VARIABILIDADE ESPACIAL DE PROPRIEDADES FÍSICAS E MECÂNICAS DE MASSA ROCHOSA NO CAZAQUISTÃO CENTRAL**SPATIAL VARIABILITY OF PHYSICAL AND MECHANICAL PROPERTIES OF ROCK MASS IN CENTRAL KAZAKHSTAN****ПРОСТРАНСТВЕННАЯ ИЗМЕНЧИВОСТЬ ФИЗИКО-МЕХАНИЧЕСКИХ СВОЙСТВ МАССИВА ГОРНЫХ ПОРОД В ЦЕНТРАЛЬНОМ КАЗАХСТАНЕ**

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RESUMO

A eficiência e a qualidade das operações de perfuração e detonação no desenvolvimento de depósitos minerais dependem amplamente da variabilidade das propriedades da massa rochosa. Graças a um estudo detalhado de vários objetos geológicos – depósitos minerais ou elementos estruturais da crosta terrestre de qualquer ordem, até uma camada ou bloco, é possível criar os modelos digitais tridimensionais. O artigo considera a possibilidade de variabilidade espacial das propriedades da massa rochosa no Cazaquistão Central utilizando a modelagem matemática com uso de dados do arquivo de controle de processo de perfuração do sistema Aquila da unidade de perfuração DH-M. Para avaliar a estrutura e condição do maciço, foi construído um modelo digital de uma seção bidimensional, que mostra a variabilidade espacial das propriedades físicas e mecânicas. A aproximação pseudo-linear entre os vetores dos arquivos foi selecionada como um método para construir seções bidimensionais. O artigo mostra a aplicação do método matemático de interpolação pseudo-linear, que permite avaliar com um grau suficiente de confiabilidade as propriedades físicas e mecânicas (PFM) das rochas no espaço entre poços e entre linhas da massa rochosa. Com base nos resultados do estudo, foi desenvolvido um software que fornece uma avaliação rápida e eficaz das propriedades físicas e mecânicas da massa rochosa, com visualização do resultado. A abordagem e o software desenvolvidos podem ser usados para aumentar a eficiência das operações de perfuração e detonação pelas empresas de mineração existentes, bem como na implementação de estudos científicos e industriais sobre modelagem matemática do estado da massa rochosa.

Palavras-chave: *previsão de variabilidade espacial, propriedades físicas e mecânicas de rochas, modelo geológico tridimensional digital, processo de perfuração, software.*

ABSTRACT

The efficiency and quality of drilling and blasting operations in the development of mineral deposits largely depend on the variability of the properties of the rock mass. With a detailed study of various geological objects—mineral deposits or structural elements of the earth's crust of any order, up to a single layer or block, it is possible to create three-dimensional digital models. The article considers the possibility of spatial variability of rock mass properties in Central Kazakhstan by means of mathematical modeling using data from the drilling process control file of the Aquila system of a DH-M drilling rig. To assess the structure and condition of the rock mass, it was constructed two-dimensional sections digital model, which shows the spatial variability of physical and mechanical properties. A pseudolinear approximation between the file vectors is chosen as a method for constructing two-dimensional sections. The application of the mathematical method of pseudo-linear interpolation is shown, which allows with a sufficient degree of reliability to evaluate the physical and mechanical properties (PMP) of the rocks in the inter-well and inter-interval space of the rock mass. Based on the results of the study, the software was

developed that provides an operational and effective assessment of the physical and mechanical properties of the rock mass with the visualization of the result. The developed approach and software can be used to improve the efficiency of drilling and blasting operations of existing mining enterprises, as well as in the implementation of scientific and industrial research on mathematical modeling of state of the rock mass.

Keywords: *prediction of spatial variability, physical and mechanical properties of rocks, digital three-dimensional geological model, drilling process, software.*

АННОТАЦИЯ

Эффективность и качество буровзрывных работ при разработке месторождений полезных ископаемых во многом зависят от изменчивости свойств горного массива. Благодаря детальному изучению различных геологических объектов – месторождений полезных ископаемых или структурных элементов земной коры любого порядка, вплоть до одного слоя или блока, можно создавать трехмерные цифровые модели. В статье рассматривается возможность пространственной изменчивости свойств горного массива в Центральном Казахстане с помощью математического моделирования с использованием данных из файла управления процессом бурения системы Aquila буровой установки ДН-М. Чтобы оценить структуру и состояние горного массива, была построена цифровая модель двумерной секция, которая показывает пространственную изменчивость физико-механических свойств. Псевдолинейное приближение между векторами файлов выбрано в качестве метода построения двумерных сечений. Показано применение математического метода псевдолинейной интерполяции, который позволяет с достаточной степенью достоверности оценивать физико-механические свойства (ФМП) горных пород в межскважинном и междурядном пространстве массива горных пород. На основе результатов исследования было разработано программное обеспечение, которое обеспечивает оперативную и эффективную оценку физико-механических свойств горного массива с визуализацией результата. Разработанный подход и программное обеспечение могут быть использованы для повышения эффективности буровзрывных работ действующих горных предприятий, а также при осуществлении научных и промышленных исследований по математическому моделированию состояния массива горных пород.

Ключевые слова: *прогноз пространственной изменчивости, физико-механические свойства горных пород, цифровая трехмерная геологическая модель, процесс бурения, программное обеспечение.*

1. INTRODUCTION

The efficiency and quality of drilling and blasting operations in the development of mineral deposits largely depend on the variability of the properties of the rock mass (Liu *et al.*, 2017; Chicco *et al.*, 2019). Since the study of natural and technogenic variability of the rock mass is carried out mainly by a selective method, using a network of natural and artificial outcrops, the set of initial data is limited and has a random nature (Chen *et al.*, 2017; Jiang *et al.*, 2017; Uyanik *et al.*, 2019). The limited experimental data and discreteness of observation networks lead to the need to create generalized models (Zhang *et al.*, 2018; Menegoni *et al.*, 2019; Xu *et al.*, 2019). The peculiarity of modeling of mining and geological structures that are modeled, but representations about them or variability of properties observed at the studied level of the object structure. The complexity of the spatial variability of physical and mechanical properties and limitations of empirical data prevent the direct application of deterministic models (Qiu *et al.*, 2019; He *et al.*, 2019). The manifestation of

random variability of the studied properties testifies not to the absence of geological regularities, but only the lack of knowledge at this stage of the study of the subsoil (Grib, 1999; Mordensky *et al.*, 2018; Baldík *et al.*, 2018).

With a detailed study of various geological objects-mineral deposits or structural elements of the earth's crust of any order, up to a single layer or block, it is possible to create three-dimensional digital models (Zhang *et al.*, 2019; Zhang and Shen, 2019; Wu *et al.*, 2019). Their development significantly facilitates the research process by visualizing the structure of the contact surfaces of individual layers and sections in any planes, and adjusting the model as new mining and geological information is obtained (Alekseev *et al.*, 2003; Grib, 2013; Grib and Kuznetsov, 2018; Kovaleva *et al.*, 2018).

In the general case, the creation of the model is carried out in the following sequence: models of the relief of the earth's surface of the blasting block and the network of wells are formed, then the places of their intersection of geological

boundaries and heterogeneities are attached to the wellbores. Interpolation predicts the configuration of boundaries in the inter-well space within the block, which is a connected area. At the final stage of development, it becomes possible to create two-dimensional cross-sections of the variability of physical and mechanical properties with depth and strike, in this case, on the basis of the vectors of the “drilling process control files” F of the “Aquila” system of the DM-H drilling rig obtained by drilling blast holes, further files (Martynov, 2011).

2. MATERIALS AND METHODS

Modern computer technologies allow visualizing the values of physical and mechanical properties corresponding to each point of a two-dimensional section of a geological body. For this reason, to assess the structure and condition of the rock mass, it was sufficient to construct not a complete three-dimensional digital model, but its approximation by two-dimensional sections, which show the spatial variability of physical and mechanical properties: the velocity of elastic waves (V_p), the pressure strength (σ_p), the tension strength (σ_t), the volume density (ρ).

A pseudolinear approximation between the file vectors is chosen as a method for constructing two-dimensional sections: let Y_1 and Y_2 be the observed values at points X_1 and X_2 , and the authors want to estimate the values of Y' at point X' . Considering the dependence in the intermediate points linear, the calculations are carried out according to Equation 1 (Korobeynikov, 2009; Martynov, 2011; Kuznetsov *et al.*, 2018).

It is assumed that the difference of values in two adjacent points is a linear function of the distance between them. The method gives satisfactory results when the number of evenly spaced points and observation points is approximately equal. If the source data is irregular and intermediate values have to estimate for each pair of observations, then linear interpolation is justified. In previous studies (Alekseev *et al.*, 2003), linear interpolation was used to construct models of homogeneous blocks.

Analyzing the experimental results (Alekseev *et al.*, 2003) establishing links and dependencies between the physical and mechanical properties of the drilled rocks and the values of the “drilling process control file” F, which were performed for overburden rocks of the Centralny Kazakhstansky, Slavgorodsky, Kenterlausky sections at the experimental site,

where 15 wells were drilled at a distance of 1 m from each other, and wells 1, 3, 5, 7, 9, 11, 13 and 15 were drilled by the DM-H drilling rig using the Aquila system, and wells 2, 4, 6, 8, 10, 12, 14 were drilled by the exploration drilling machine with full core selection throughout the trunk wells. The study of physical and mechanical properties of core samples from wells 12-14 was carried out in the petrophysical laboratory. According to the test results, the following parameters of physical and mechanical properties were obtained:

- the velocity of propagation of elastic longitudinal waves in rock V_p , km/sec;
- compressive strength of rocks on compression σ_p and tension σ_t MPa;
- volume density of rocks, δ_o g/cm³;
- porosity coefficient, %;
- humidity, %.

Next, a regression analysis was performed between the data of the “drilling process control file” F, and the experimentally obtained values of physical and mechanical properties (Alekseev *et al.*, 2003).

The established correlations are described by the regression equation of the form (Equation 2).

The correlation coefficient between the parameters of the file vectors varies within $0.5 < R < 0.95$, the coefficient of variation $S/X \leq 0.2$ – with confidence not lower than 95%. Where: S is the mean square deviation, X is the mean. This was the basis to consider the exploding block as a connected geological body, and to build its model in the form of a smooth manifold of dimension ≥ 3 . The spread of the parameters of the file vectors is explained by the natural heterogeneity and technogenic fracturing of rocks. The regression of the species is constructed (Equation 3).

Where: X-the average values of the studied parameters calculated for all analyzed wells, and Y-the variance estimate of the average values of the studied parameters; parameters: $A = -0.1941769$, $B = 3.225438 \cdot 10^{-2}$. The correlation coefficient for this regression is $R = 0.9014131$, which according to the theory of A.B. Vistelius (1980), means that the entire exploding block was formed as a result of one process and that the block is of one petrographic composition.

Processing, interpretation, modeling of the structure, composition, and state of the rock mass are based on the assumption that the three-dimensional geological space can be successfully

approximated by a system of two-dimensional layers (Matheron, 2009; Demyanov, 2010). In work (Alekseev *et al.*, 2003), the method of approximation of three-dimensional distributions of parameters of physical and mechanical properties by means of two-dimensional sections is offered. Not random variability allows estimating the character of anisotropy of properties of a massif of rocks and consistency of its structure in various directions. Its characteristic serves as the basis for the choice of distances between wells and the design of the charge mass. Random variability has a direct impact only on the statistical evaluation of the average characteristics of physical and mechanical properties, the anisotropy of which is manifested in the difference of non-random variability of the studied parameter in different directions. A measure of the anisotropy I is the ratio of the average number n of heterogeneity elements intersected by lines drawn in a given direction to the lengths of these lines l within the studied block (Equation 4).

Thus, the ideas about the anisotropy of the properties and structure of the rock mass depend on their natural properties and on the location of the wells of the drilled network. As a tool for constructing two-dimensional sections, it was used the Delphi programming environment and the package of functions of the OpenGL open graphic library (Flenov, 2011; Ginsburg and Purnomo, 2015), the capabilities of which allow us to obtain reliable and visual pictures of sections. The developed application program allows to quickly build two-dimensional sections and maps of arbitrary orientation to determine the most informative types of sections and maps. In addition, the current version of the program allows to visualize cracks in blocks (provided that the angle of incidence of the crack is not very large (Equation 5), where h is the distance between the horizontal lines, l is the distance between the wells).

3. RESULTS AND DISCUSSION:

The program of an automated geological modeling system divided into three blocks: input-output, matrix transformation, and forecasting geological boundaries. The input-output unit writes to the matrix information file about one section on the same network: a series of wells, output of this information, and construction of sections of iso-areas. At the same time, control is carried out on the minimum and maximum permissible values to eliminate typical errors. In the construction of contours, interpolation is applied using the method of rectangles and trapeziums. Variation in scale,

additional smoothing of the boundaries of iso-areas, their display in different colors are provided. In the absence of information in certain parts of the numerical model on the map of iso-areas, these areas are marked in white.

When visualizing strength sections, the interpolation points are evenly spaced at a specified interval. At these points, the values of physical and mechanical properties are determined by interpolation and used as initial points within a square with a side equal to the step of the network model. The data conversion unit is developed on the basis of the established correlations of physical and mechanical properties of rocks with the data of the file F. Regression equations are obtained for four parameters: σ_P , σ_T , V_p , δ_o (Equations 6-9).

Where F_n -values was taken from the drilling control file of the machine "Aquila" (Vistelius, 1980; Korobeynikov, 2009). The program provides data input from a file and from the keyboard. The data is presented in the form of Table 1, in which the first column is the parameter F_n (entered by the user), the columns (σ_P , σ_T , V_p , δ_o) are calculated according to the above formulas.

In practice, three methods are used to describe maps of isolines: coordinates of vertices of polygons approximating the isolines; coordinates of characteristic points of the surface; numerical matrices, the elements of which are the depths or altitudes of the described surface in the nodes of a rectangular network. Based on the data obtained from the "drilling process control file" of the Aquila system, we adopted the third method-the description of the contour maps in the nodes of the rectangular network. At the same time, the maximum angle of bodies described by contact surfaces cannot exceed (Equation 10), where ΔH is the depth difference at neighboring points, ΔL is the horizontal step of the network.

It should be emphasized that the contact surfaces can be described large relative to the applied network of the body, as with a small number of points that fall into the contour, the approximation may be unacceptably rough. The density of the network should ensure satisfactory accuracy of determining the contact surface at any point of the site by linear interpolation of the matrix elements corresponding to the nodes of the cell within which the defined point is located (Matheron, 2009; Demyanov, 2010; Martynov, 2011; Moiseev, 2011). When interpolating, the following technique will be applied. It is natural to assume that in the center of the cell, the value is equal to the average of the values specified in the

nodes (Matheron, 2009; Demyanov, 2010; Martynov, 2011; Moiseev, 2011). Based on this in the center of the cell (Figure 1), set a point with a value equal to the average value. Now it is possible to form four triangles in a cell, and within each of them, it is possible to determine the values by drawing planes through the known values at the vertices of the triangle.

Let's introduce notation (Equation 11). Then it is easy to get expressions to calculate the values within the first – fourth triangle, respectively (Equations 12-15).

Similar results can be obtained by linear interpolation in two mutually perpendicular directions. The resulting surface will not be a plane, so the interpolation is called pseudolinear (Matheron, 2009; Demyanov, 2010; Martynov, 2011; Moiseev, 2011). Experience shows that depending on the complexity of the relief of contact surfaces, the density of the network should be sufficiently detailed (Alekseev *et al.*, 2003; Kutuzov, 2011; Fokin, 2015). Therefore, to display two-dimensional vertical sections along the rows of wells, a uniform network with a step of $Y = 8\text{m}$ and a step of $X = 0.5\text{ m}$ is used, condensed to a uniform network with steps of 0.05 m and 0.1 m along the x and Y axes, respectively. This thickening of the network avoids complex and time-consuming calculations of the passage of isolines and proceed to the direct display of the values of the resulting matrix pixel by pixel, using the capabilities of OpenGL (Figure 2).

4. CONCLUSIONS:

To represent a three-dimensional image of the results of calculations of the interpolation of physical and mechanical properties of rocks, a cube is used, the faces of which are two-dimensional sections along the rows of the block under study. Each pair of opposite faces contains the same number of displayed wells. A separate image is constructed for each pair of wells. After building images of all pairs of wells displayed in the volumetric figure, the resulting textures are superimposed on the faces of the cube in accordance with the spatial position. After displaying the image on the screen, the user can rotate the cube with the cursor keys, studying in detail the images of each pair or three faces, which allows to more clearly represent the spatial variability of physical and mechanical properties in the block under study.

Thus, using the data file management system drilling Aquila drilling machine DH-M, their correlation with the strength and elastic

characteristics of rocks, pseudobinary mathematical method of interpolation and the proposed software may reliably predict physical and mechanical properties of rocks in the interwell space that is prepared for the explosion of the unit, to determine the optimal conditions for blasting.

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$$Y' = \frac{(Y_2 - Y_1)(X' - X_1)}{X_2 - X_1} + Y_1 \quad (\text{Eq. 1})$$

$$y = \frac{x}{a + b \cdot x} \quad (\text{Eq. 2})$$

$$Y = f(X) = \frac{X}{A + BX} \quad (\text{Eq. 3})$$

$$l = n// \quad (\text{Eq. 4})$$

$$\text{tg}\varphi \leq h// \quad (\text{Eq. 5})$$

$$\sigma_P = 165,313 \cdot \text{EXP} \left(\frac{27,11714}{F_n} \right) + 2,018, \text{ (MPa)}, \quad (\text{Eq. 6})$$

$$Vp = 5,112606 \cdot \frac{F_n}{(13,34124 + F_n)} + 0,08, \text{ (km/c)}, \quad (\text{Eq. 7})$$

$$\delta_0 = \frac{F_n}{(1,204589 + 0,3686797 \cdot F_n)} - 0,0576, \text{ (g/cm}^3\text{)}, \quad (\text{Eq. 8})$$

$$\sigma_T = \left(\frac{F_n}{(3,255616 + 0,688612 \cdot F_n)} \right)^6 + 1,299, \text{ (MPa)}, \quad (\text{Eq. 9})$$

$$\alpha = \text{arctg} (\Delta H / \Delta L), \quad (\text{Eq. 10})$$

$$\beta = (y - y_1) / \Delta y \quad (\text{Eq. 11})$$

$$F_1(x, y) = (1 - \alpha - 0,5\beta)F_1 - (\alpha - 0,5\beta)F_4 + 0,5\beta F_3 + 0,5\beta F_2 \quad (\text{Eq. 12})$$

$$F_2(x, y) = (1 - \beta - 0,5\alpha)F_1 - (\beta - 0,5\alpha)F_2 - 0,5\alpha F_3 + 0,5\alpha F_4, \quad (\text{Eq. 13})$$

$$F_3(x, y) = (\alpha + \frac{1+\beta}{2})F_3 + (\alpha - \frac{1-\beta}{2})F_2 + \frac{1-\beta}{2}F_1 + \frac{1-\beta}{2}F_4, \quad (\text{Eq. 14})$$

$$F_4(x, y) = (\beta - \frac{1+\alpha}{2})F_3 - (\beta - \frac{1-\alpha}{2})F_4 + \frac{1-\alpha}{2}F_1 + \frac{1-\alpha}{2}F_2, \quad (\text{Eq. 15})$$

Table 1. Example of calculation of physical and mechanical properties

F_n	V_p, km/sec	δ₀g/cm³	σ_P, MPa	σ_T, MPa
7.84	1.968	1.914	7.220	0.653
18.96	3.077	2.313	41.570	2.566
48.64	4.088	2.541	96.682	5.476
29.71	3.604	2.443	68.379	3.967
44.08	4.000	2.525	91.377	5.189
33.338	3.727	2.470	75.309	4.332

Figure 1. The interpolation cell; 1-4 the numbers of triangles

Figure 2. General view of the running program window