INVESTIGAÇÃO DA DINÂMICA DE SISTEMAS MECÂNICOS NÃO LINEARES COM LONGAS LINHAS DE ALIMENTAÇÃO ATRAVÉS DA MODELAGEM DIGITAL**INVESTIGATION OF THE DYNAMICS OF NONLINEAR MECHANICAL SYSTEMS WITH LONG POWER LINES THROUGH DIGITAL MODELING****ИССЛЕДОВАНИЕ ДИНАМИКИ НЕЛИНЕЙНЫХ МЕХАНИЧЕСКИХ СИСТЕМ С ДЛИННЫМИ ЛИНИЯМИ ПИТАНИЯ ЧЕРЕЗ ЦИФРОВОЕ МОДЕЛИРОВАНИЕ**

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RESUMO

Muitos dos mecanismos usados na indústria contêm links de entrada e saída conectados por longas linhas de força. Aumentar a eficiência e a vida útil dos sistemas mecânicos com longas filas é de grande importância para a economia do país. Para um uso mais racional desses dispositivos, é importante manter esses modos de operação com a máxima precisão, geralmente incluindo a velocidade necessária do atuador e a tensão nas linhas. Tais parâmetros podem mudar espontaneamente, dependendo das condições de operação do sistema. Na presença de várias influências, tarefas semelhantes para determinar os regimes e parâmetros marcados, indicando a necessidade de sua mudança, só podem ser resolvidas com a ajuda da teoria e dos métodos de pesquisa correspondentes. O artigo apresenta os problemas e o método de estudar sistemas mecânicos de duas camadas com um número infinito de graus de liberdade com base nas equações de momento e momento de momento em forma diferencial. São propostas transformações com o uso de equações de ondas conhecidas, que possibilitam levar explicitamente em consideração as oscilações das velocidades de movimento e tensões nas linhas de força dos sistemas mecânicos ao descrever processos dinâmicos. A solução de sistemas de equações diferenciais parciais é dada usando a transformada de Laplace, que possibilitou obter equações gerais de movimento e, após algumas simplificações, proceder a equações diferenciais ordinárias que levam em consideração as características dinâmicas de sistemas com parâmetros distribuídos. O método Runge-Kutta modernizado obteve soluções e realizou simulação numérica de processos transitórios no acionamento hidráulico, cujos resultados têm boa convergência com experimentos em larga escala.

Palavras-chave: equações diferenciais, parâmetros distribuídos, sistema mecânico, atuador hidráulico, características de frequência.

ABSTRACT

Many of the mechanisms used in industry contain input and output links connected by long lines of force. Increasing the efficiency and service life of mechanical systems with long lines is of great importance for the country's economy. For a more rational use of these devices, it is important to maintain these operating modes with maximum accuracy, usually including the required speed of the actuator and the voltage in the lines. Such parameters can spontaneously change depending on the operating conditions of the system. In the presence of various influences, similar tasks to determine the marked regimes and parameters indicating the

need for their change can be solved only with the help of the corresponding theory and research methods. The article presents the problems and the method of studying two-tier mechanical systems with an infinite number of degrees of freedom on the basis of the equations of momentum and moment of momentum in differential form. Transformations with the use of well-known wave equations are proposed, which made it possible to explicitly take into account the oscillations of the speeds of motion and stresses in the force lines of mechanical systems when describing dynamic processes. The solution of systems of partial differential equations is given using the Laplace transform, which made it possible to obtain general equations of motion and, after some simplifications, proceed to ordinary differential equations that take into account the dynamic features of systems with distributed parameters. The modernized Runge-Kutta method obtained solutions and carried out numerical simulation of transient processes in the hydraulic drive, the results of which have good convergence with full-scale experiments.

Keywords: *differential equations, distributed parameters, mechanical system, hydraulic drive, frequency characteristics.*

АННОТАЦИЯ

Многие из используемых в промышленности механизмов содержат входные и выходные звенья, соединенные длинными силовыми линиями. Такие механизмы широко используются в технологии глубокого сверления в коллекторах теплообменников, гребных валов, турбо-сверлильных валов с длиной отверстий до семи метров и других критических конструкций, а также в процессах бурения нефтяных и газовых скважин длиной вращающихся линий электропередач, достигающих нескольких километров. Повышение эффективности и срока службы механических систем с длинными линиями имеет большое значение для экономики страны. Для более рационального использования этих устройств важно поддерживать эти режимы работы с максимальной точностью, обычно включая требуемую скорость привода и напряжение в линиях. Такие параметры могут самопроизвольно меняться в зависимости от условий работы системы. При наличии различных воздействий аналогичные задачи по определению отмеченных режимов и параметров, указывающих на необходимость их изменения, могут быть решены только с помощью соответствующей теории и методов исследования. В статье представлены проблемы и методика изучения двухуровневых механических систем с бесконечным числом степеней свободы на основе уравнений импульса и момента импульса в дифференциальной форме. Предложены преобразования с использованием известных волновых уравнений, которые позволили при учете динамических процессов явным образом учитывать колебания скоростей движения и напряжений в силовых линиях механических систем. Решение систем дифференциальных уравнений в частных производных дается с помощью преобразования Лапласа, что позволило получить общие уравнения движения и после некоторых упрощений перейти к обыкновенным дифференциальным уравнениям, учитывающим динамические особенности систем с распределенными параметрами. Модернизированный метод Рунге-Кутты позволил получить решения и провести численное моделирование переходных процессов в гидроприводе, результаты которого хорошо сходятся с полномасштабными экспериментами.

Ключевые слова: *дифференциальные уравнения, распределенные параметры, механическая система, гидравлический привод, частотные характеристики.*

1. INTRODUCTION

Used in industry, many mechanisms are mechanical systems containing input and output links interconnected by long force lines. Such mechanisms are widely used in the technology of deep drilling of holes in the headers of heat exchangers, propeller shafts, turbodrill shafts with hole lengths up to seven meters and other responsible structures, as well as in oil and gas well drilling processes when the length of rotating force lines reaches several kilometers. The given examples show that the increase in efficiency and service life of mechanical systems with long lines is of great importance for the country's economy.

For more rational use of these devices, it is important to maintain the specified (target) modes of operation with maximum accuracy, usually including the required speed of movement of the executive body and the voltage (pressure) in the lines. Such parameters may spontaneously change depending on the system operating conditions. In the presence of various impacts, similar tasks by definition of the noted modes and parameters, indicating the need for their changes, can be solved only with the appropriate theory and research methods (Yasnitskii, 1989). While designing such structures, the dynamic properties of the system can be assessed using traditional methods, for example, frequency analysis. However, for the implementation of technological

operations, it is important to know the regularity of the movement of the executive body (cutter, cutting part of the drill, etc.), as well as stresses arising in structural elements (Kondratenko and Mironova, 2018a). In addition, for their more effective application, it is necessary to maintain rational modes of operation with maximum accuracy, for example, maintain the required speed the executive body at given loads.

The design of two-link mechanisms often constitute the input link, which is the engine, and the output link is the executive body (the cutting part of the drill, chisel, mill, etc.), rigidly interconnected by force lines. Such simple mechanisms Babakov I.M. refers to systems with an infinite number of degrees of freedom (Babakov, 1958). The study of processes in the long force lines of mechanical systems usually described using differential equations in partial derivatives, has been the subject of a number of papers, for example, (Sirazijeva *et al.*, 2004, Kondratenko and Mironova, 2018a; Kondratenko and Mironova, 2018b; Kondratenko and Mironova, 2018c; Babakov, 1958; Popov, 1977; Sedov, 1970; Voronov, 1981; Frolov, 2000; Markin, 2010; Kondratenko, 2005, Vlasova *et al.*, 2016). However, only in (Kondratenko and Mironova, 2018a); Popov, 1977; Kondratenko and Mironova, 2018c), the authors take into account the presence of a load on the output. Traditionally, the description is constructed using partial differential equations.

Due to the fact that significant oscillatory phenomena occur in systems with long lines, it is very important to find a way to calm the oscillations. This path is associated with overcoming serious difficulties. To solve problems of this kind, it was proposed in (Berezyanskij and Kondratev, 1988) to use the expansion theory in the form of an infinite family of commuting self-adjoint operators with the corresponding projection spectral theorem. With this approach, theorems of representation of positive definite functions of an infinite number of variables in a layer are established and the infinite-dimensional problem of moments is studied. The role of operators is played by a countable family of differentiations with respect to various variables, infinite families of shifts, etc. However, the application of this theorem very often gives negative answers. The main question was investigated only in certain particular cases when in this way the values of optimal control actions were obtained, both for the problems of program control and for certain problems of the synthesis of systems with feedback. In addition to

the above-mentioned studies related to the use of results relating to the problem of moments, problems are known about controlling linear parabolic and hyperbolic systems (Godunov and Ryabenkij, 1977). However, such decisions are justified so far only for individual special cases.

In some cases, similar problems are solved by an approximate method with the replacement of the corresponding functional equations by suitable finite-dimensional difference schemes. As a result, the problem of optimal control of an approximating system described by equations in finite differences or by a system of ordinary differential equations is obtained. Wherein, for a finite-dimensional system, it becomes necessary to consider the maximum principle and estimate the convergence of approximating controls. However, the questions of substantiating such a finite difference approximation have not been studied enough. In mechanical engineering, it is quite rare to find situations when the calculation of oscillations (vibrations) of a particular structural element can be carried out accurately. As a rule, technical calculations are approximate. The main assumptions are made when choosing a calculation scheme. At the same time, the insignificant features of the system are ignored and only its main parameters that determine the nature of the phenomenon are highlighted, although in some cases minor features can have a great influence on the operation of the mechanism. In many cases, systems with distributed masses are replaced by systems with concentrated masses, parts of complex geometric shapes (springs, crankshafts, etc.) are usually reduced to an equivalent straight beam, nonlinear elastic elements are often replaced by equivalent linear ones, etc. Simulation of such systems is also possible with the help of analog machines. However, the complexity of creating a model, low accuracy of results, and low mobility of machines significantly limit the spread of this method.

In most cases, mechanical systems operate under the action of variable external influences in the presence of various nonlinearities, including significant ones (backlash, pressure and displacement restrictions, dead band). Periodic effects can be approximated by harmonic functions, the response of the system to which is conveniently described by frequency characteristics. Often the impacts are sporadic. For example, during deep drilling, getting a carbide particle under the cutting blade will cause a sharp change in the

moment of resistance, which in turn may distort the hole trajectory or disturb the surface layer. It is very difficult to describe the behavior of the system in such cases using frequency methods. It requires calculations that take into account fluctuations in the time domain. Therefore, the study of issues of dynamics and the determination of the actual values of speeds and voltages, characterizing the features of power transmission, is very important for developing solutions to improve the efficiency of these devices. An attempt to solve the noted problems was undertaken in the proposed article.

2. MATERIALS AND METHODS

In this question, the performance of the devices in question is determined by the transmitted power, i.e., the speed of movement of the executive body and the stress in the force line. When these parameters deviate from rational values, there is a need to study the Equations 1, 2 (Kondratenko *et al.*, 2018; Babakov, 1958). Here Ω is the speed of movement of the technological object; t is the time; ζ_i is the coordinates of the system; f_i is the speed of change of coordinates. In this case, the study is reduced to solving Equation 3. Usually, in the study of dynamic processes in mechanical systems, the tasks of determining changes in coordinates, vibration shapes (Kolovskij, 1989; Ualiev and Ualiev, 2006; Vulfson, 1990) are solved. For this purpose, the methods of the theory of elasticity are used, for example, they use the Equation 4 (Kondratenko *et al.*, 2018; Frolov, 1995; Yavorskij and Detlaf, 1974) the solution of which is sought in Equation 5.

Here v , E is the mass and elastic characteristics of the mechanical line; f_k is the section area; Q_H is the intensity of the external load; H_k , θ_k , p_k , a_k is constants determined from the initial conditions. In the case of using the Lagrange equation of the 2nd kind (Equation 6) considered are the kinetic (T) and potential energy (U) oscillations, taking into account Equation 7 where q is the generalized coordinate.

The parameters of the movement of the mechanism from this equation are determined after some transformations. The indicated equations that underlie many papers on the dynamics of machines, for example (Babakov, 1958; Kolovskij, 1989; Ualiev and Ualiev, 2006; Vulfson, 1990), allow, for given boundary conditions, to estimate the change in the displacements of the elementary volume of a bar, column, and other parameters in time and space.

However, such information, if it is necessary to take into account the interrelationship of a large number of factors, is, on the one hand, redundant, since it is often enough to know under what conditions self-oscillations occur of the rotation frequency of a technological object (executive body), i.e. stability is lost, and under what conditions the system is stable. On the other hand, without explicitly taking into account the second variable (stress), complete information about the dynamics of the stress state in the course of work cannot be received, which makes it difficult to estimate the likelihood of a part breaking or disrupting the mechanism. In the case of using systems with long force lines, there is also the problem of describing the behavior of the mechanism under standard disturbance (impacts), for example, step disturbance.

Studies show, for example (Postnikov, 1974; Reiner, 1958), that an intermediate variable (stress) is not always linearly related to displacement. However, the described methods do not take this feature into account. In addition, depending on the stresses, the position of the executive body may change, which will result in changes in indicators boring of aperture, drilling of wells for oil and gas. In this regard, there is a need to develop a method where the oscillations of both speeds and stresses, as well as the influence of various nonlinearities, are explicitly taken into account.

3. RESULTS AND DISCUSSION:

3.1. The Conclusion of the Original Equations

To solve this problem, when considering longitudinal oscillations in a straight solid rod in the absence of mass forces, the equation of momentum in differential form was used (Equation 8) (Sedov, 1970) and the equation of longitudinal vibrations (Equation 9) where Equation 10; u is moving along the x axis; σ is longitudinal (normal) voltage; ρ is the density of the material; E is the modulus of elasticity. From Equations 8, 9 the Equation 11 is obtained. Integrating Equation 3 over x , assuming that for $x = 0$, $\sigma = \text{const}$, and then differentiating over t the result obtained, the expression is arrived at Equation 12 (Kondratenko, 2005).

Denote the current value of the stress σ_x by σ . Given that the surface forces acting on each point of the cross section of an elementary volume are directed in the opposite direction from the direction of the speed of movement,

Equations 8, 12 are rewritten in the form Equations 13, 14. When considering torsional vibrations, the deformation of sections is assumed to be absent. Then the elastic oscillations are described by the Equation 15 (Kondratenko, 2005; Kondratenko *et al.*, 2017b) where φ , x is the angle of rotation of the section line and the coordinate with zero in the center of the driving link; G is material shear modulus. From the equation of angular momentum for a finite volume of a continuous medium in the absence of viscous friction in the material (Sedov, 1970), after the known transformations (Kondratenko, 2005; Kondratenko *et al.*, 2017b), it is possible to obtain for the elementary volume of the rod with an outer radius r with $\rho = \text{const}$ the equation of angular momentum in the differential form Equation 16 where Ω is rotation speed, Equation 17; τ is the maximum tangential stresses in torsion. From Equations 14, 16 the Equation 18 is arrived at. After integrating Equation 18 along the x coordinate and subsequent differentiation by t , the Equation 19 is gotten. The system of Equations 16, 19 makes it possible to describe the changes in the tangential stresses on the outer cylindrical surface in an elementary volume and the speed of movement of the elementary sections of the trunk.

Assuming that $G = \text{const}$, $\rho = \text{const}$ and performing the one-dimensional Laplace transform of these equations with zero initial conditions (Ivanov *et al.*, 1971), Equation 20 is obtained (Equation 21) where s is a complex variable, Equation 22, ω is the frequency of circular oscillation. Suppose that the process of transferring motion during a shift is similar to the process of transferring motion during longitudinal vibrations. In this case, following the phenomenological Zener model (Reiner, 1958) for the elementary volume, the equation of motion can be written in the form Equation 23 where θ , η is deformation and viscosity; G_1 , G_2 is isothermal and adiabatic shear modulus. If Equation 23 is rewritten in operator form and then using the Laplace transform with zero initial conditions, then Equation 24 can be obtained where a_e is the relaxation constant (Equations 25, 26). From here for the generalized modulus of elasticity can be written (Equation 27). Here G_u , G_v are, respectively, the ordinates of the complex function $G_\omega(j\omega)$ along the real and imaginary axes.

It is further assumed that the features of the interaction of transverse waves with the interface are similar to the interactions of longitudinal waves and are largely determined by

the wave resistance (Equations 28, 29) (Yavorskij and Detlaf, 1974) where a_2 is the speed of movement of the transverse waves. If to consider the harmonic oscillations propagating along the line, it can be introduced into consideration the wave resistance in operator form – impedance $Z_b(j\omega)$ (Equation 30) (Kondratenko and Mironova, 2018c; Kondratenko *et al.*, 2017a). Assuming the material density is constant with regard to Equations 27–29, the transformed Equation 30 is written in the form Equation 31. Here are the following ratios (Equation 32) where $\psi(\omega)$ is the generalized function of internal friction. Internal friction ψ in solids can play a significant role. It is known, for example, that magnesium alloys and a number of other materials have very good vibration-insulating properties, largely due to internal friction. At the same time, for steels, because of the smallness, this value is often neglected. Then Equation 33, $\psi(\omega) = 0$. Dynamic features of force line with parameters distributed along the length are characterized by the operator coefficient of wave propagation, which, based on Equation 20 in Laplace images with these assumptions, can be written in the form Equation 34.

Differentiating Equation 16 with respect to the x coordinate, then eliminating the derivative $d\Omega(s)/dx$ using Equation 19 and applying Equation 34, Equation 35 is obtained (Kondratenko, 2005; Kondratenko *et al.*, 2017b). This equation is a 2nd order differential equation with constant coefficients. Its solution after the introduction of hyperbolic functions (Kondratenko, 2005; Kondratenko *et al.*, 2017b) under boundary conditions for $x = 0$ (Equations 36, 37), has the form Equation 38. Having solved the system of Equations 16, 19 in the manner described above with respect to $\Omega(s, x)$, Equation 39 is obtained and for the boundary conditions for: $x = 0$ (Equations 40, 41) finally, there will be Equation 42. If in the process of oscillation, the output link does not miss the angular momentum that is supplied to this link, then disturbance waves are reflected from the end of line. The case of a matched load will be considered when there are no reflected waves in the system. In this case, the boundary conditions are the following relations (Equations 43–45) (Kondratenko, 2005; Kondratenko and Mironova, 2018c). Here, W_{p2} is the geometric polar moment of the resistance of the section of the line of the executive body with the moment of inertia J ; h_k , M_c is the coefficients of friction loss and moment of resistance acting on the executive body; l is the length of the line. The coefficient h_k takes into account only friction

losses proportional to the speed of the load.

Jointly solving Equations 38, 42 taking into account the specified boundary conditions, the equation of motion of the executive body of the system is obtained (Equations 46, 47). Having performed similar transformations as applied to longitudinal vibrations in solid lines, Equations 11, 12, the equation of motion is obtained (Kondratenko, 2005; Ivanov *et al.*, 1971), which has almost the same form as Equation 46.

If to carry out transformations similar to those described for a volumetric hydraulic actuator (Popov, 1977; Kondratenko, 2005), then, assuming that the leakage through the hydraulic motor is significantly higher than the leakage through the pump, the equation of motion can be obtained (Equation 48) (Kondratenko *et al.*, 2017a; Rabinskiy and Tushavina, 2019a; Rabinskiy and Tushavina, 2019b; Kamke, 1959; Nikitin *et al.*, 2019). Here are the following ratios (Equations 49, 50) where τ is tightness criterion of volumetric hydraulic actuator; g_{0s} is the coefficient of elasticity; $E_g(s)$, $\Psi(s)$ are polynomials obtained during transformations; w is the volumetric constant of the hydraulic motor; Q is pump flow rate; $\theta_{01}(s)$ is the operator coefficient of wave propagation in the pressure line; J is the moment of inertia of rotating parts; c is the coefficient of friction loss proportional to the pressure differential; h is the coefficient of friction loss proportional to the speed of movement; ρ_0 , κ is the density and bulk modulus of elasticity of the liquid; Ω is the frequency of rotation of the output shaft; M_r - the moment of resistance; L is the length of the hydraulic line; K_u is coefficient leakages in the volumetric engine. In this case, the boundary conditions were determined by the balance of the flows rate fluid entering the pipelines and resulting from them.

3.2. Solving Systems of Equations

Since the mechanisms usually have nonlinearities, including substantial ones, the solution of such problems is possible only by a numerical method. Considering the similarity of equations describing the dynamics of mechanical and hydraulic systems (Kondratenko, 2005), as well as a simpler fixation of oscillations in the hydraulic drive, the development of a method for numerical simulation of dynamic processes in mechanical systems was based on comparing simulation results with field tests of hydraulic drive. To solve this problem, the Equation 48 has to be algebraically decomposed into Equations

51, 52. Here p is the differential pressure on the hydraulic motor. The correctness of the decomposition is checked by the reverse decision. From the Equation 31 the transfer function of the influence of oscillations of the pump flow rate is obtained on the pressure drop at $M_r = 0$ (Equation 53). The complex functions $\Psi(s)$ and $\mathfrak{G}(s)$ in Equation 32 will be replaced by their real parts $\Psi(\omega)$ and $\mathfrak{G}(\alpha)$ (Equation 54).

Figure 1 shows the calculated ratios of the modules of real and imaginary parts of the functions $\mathfrak{G}(i\omega)$, $\Psi(i\omega)$. The calculations were carried out for hydraulic IID. It can be seen from the graphs that the real parts of the functions significantly exceed their imaginary parts in modulus in almost the entire frequency range. Turning to the originals of Equations 51, 52, Equations 55, 56 are obtained. Simulation was carried out for electrohydraulic drive with parameters: $J = 34 \text{ N} \cdot \text{cm} \cdot \text{s}^2$; $w = 11.3 \text{ cm}^3 / \text{rad}$; $h = 17 \text{ N} \cdot \text{cm} \cdot \text{s}$; $\alpha = 1.4 \cdot 10^3 \text{ MPa}$; $\rho_0 = 870 \text{ kg} / \text{m}^3$; $f_1 = f_2 = 3.13 \text{ cm}^2$; $\tau = 0.05 (\text{N} \cdot \text{cm} \cdot \text{s})^{-1}$; $a_{1m} = 5.36 \cdot 10^{-2} \text{ s}$; $a_{2m} = 5.76 \cdot 10^{-4} \text{ c}^2$; $K_{mu} = 0.02 \text{ l} / (\text{B})$. Safety valves in both lines are set to a pressure of $p = 15 \text{ MPa}$. The length of each of the two hydraulic lines was $l = 22 \text{ m}$. For example, taking into account Equations 55, 56 for the hydraulic drive, a system of Equations 57-60.

Here $U(t)$, a_{1m} , a_{2m} is respectively, the control voltage, the coefficients of the control mechanism; γ is angle of rotation of the control mechanism roller; p_1 , p_2 is respectively, the pressure in the pressure and make-up lines; Δp is pressure differential on the hydraulic motor, Equation 61; Q_T is theoretical maximum flow rate of the pump; Q_{k1} , Q_{k2} is leakage through valves; V_1 is volume of fluid in the line. The Equation 35 was solved using the 4th order Runge-Kutta method (Kondratenko *et al.*, 2018; Kamke, 1959).

3.3. Simulation Results

Modeling was carried out using a PC with counting steps $H = 10^{-4}$; $2 \cdot 10^{-4} \text{ sec}$. The error did not exceed 10%. The control signal was changed according to the law (Equation 62). For each settable frequency ω with $U_0 = 10 \text{ V}$, $U_a = 5 \text{ V}$ for 10 sec. the counts calculated the values of the input coordinate U , as well as the liquid flow Q supplied to the system and the values of the other variables. After analyzing the obtained numerical values, the amplitude of the oscillations of the pressure differential A_p and the phase shift φ of forced oscillations the differential pressure relative to the oscillations of the flow were determined. Then calculated the value of

Equation 63. In addition, the frequency response was calculated from the transfer function (Equation 53) using complex functions. Also for the marked hydraulic drive, experimental frequency characteristics were obtained. The results of all calculations and natured experiments are shown in Figure 2. It can be seen from them that the numerical simulation performed by the method described above very accurately reflects the processes in the system with distributed parameters of elastic lines (Kondratenko and Mironova, 2018c).

4. CONCLUSIONS:

The proposed method for solving systems of nonlinear differential equations that take into account the distributed parameters of the power lines of the mechanisms allows us to conduct various numerical experiments to study the parameters of motion and the stressed state in various mechanical systems and obtain adequate results. Here, with a constant counting step each of the four stages of the calculation cycle, nonlinear factors were taken into account (friction, unidirectional rotation, pressure limitation). For each input frequency of forced oscillations ω was recalculated the values of α and the functions $\Psi(\omega)$, $\Theta_s(\alpha)$. If to consider that during the operation of virtually any mechanism oscillations are generated, the frequency of which depends on the speed of movement of the executive body, then the proposed method can be used to obtain not only frequency characteristics but also oscillograms of transients in the considered systems.

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$$\frac{\partial \Omega}{\partial t} = \sum_{i=1}^n \frac{\partial \Omega}{\partial \zeta_i} f_i \quad (\text{Eq. 1})$$

$$f_i = \frac{d\zeta_i}{dt} \quad (\text{Eq. 2})$$

$$\frac{d\zeta_i}{dt} = f_i(t, \zeta_1, \zeta_2, \dots, \zeta_n) \quad (\text{Eq. 3})$$

$$v \frac{\partial^2 u}{\partial t^2} - \frac{\partial [Ef_k(\partial x / \partial t)]}{\partial x} = Q_n(x, t) \quad (\text{Eq. 4})$$

$$u(x, t) = \sum_{k=1}^{\infty} H_k \theta_k(x) \sin(p_k t + \alpha_k) \quad (\text{Eq. 5})$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} = Q \quad (\text{Eq. 6})$$

$$Q = - \frac{dU}{dq} \quad (\text{Eq. 7})$$

$$\rho \frac{\partial v}{\partial t} = \frac{\partial \sigma}{\partial x} \quad (\text{Eq. 8})$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 u}{\partial x^2} \quad (\text{Eq. 9})$$

$$v = \frac{\partial u}{\partial t} \quad (\text{Eq. 10})$$

$$\frac{1}{\rho} \frac{\partial \sigma}{\partial x} = \frac{E}{\rho} \frac{\partial^2 u}{\partial x^2} \quad (\text{Eq. 11})$$

$$\frac{1}{E} \frac{\partial \sigma_x}{\partial t} = \frac{\partial v}{\partial x} \quad (\text{Eq. 12})$$

$$\rho \frac{\partial v}{\partial t} = - \frac{\partial \sigma}{\partial x} \quad (\text{Eq. 13})$$

$$\frac{1}{E} \frac{\partial \sigma}{\partial t} = - \frac{\partial v}{\partial x} \quad (\text{Eq. 14})$$

$$\frac{\partial^2 \varphi}{\partial t^2} = \frac{G}{\rho} \frac{\partial^2 \varphi}{\partial x^2} \quad (\text{Eq. 15})$$

$$\rho r \frac{\partial \Omega}{\partial t} = - \frac{\partial \tau}{\partial x} \quad (\text{Eq. 16})$$

$$\Omega = \frac{\partial \varphi}{\partial t} \mathbf{d} \quad (\text{Eq. 17})$$

$$G \frac{\partial^2 \varphi}{\partial x^2} = - \frac{1}{r} \frac{\partial \tau}{\partial x} \quad (\text{Eq. 18})$$

$$rG \frac{\partial \Omega}{\partial x} = - \frac{\partial \tau}{\partial t} \quad (\text{Eq. 19})$$

$$\rho r s \Omega(s) = - \frac{d\tau(s)}{dx} \quad (\text{Eq. 20})$$

$$rG \frac{d\Omega(s)}{dx} = -s\tau(s) \quad (\text{Eq. 21})$$

$$s = j\omega \mathbf{g} \quad (\text{Eq. 22})$$

$$\tau + \frac{\eta}{G_2} \frac{d\tau}{dt} = G_1 \theta + \eta \frac{d\theta}{dt} \quad (\text{Eq. 23})$$

$$\tau(s) \left(s + \frac{1}{a_\varepsilon} \right) = \theta(s) G_2 \left(s + \frac{1}{k_b \alpha_\varepsilon} \right) \quad (\text{Eq. 24})$$

$$a_\varepsilon = \frac{\eta}{G_2} \quad (\text{Eq. 25})$$

$$k_b = \frac{G_2}{G_1} \quad (\text{Eq. 26})$$

$$G_{\omega}(s) = \frac{\tau(s)}{\theta(s)} = G_u(\omega) + jG_v(\omega) \quad (\text{Eq. 27})$$

$$-\frac{\tau}{\Omega} = \rho a_2 \quad (\text{Eq. 28})$$

$$a_2 = \sqrt{\frac{G}{\rho}} \quad (\text{Eq. 29})$$

$$Z_b(j\omega) = \frac{\tau(j\omega)}{\Omega(j\omega)} \quad (\text{Eq. 30})$$

$$Z_b(j\omega) = \frac{j\omega G_u(\omega) \sqrt{\rho / G_u(\omega)} \sqrt{1 + j \frac{G_v(\omega)}{G_u(\omega)}}}{j\omega} = \frac{G_u(\omega) \theta(j\omega)}{j\omega} j \quad (\text{Eq. 31})$$

$$\theta(j\omega) = \pm \sqrt{\frac{j\omega}{G_u(\omega)} [\rho j\omega + \psi(\omega)]} \quad (\text{Eq. 32})$$

$$G_u(\omega) = G = \text{const} \quad (\text{Eq. 33})$$

$$\theta(s) = \pm s \sqrt{\frac{\rho}{G}} \quad (\text{Eq. 34})$$

$$\frac{\partial^2 \tau(s)}{\partial x^2} - \theta^2(s) \tau(s) = 0 \quad (\text{Eq. 35})$$

$$\tau(s, x) = \tau_1(s, 0) \quad (\text{Eq. 36})$$

$$\frac{\partial \tau(s, x)}{\partial x} = -\frac{rG}{s} \theta^2(s) \Omega_1(s, 0) \quad (\text{Eq. 37})$$

$$\tau(s, x) = \tau_1(s, 0) ch \left[\theta(s) x \right] - \frac{rG}{s} \theta(s) \Omega_1(s, 0) sh \left[\theta(s) x \right] \quad (\text{Eq. 38})$$

$$\frac{\partial^2 \Omega(s)}{\partial x^2} - \theta^2(s) \Omega(s) = 0 \quad (\text{Eq. 39})$$

$$\Omega(s, x) = \Omega_1(s, 0) \quad (\text{Eq. 40})$$

$$\frac{\partial \Omega(s, x)}{\partial x} = -\frac{s \tau_1(s, 0)}{rG} \quad (\text{Eq. 41})$$

$$\Omega(s, x) = \Omega_1(s, 0)ch[\theta(s), x] - \frac{s\tau_1(s, 0)sh[\theta(s)x]}{\theta(s)rG} \quad (\text{Eq. 42})$$

$$\Omega(s, 0) = \Omega_1(s); \Omega(s, l) = \Omega_2(s) \quad (\text{Eq. 43})$$

$$\tau(s, 0) = \tau_1(s); \tau(s, l) = \tau_2(s) \quad (\text{Eq. 44})$$

$$\tau_2(s) = \frac{M_c(s) + h_k\Omega_2(s) + Js\Omega_2(s)}{W_{p2}} \quad (\text{Eq. 45})$$

$$\Omega_2(s)[1 + h_k\vartheta_k(s)s + J\vartheta_k(s)s^2] = \frac{\Omega_1(s)}{chA} - M_c(s)\vartheta_k(s)s \quad (\text{Eq. 46})$$

$$\vartheta_k(s) = \vartheta_{k0}Z_n(s); \vartheta_{k0} = \frac{l}{GrW_{p2}}; Z_k(s) = \frac{thA}{A}; A = \theta(s)l \quad (\text{Eq. 47})$$

$$\begin{aligned} \Omega(s)\{s^2J\vartheta_s(s) + s[J\tau_k + \vartheta_s(s)h] + 1 - c + \tau_k h\} = \\ = Q(s)(1 - c)\Psi(s) - M_r(s)[\vartheta_s(s)s + \tau_k] \end{aligned} \quad (\text{Eq. 48})$$

$$\vartheta_s(s) = \frac{1}{E_g(s)}\vartheta_{0s}(s)Z(s); \tau_k = \frac{K_u}{w^2}; \vartheta_{0s} = \frac{Lf_1}{\kappa w^2} \quad (\text{Eq. 49})$$

$$Z(s) = \frac{thA_1}{A_1}; \vartheta_{01}(s) = \sqrt{\frac{\rho_0}{\kappa}}; A_1 = \vartheta_{01}(s)L \quad (\text{Eq. 50})$$

$$Q(s)\Psi(s) = w\Omega(s) + \tau_k w^2 p(s) + \vartheta_s(s)w^2 sp(s) \quad (\text{Eq. 51})$$

$$p(s)w(1 - c) = M_r(s) + h\Omega(s) + Js\Omega(s) \quad (\text{Eq. 52})$$

$$W_{pq}(s) = \frac{P(s)}{Q(s)} = \frac{(Js + h)\Psi(s)}{s^2J\vartheta_s(s) + s[J\tau_k + \vartheta_s(s)h] + 1 - c + \tau_k h} \quad (\text{Eq. 53})$$

$$\alpha = l\omega\sqrt{\frac{\rho}{\kappa}} \quad (\text{Eq. 54})$$

$$Q(t)\Psi(\omega) = w\Omega(t) + \tau w^2 p(t) + \vartheta(\alpha)w^2 \frac{dp}{dt} \quad (\text{Eq. 55})$$

$$p(t)w(1 - c) = M_c(t) + h\Omega(t) + J\frac{d\Omega}{dt} \quad (\text{Eq. 56})$$

$$\frac{V_1}{\kappa}Z_1(\omega)\frac{dp_1}{dt} = \gamma(t)Q_r\Psi(\omega) - w\Omega(t) + Q_{k2} - Q_{k1} - \tau_k w^2 \Delta p(t) - 0,1\tau_k w^2 p_2(t) \quad (\text{Eq. 57})$$

$$\frac{V_1}{\kappa} Z_1(\omega) \frac{dp_1}{dt} = -\gamma(t) Q_T \Psi(\omega) + w \Omega(t) - Q_{k2} + Q_{k1} - \tau_k w^2 p_1(t) \quad (\text{Eq. 58})$$

$$\Delta p w = M_c(t) + h \Omega(t) + J \frac{d\Omega}{dt} \quad (\text{Eq. 59})$$

$$K_u U(t) = \gamma(t) + a_{1m} \frac{d\gamma}{dt} + a_{2m} \frac{d^2\gamma}{dt^2} \quad (\text{Eq. 60})$$

$$\Delta p = p_1 - p_2 f \quad (\text{Eq. 61})$$

$$U = U_0 + U_a \sin(\omega t) f \quad (\text{Eq. 62})$$

$$L = 20 \lg \left(\frac{A_p}{Q_a} \right) f \quad (\text{Eq. 63})$$

Figure 1. The ratio of the modules of the real (1) and imaginary (2) parts of the functions $\Psi(i\omega)$, $\Psi(i\omega)$ for the IID hydraulic drive

Figure 2. Logarithmic amplitude (L) and phase (ϕ) frequency characteristics of the hydraulic drive: curves 1, 2 is calculated curves of the simulation model; curves 3, 4 is curves obtained by experiment