ESTUDO DA INFLUÊNCIA DE EFEITOS TÉRMICOS E CLIMÁTICOS NO DESEMPENHO DA PROTEÇÃO TÉRMICA DE TELHAS DE VEÍCULOS ESPACIAIS**INVESTIGATION OF THE INFLUENCE OF THERMAL AND CLIMATE EFFECTS ON THE PERFORMANCE OF TILED THERMAL PROTECTION OF SPACECRAFT****ИССЛЕДОВАНИЕ ВЛИЯНИЯ ТЕПЛОВЫХ И КЛИМАТИЧЕСКИХ ВОЗДЕЙСТВИЙ НА РАБОТОСПОСОБНОСТЬ ПЛИТОЧНОЙ ТЕПЛОВОЙ ЗАЩИТЫ КОСМИЧЕСКИХ ЛЕТАТЕЛЬНЫХ АППАРАТОВ**RABINSKIY, Lev N.^{1*}; TUSHAVINA, Olga V.²;¹ Moscow Aviation Institute (National Research University), Institute of General Engineering Training, Moscow – Russian Federation² Moscow Aviation Institute (National Research University), Institute of Aerospace, Moscow – Russian Federation** Correspondence author
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RESUMO

O artigo discute as questões de proteção térmica de veículos espaciais sob vários tipos de efeitos térmicos e climáticos. Durante a preparação para o lançamento, vários tipos de formação de gelo são possíveis na superfície da estrutura telhada, consistindo em telhas de quartzo com proteção contra o calor. É muito importante durante a operação determinar as tensões normais e tangenciais durante a formação de gelo e o efeito do gelo formado nas lacunas entre as telhas. É necessário calcular a durabilidade e os indicadores da proteção térmica de telhas nos estágios ativos do voo da nave espacial. Foi criado um modelo matemático que permite, com um grau de precisão suficiente, estudar a estrutura telhada quanto à resistência, cisalhamento e rasgo das telhas quando exposto ao gelo. A resistência, cisalhamento e rasgo das telhas sob a influência do gelo foram calculados. Estudos teóricos e experimentais foram comparados. Os cálculos realizados permitiram mostrar que as telhas não entrariam em colapso sob a carga considerada. A resistência do substrato suporta as forças de cisalhamento emergentes. Foram obtidos os valores da resistência de aderência do gelo que aparece na superfície do revestimento protetor térmico de fibra de quartzo. Foi demonstrado que a proporção de tangentes no estado estressado das telhas não excede 10%. O modelo matemático proposto para o estudo das características da proteção térmica de telhas de uma veículo espacial descreve adequadamente os processos em estudo. Estudos experimentais mostraram concordância satisfatória com os resultados teóricos. Os cálculos realizados para determinar as tensões de cisalhamento finais no substrato mostraram que a resistência do substrato suporta as forças de cisalhamento emergentes. A consideração das condições climáticas pode reduzir significativamente os riscos no projeto de proteção térmica de aeronaves modernas. Os resultados deste artigo podem ser usados em pesquisas adicionais e também podem ser levados em consideração ao projetar aeronaves.

Palavras-chave: *veículo espacial, processo de formação de gelo, telhas de quartzo com proteção térmica, período de preparação antes do lançamento, lacunas entre as telhas.*

ABSTRACT

The article discusses the thermal protection of spacecraft under various types of thermal and climatic influences. During the pre-launch preparation, several types of icing are possible on the surface of the tiled structure, consisting of quartz heat-shielding tiles. It is very important in the process of operation, to determine normal and shear stresses during icing and the impact of ice formed in the tile gaps. It is necessary to calculate the strength and performance of the tiled thermal protection in active stages of flight of spacecraft. A mathematical model was constructed that allows, with a sufficient degree of accuracy, to research the tiled structure for strength, shear and tearing of the tile when exposed to ice. Calculations for strength, shear, and tearing of tiles under the influence of ice were performed. Theoretical and experimental studies were compared. The performed calculations made it possible to show that the tile will not collapse under the considered load.

The strength of the substrate withstands the emerging shear forces. The values of the adhesion strength of ice appearing on the surface of quartz fiber heat-protective coating were obtained. It was demonstrated that the proportion of tangents in the stressed state of the tile does not exceed 10%. The proposed mathematical model for studying the characteristics of the tiled thermal protection of a spacecraft adequately describes the processes under study. Experimental studies have shown satisfactory agreement with theoretical results. The calculations performed to determine the ultimate shear stresses in the substrate showed that the strength of the substrate withstands the emerging shear forces. Consideration of climatic conditions can significantly reduce risks in the design of thermal protection of modern aircraft. The results of this study can be used in further studies, and they can also be taken into account in the construction of aircraft.

Keywords: *spacecraft, icing process, quartz heat-shielding tiles, prelaunch period, tile gaps.*

АННОТАЦИЯ

В статье рассматриваются вопросы теплозащиты космических аппаратов при различных видах тепловых и климатических воздействий. Во время подготовки к запуску возможны несколько видов обледенения на поверхности черепичной конструкции, состоящей из кварцевых теплозащитных плиток. Очень важно в процессе эксплуатации определять нормальные и касательные напряжения при обледенении и воздействие льда, образующегося в зазорах плитки. Необходимо рассчитать прочность и показатели плиточной теплозащиты на активных этапах полета космического корабля. Была построена математическая модель, которая позволяет с достаточной степенью точности исследовать плиточную структуру на прочность, сдвиг и разрыв плитки при воздействии льда. Были выполнены расчеты на прочность, сдвиг и разрыв плитки под воздействием льда. Теоретические и экспериментальные исследования были сопоставлены. Выполненные расчеты позволили показать, что плитка не будет разрушаться при рассматриваемой нагрузке. Прочность подложки выдерживает возникающие сдвиговые усилия. Получены значения адгезионной прочности льда, появляющегося на поверхности термозащитного покрытия из кварцевого волокна. Было продемонстрировано, что доля касательных в напряженном состоянии плитки не превышает 10%. Предложенная математическая модель для изучения характеристик плиточной тепловой защиты космического аппарата адекватно описывает изучаемые процессы. Экспериментальные исследования показали удовлетворительное согласие с теоретическими результатами. Расчеты, выполненные для определения предельных напряжений сдвига в подложке, показали, что прочность подложки выдерживает возникающие сдвиговые силы. Учет климатических условий позволяет значительно снизить риски при проектировании тепловой защиты современных самолетов. Результаты этой статьи могут быть использованы в дальнейших исследованиях, а также могут быть приняты во внимание при конструировании самолетов.

Ключевые слова: *космический летательный аппарат, процесс обледенения, кварцевые теплозащитные плитки, период предстартовой подготовки, межплиточные зазоры.*

1. INTRODUCTION

In the process of operation, the surface of the spacecraft is exposed to various physical and mechanical fields that adversely affect flight performance. This increases the risk of deterioration of aerodynamic characteristics, increase in passive mass, likelihood of individual tiles failing or tearing off the structure under the action of forces when ice is formed in the tile gaps of systems and aggregates, as well as the possibility of destruction of the thermal barrier coating. The causes of ground icing of bodies and structures in natural climatic conditions are fairly well understood (Bogorodsky and Gavrilov, 1980; Trude, 1983; Trunov, 1983; Afanasyev and Tushavina, 2016), and the main mechanisms of ice formation during prelaunch preparation are identified (Sandu *et al.*, 2018).

The study of the influence of climatic

effects such as radiation, icing, rain, fog during the operation of spacecraft on performance of structural elements with porous heat-shielding material is of paramount importance (Orlov *et al.*, 2003; Shejko *et al.*, 2016, Daus and Kharchenko, 2018). Taking into account the above-mentioned negative features in operation, the adhesive strength and stiffness of casing of modern spacecraft were studied.

To solve this problem, a mathematical model has been developed that correctly describes the stress-strain state, which allows us to examine tiled structure for strength, shear, and tear. The development of mathematical model that takes into account unsteady thermal processes under the influence of physical and mechanical fields of various physical nature was considered in works (Kolesnik *et al.*, 2015; Kuznetsova *et al.*, 2015; Lurie *et al.*, 2015; Afanasyev and Tushavina, 2016; Danilin *et al.*,

2016; Prokofiev *et al.*, 2016; Babaytsev *et al.*, 2017; Lomakin *et al.*, 2017; Kakhramanov *et al.*, 2017; Formalev and Kolesnik, 2017a; Formalev and Kolesnik, 2017b; Lurie *et al.*, 2017a; Lurie *et al.*, 2017b; Bulychev *et al.*, 2018; Formalev *et al.*, 2018a; Formalev *et al.*, 2018b; Kuznetsova *et al.*, 2018; Lomakin *et al.*, 2018; Rabinskiy and Tushavina, 2019; Rabinskiy *et al.*, 2019).

2. MATERIALS AND METHODS

The installation diagram of heat-shielding tiles on the glider surface is shown in Figure 1 which schematically displays the method of tiled thermal protection of spacecraft from various types of thermal fields of various physical nature. In this case, the thermal protection of the airframe consists of quartz tile 1 with coating 2 lying on damping substrate 3. Thermal protection insulation tile consists of quartz ceramics with significant porosity of up to 90%. There is a gap 4 between the tiles, which can be filled with liquid that turns into ice. Between the quartz tile 1 and substrate 3 is an adhesive layer 5. The entire heat-protection structure rests on the casing 6.

The need to study the stress-strain state and strength of heat-protection tiles during icing is one of the most important during storage and transportation of the product in open atmospheric conditions, since the moisture contained in the pores of thermal insulation during freezing forms significant forces that can not only cause damage but also destroy the thermal protection system, then, during aerodynamic heating in flight, such damage can cause an emergency or complete destruction of the spacecraft.

Thus, the need to study the stress-strain state and strength of heat-protective tiles during icing is one of the most important issues during the storage and transportation of the product in open atmospheric conditions (Trude, 1983). The finding of the stress-strain state and strength of heat-shielding tiles is divided into two stages — the determination of spacer stresses in a heat-protection coating and also normal, and shear stresses in the substrate-tile system. Installation of heat-shielding tiles on the glider casing using Figure 1. presented in the form (Figure 2).

In accordance with the design scheme, a tile with a layer of ice in load conditions can be represented in the form of a beam, shown in Figure 3. Then, due to the freezing of the gap, the beam will be extended by an amount $\varepsilon_0 \delta$ and total length $l_{pl} + \delta(1 + \varepsilon_0)$ will be decreased by this

amount due to compression forces N_s (Equation 1). In the equation E_{pl}, E_l, F_{pl}, F_l are modules of elasticity and cross-sectional area of tiles and ice, respectively, ε_0 — linear relative expansion of water during freezing. Let's transform Equation 1 to the form of Equation 2. Using well-known inequality in Equation 2 $\frac{E_l F_l l_{pl}}{E_{pl} F_{pl} \delta} \leq 1 + \varepsilon_0$ we'll get

Equation 3. Then the expression for spacer stresses will be expressed like Equation 4.

3. RESULTS AND DISCUSSION:

3.1. Determination of normal and tangential stresses in the substrate-tile system

A scheme of the shear stresses arising in the damping substrate made of felt material and located between the heat-shielding tiles and the structure of the spacecraft is shown in Figure 4. To determine the stress-strain state in the substrate-tile system, a beam element has to be cut at a distance x from the axis of symmetry y . Equilibrium Equation 5 for a tile with respect to the longitudinal axis x will have the look of Equations 5, 6. In these expressions $\sigma(x), \tau(x)$ are normal and tangential stresses in section, σ_0 is stress at the origin. Then the longitudinal movement in the tile will have a look of Equation 7 and the angular displacement will be Equation 8. As a result, the tangential stress in felt substrate will take the form of Equation 9.

Equation 9 will be differentiated twice with respect to the variable x , the differential equation is obtained for distribution of tangential stresses along the substrate Equations 10, 11. G_f, ν are the shear modulus of the felt and Poisson's ratio respectively. The solution of the Equation 10 will have the form of Equation 12. Constants of integration C_1, C_2 and unknown voltage σ_0 at the origin are taken from boundary conditions (Equations 13-15).

By substituting Equations 6-9 into Equations 13-15, taking into account Equation 12, a system of equations is gotten for determining the constants C_1, C_2 and σ_0 (Equations 16-18). Using the solution of Equations 16-18, the expressions for the constants are gotten (Equations 19-21). Then, using Equations 19-21, the analytical expressions are determined for normal and tangential stresses for quartz tile and damping substrate

(Equations 22, 23). Using Equation 11 finally Equations 24, 25 are gotten. The fraction of shear stresses in the stress state of the tile can be written in the form of relations (Equation 26). Longitudinal displacements $u(x)$ of heat-protective tiles will be determined via known relations (Equation 27). Figures 6-7 show the relationship between the ratio of shear stresses to normal and the distribution of displacements depending on the length of the heat-shielding tile. The following values of the parameters included in Equations 26-27 are accepted.

3.2. Experimental research of tiled thermal protection of spacecraft

The schematic diagram of the installation for studying the adhesion strength of ice is presented in Figure 8. Cooling and temperature control of model 7 was done in a climatic chamber 6 by nitrogen vapor at atmospheric pressure. The dimensions of the climate chamber were $560 \times 240 \times 230$ mm. Nitrogen was directed to the thermostat from Dewar 1 vessel. The temperature at thermostat was controlled using the thermostat 5 before they entered climate chamber 6. Sample 7 was installed in grips 8, mounted on power frame. Loading of the sample is performed using the movement of beam 10 moving along the power columns 9. Dimensions of test samples are $195 \times 10 \times 3$ mm.

During the experiment, the gas temperature in the thermostat was measured with a thermometer with measuring scale from -50°C to 0°C and a division value of 0.1 degrees. The temperature in chamber was controlled by thermostat 5. The sample was cooled by the gas temperature in the thermostat of 279 K. Thermostating was done for at least 30 min (Figures 9-11).

4. CONCLUSIONS:

Hereby:

- The performed calculations on the strength of tile under the conditions of ice formation in inter-tile gaps made it possible to show that the tile will not collapse under the considered load.
- The calculations performed to establish the ultimate shear stresses in the substrate showed that the strength of the substrate withstands the emerging shear forces.
- The calculations of stresses arising due to freezing of moisture in the heat-insulating layer of damping substrate made it possible to confirm

the tensile strength of the substrate.

- Based on the results of experimental research on a test bench, the values of the adhesion strength of ice appearing on the surface of quartz fiber heat-protective coating were obtained.
- It was demonstrated that the proportion of tangents in the stressed state of the tile does not exceed 10%.
- The suggested mathematical model for the study of performance of tiled thermal protection of spacecraft adequately describes the processes under study.
- The performed calculations have demonstrated the ability to predict the performance of promising aerospace products.
- The experimental studies have demonstrated a satisfactory agreement with theoretical results.
- The calculations performed to determine the ultimate shear stresses in the substrate showed that the strength of substrate withstands the arising shear forces
- Consideration of climatic conditions allows to significantly reduce risks in the design of thermal protection of modern aircraft

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$$\frac{N_s l_{pl}}{E_{pl} F_{pl}} + \frac{N_s (1 + e_0) d}{E_l F_l} = e_0 d \quad (\text{Eq. 1})$$

$$N_s = \frac{\varepsilon_0 E_l F_l}{1 + \varepsilon_0 + \frac{E_l F_l l_{pl}}{E_{pl} F_{pl} \delta}} \approx \varepsilon_0 E_{pl} F_{pl} \frac{\delta}{l_{pl}} \quad (\text{Eq. 2})$$

$$N_s \approx \varepsilon_0 E_{pl} F_{pl} \frac{\delta}{l_{pl}} \quad (\text{Eq. 3})$$

$$\sigma_R = \frac{N_s}{F_{pl}} = \varepsilon_0 \frac{N_s}{F_{pl}} = \varepsilon_0 E_{pl} \frac{\delta}{l_{pl}} \quad (\text{Eq. 4})$$

$$\Sigma x = 0 \quad \sigma(x) H_{pl} b - \int_0^x \tau(x) b dx - \sigma_0 H_{pl} b = 0 \quad (\text{Eq. 5})$$

$$\sigma(x) = \frac{1}{H_{pl}} \int_0^x \tau(x) dx + \sigma_0 \quad (\text{Eq. 6})$$

$$\delta(x) = \int_0^x \frac{\sigma(x)}{E_{pl}} dx = \frac{1}{E_{pl}} \int_0^x \left[\frac{1}{H_{pl}} \int_0^x \tau(x) b dx + \sigma_0 \right] dx \quad (\text{Eq. 7})$$

$$\gamma(x) = \frac{\delta(x)}{H_f} \quad (\text{Eq. 8})$$

$$\tau(x) = G_f \gamma(x) = \frac{G_f}{E_{pl} H_{pl} H_f} \int_0^x \left[\int_0^x \tau(x) dx + \sigma_0 H_n \right] dx \quad (\text{Eq. 9})$$

$$\frac{d^2 t(x)}{dx^2} = k^2 t(x) \quad (\text{Eq. 10})$$

$$k^2 = \frac{G_f}{E_{pl} H_{pl} H_f} \quad (\text{Eq. 11})$$

$$t(x) = C_1 e^{kx} + C_2 e^{-kx} \quad (\text{Eq. 12})$$

$$\tau(x) \Big|_{x=0} = 0 \quad (\text{Eq. 13})$$

$$\sigma(x) \Big|_{x=\frac{l_{pl}}{2}} = \sigma_R \quad (\text{Eq. 14})$$

$$\delta(x) \Big|_{x=\frac{l_{pl}}{2}} = H_f \gamma(x) \Big|_{x=-\frac{l_{pl}}{2}} \quad (\text{Eq. 15})$$

$$C_1 + C_2 = 0 \quad (\text{Eq. 16})$$

$$\frac{1}{H_n k} \left(C_1 e^{k \frac{l_{pl}}{2}} - C_2 e^{-k \frac{l_{pl}}{2}} - C_1 + C_2 \right) + \sigma_0 = \sigma_R \quad (\text{Eq. 17})$$

$$\frac{1}{E} \int_0^x \left[\frac{1}{H_{pl}} \int_0^x \tau(x) b dx + \sigma_0 \right] dx \Big|_{x=\frac{l_{pl}}{2}} = \frac{C_1 e^{kx} + C_2 e^{-kx}}{G} \Big|_{x=\frac{l_{pl}}{2}} \quad (\text{Eq. 18})$$

$$C_1 = \sigma_R \frac{l_{pl} H_{pl} k^2 G_f}{2(H_{pl} H_f k^2 E_{pl} - G_f) \left(e^{k \frac{l_{pl}}{2}} - e^{-k \frac{l_{pl}}{2}} \right) + k l_{pl} G_f \left(e^{k \frac{l_{pl}}{2}} - e^{-k \frac{l_{pl}}{2}} \right)} \quad (\text{Eq. 19})$$

$$C_2 = -\sigma_R \frac{l_{pl} H_{pl} k^2 G_f}{2(H_{pl} H_f k^2 E_{pl} - G_f) \left(e^{k \frac{l_{pl}}{2}} - e^{-k \frac{l_{pl}}{2}} \right) + k l_{pl} G_f \left(e^{k \frac{l_{pl}}{2}} - e^{-k \frac{l_{pl}}{2}} \right)} \quad (\text{Eq. 20})$$

$$\sigma_0 = 2\sigma_R \frac{\left(e^{kl_{pl}} - e^{-kl_{pl}} \right) (H_f H_n k^2 E_{pl} - G_f) + k l_{pl} G_f}{\left(e^{kl_{pl}} - e^{-kl_{pl}} \right) (2H_f H_n k^2 E_{pl} - 2G_f) + k l_{pl} G_f (e^{kl_{pl}} + e^{-kl_{pl}})} \quad (\text{Eq. 21})$$

$$\tau(x) = \sigma_R \frac{l_{pl} H_n k^2 \left[e^{k \left(\frac{l_{pl}}{2} + x \right)} - e^{k \left(\frac{l_{pl}}{2} - x \right)} \right]}{2 \left(e^{kl_{pl}-1} \left(H_f H_n k^2 \frac{E_{pl}}{G_f} - 1 \right) + k l_{pl} (e^{kl_{pl}} + 1) \right)} \quad (\text{Eq. 22})$$

$$\sigma(x) = \sigma_R \frac{2 \left(e^{kl_{pl}} - 1 \right) \left(H_f H_n k^2 \frac{E_{pl}}{G_f} - 1 \right) + k l_{pl} \left[e^{k \left(\frac{l_{pl}}{2} + x \right)} + e^{k \left(\frac{l_{pl}}{2} - x \right)} \right]}{2 \left(e^{kl_{pl}} - 1 \right) \left(H_f H_n k^2 \frac{E_{pl}}{G_f} - 1 \right) + k l_{pl} (e^{kl_{pl}} + 1)} \quad (\text{Eq. 23})$$

$$\tau(x) = \sigma_R \frac{H_{pl} k \left[e^{k \left(\frac{l_{pl}}{2} + x \right)} - e^{k \left(\frac{l_{pl}}{2} - x \right)} \right]}{e^{kl_{pl}} + 1} \quad (\text{Eq. 24})$$

$$\sigma(x) = \sigma_R \frac{e^{k \left(\frac{l_{pl}}{2} + x \right)} + e^{k \left(\frac{l_{pl}}{2} - x \right)}}{e^{kl_{pl}} + 1} \quad (\text{Eq. 25})$$

$$\frac{\tau(x)}{\sigma(x)} = H_{pl} k \frac{1 - e^{-2kx}}{1 + e^{-2kx}} \quad (\text{Eq. 26})$$

$$u(x) = \int_0^x \frac{\sigma(x)}{E_{pl}} dx = \frac{\sigma_R}{E_{pl}(e^{kl_{pl}} + 1)} \int_0^x \left[e^{k\left(\frac{l_{pl}}{2} + x\right)} + e^{k\left(\frac{l_{pl}}{2} - x\right)} \right] dx = \frac{2\sigma_R sh(kx)}{E_{pl}k(1 + e^{-kl_{pl}})} \quad (\text{Eq. 27})$$

Figure 1. Location of heat-shielding tiles on the airframe

Figure 2. Layout of heat-protective tiles on the airframe

Figure 3. Design scheme for determining spacer stresses

Figure 4. Stress state in substrate-tile system

Figure 5. Beam element for the equilibrium equation

Figure 6. Tile shear ratio in stresses state

Figure 7. The longitudinal movement of the tile

Figure 8. Schematic diagram of installation for studying adhesion strength of ice: 1 – dewar vessel with nitrogen; 2 – gear; 3 – pressure gauge; 4 – hose for supplying nitrogen to chamber; 5 – thermostat; 6 – climate chamber; 7 – test sample; 8 – captures; 9 – power columns; 10 – load traverse; 11 – direction of movement of traverse; 12 – data transmission cable, 13 – information output and processing monitor

Figure 9. Picture of the test sample

Figure 10. Picture of the sample in installation

Figure 11. Picture of the sample during testing