased, the stresses in the embedment zone increased. When solving the task, the effect of the transition of form of loss of stability from local (at short times and high-temperature gradients) to general (at large times and general heating of the structure) was revealed. The various first forms of buckling are demonstrated in Figures 3-4.

The results of solving the spectral problem have demonstrated that over time the value of critical effect decreases, and asymptotically tends to a fixed minimum value. Figure 5 illustrates typical curves for parametric interaction critical values. Based on the results obtained, the following conclusions could be made.

1) when solving the problem, allocation of the model parameter ranges is realized, in which using of given laser heat source with a fixed exposure time at a point is unacceptable ($\lambda < 1$)

2) solving the problem allows us to develop methods for predicting the behavior of thin-walled products during growth, reducing the possibility of appearing of "leashes" and changing the required geometric characteristics at the output.

4. CONCLUSIONS:

Numerical solutions were obtained for the problems of stability loss by thermoelastic thin-walled cylindrical structures during high-intensity edge heating by moving heat source. This problem is an approximation of the model of additive formation of the thin-walled structure by selective laser sintering. The questions were solved by the finite element method in three-dimensional formulation. At the same time, the source was modeled as a field of heat flux vector normal to the side surface of the shell with an amplitude that is isotropically attenuated exponentially.

The quasistatic problem corresponding to heating the structure at various points in time was considered. In the bifurcation formulation of the stability loss problem, the critical values of heat flux power were determined depending on geometric parameters of the model, and the duration of heat source influence. Zones of changes in the form of loss of stability from local to general were identified. It was demonstrated that in the process of laser synthesis, instability zones arise that do not allow using the laser sintering method. The obtained dependencies make it possible to determine the permissible time of

exposure to a laser beam of one zone, taking into account the power of the input stream and geometric dimensions of the product when developing an additive process for thin-walled structural elements based on selective laser sintering.

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$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} \frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0, \frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} + \frac{\tau_{r\theta}}{r} = 0, \frac{\partial \tau_{rz}}{\partial r} + \frac{1}{r} \frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\tau_{rz}}{r} = 0 \quad (\text{Eq. 1})$$

$$\begin{aligned} \frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} \frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} &= 0, \\ \frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} + \frac{\tau_{r\theta}}{r} &= 0, \\ \frac{\partial \tau_{rz}}{\partial r} + \frac{1}{r} \frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\tau_{rz}}{r} &= 0. \end{aligned} \quad (\text{Eq. 2})$$

$$\begin{aligned} \sigma_{rr} &= \lambda(\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{zz}) + 2\mu\varepsilon_{rr} - (3\lambda + 2\mu)\alpha T, \\ \sigma_{\theta\theta} &= \lambda(\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{zz}) + 2\mu\varepsilon_{\theta\theta} - (3\lambda + 2\mu)\alpha T, \\ \sigma_{zz} &= \lambda(\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{zz}) + 2\mu\varepsilon_{zz} - (3\lambda + 2\mu)\alpha T, \\ \sigma_{r\theta} &= 2\mu\varepsilon_{r\theta}, \sigma_{\theta z} = 2\mu\varepsilon_{\theta z}, \sigma_{rz} = 2\mu\varepsilon_{rz}. \end{aligned} \quad (\text{Eq. 3})$$

$$u_r = u_\theta = u_z = 0, z = 0 \quad (\text{Eq. 4})$$

$$\frac{\partial T}{\partial t} = a \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{Q}{\rho c}, \quad (\text{Eq. 5})$$

$$-nq = 0 \quad (\text{Eq. 6})$$

$$T = 0, t = 0 \quad (\text{Eq. 7})$$

$$q_0 = \frac{2\eta P}{\pi r_0^2} e^{-\frac{2r^2}{r_0^2}} \quad (\text{Eq. 8})$$

$$q_0 = \lambda \frac{2\eta P}{\pi r_0^2} e^{-\frac{2r^2}{r_0^2}} \quad (\text{Eq. 9})$$

Figure 1. The accepted law of distribution of amplitude of the heat flux in time

Figure 2. General view of model

Figure 3. Local form of loss of stability

Figure 4. General form of loss of stability

Figure 5. Typical curves of dependence of the critical value of amplitude of heat flux on time