DEFORMAÇÃO DE ESTRUTURAS DE ARAME DO TIPO HELICOIDAL

DEFORMATION OF THE HELICAL TYPE WIRE STRUCTURES

ДЕФОРМАЦИЯ ПРОВОЛОЧНЫХ КОНСТРУКЦИЙ СПИРАЛЬНОГО ТИПА

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RESUMO

A análise de linhas aéreas de transmissão de energia envolve a modelagem da estática e das vibrações dos condutores. A solução de tais problemas exige estritamente a devida consideração da estrutura interna do condutor, em particular para sistemas de segurança energética, e a confiabilidade do suporte de informações e telecomunicações para aeródromos e sistemas de aviação, bem como para linhas aéreas de transmissão de energia sujeitas a ventos fortes, especialmente em condições de gelo. Devido à complexidade das estruturas de arame, surgem problemas conhecidos ao avaliar suas deformações, rigidez, capacidade de carga. Por exemplo, a rigidez à flexão de um condutor pode variar significativamente dependendo de sua deformação. Portanto, a rigidez pode variar ao longo do eixo do condutor e no tempo. Este artigo propõe um novo modelo para a deformação de estruturas de arame semelhante aos condutores típicos das linhas aéreas de transmissão de energia. Tais projetos incluem não apenas condutores e cabos, mas também braçadeiras em espiral projetadas para tensionar, pendurar, conectar, proteger e reparar os condutores. Com base na média de energia, cada camada de arame é considerada como um invólucro cilíndrico anisotrópico elástico equivalente. Portanto, o condutor ou braçadeira em espiral pode ser modelado como um sistema de invólucros embutidos um no outro e interagindo por meio de pressão e atrito. No processo do estudo, foram feitos cálculos que permitiram formular equações para as matrizes de flexibilidade e rigidez das estruturas em espiral. O problema da interação da braçadeira de tensão com a camada do condutor externo é formulado e resolvido. Finalmente, o mecanismo de transferência de força da braçadeira para o condutor foi investigado.

Palavras-chave: condutor, braçadeira espiral, média de energia, invólucro cilíndrico de membrana, carga admissível.

ABSTRACT

Analysis of overhead power transmission lines (OPL) involves the simulation of statics and oscillation. Solving such problems strictly requires the proper accounting of the internal conductor structure, in particular for power safety and reliability systems of information-telecommunication supply of aerodromes and aircraft systems, as well as for overhead transmission lines subjected to intense wind, especially in icing conditions. Due to the complexity of wire structures, known issues arise in the estimates of their deformations, stiffness, bearing capacity, etc. For instance, the bending stiffness of the conductor can vary considerably. Consequently, the stiffness can vary along the conductor axis as well as in time. This paper proposes a new deformation model of wire structures similar to typical OPL conductors. Such structures include not only conductors and cables, but spiral clamps intended for tension, suspension, joining, protection, and repair of conductors. Based on energy averaging each wire layer is considered as an equivalent elastic anisotropic cylindrical shell. Therefore a conductor or a spiral clamp can be modeled as a system of shells nested into each other and interacting by means of pressure and friction forces. In the process of the study, calculations were made that made it possible to formulate equations for the matrices of flexibility and rigidity of spiral structures. The interaction problem for a tension clamp with an external wire layer of a conductor has been formulated and solved. Finally, the mechanism of the force transfer from the clamp on the conductor has been investigated.

Keywords: Conductor, spiral clamp, energy averaging, membrane cylindrical shell, bearing capacity.

АННОТАЦИЯ

Анализ воздушных линий электропередачи (ОПЛ) включает моделирование статики и колебаний проводников. Решение таких задач строго требует надлежащего учета внутренней проводниковой структуры, в частности, для систем энергетической безопасности и надежности информационно-телекоммуникационного обеспечения аэродромов и авиационных систем, а также для воздушных линий электропередачи, подверженных сильному ветру, особенно в условиях обледенения. Из-за сложности проволочных конструкций возникают известные проблемы при оценке их деформаций, жесткости, несущей способности и т.д. Например, изгибная жесткость проводника может значительно варьироваться в зависимости от его деформации. Следовательно, жесткость может изменяться как вдоль оси проводника, так и во времени. В этой статье предлагается новая модель деформации проволочных конструкций, аналогичная типичным проводникам ОПЛ. Такие конструкции включают не только проводники и кабели, но и спиральные зажимы, предназначенные для натяжения, подвешивания, соединения, защиты и ремонта проводников. На основе усреднения энергии каждый слой проволоки рассматривается как эквивалентная упругая анизотропная цилиндрическая оболочка. Поэтому проводник или спиральный зажим могут быть смоделированы как система оболочек, вложенных друг в друга и взаимодействующих посредством сил давления и трения. В процессе исследования были произведены вычисления, которые позволили сформулировать уравнения для матриц гибкости и жесткости спиральных структур. Сформулирована и решена задача взаимодействия натяжного зажима с внешним проводным слоем проводника. Наконец, механизм передачи силы от зажима на проводник был исследован.

Ключевые слова: Проводник, спиральный зажим, усреднение энергии, мембранная цилиндрическая оболочка, допустимая нагрузка.

1. INTRODUCTION

In engineering, wire structures similar to OPL conductors are considered usually as flexible uniform rods or cables with average stiffness parameters. At the same time, their internal structures are neglected. This approach is used, for instance, in computing conductors' sag between supports, or in simulating conductor vibrations under wind flows (Glazunov, 1956; Boshnyakov, 1971). However, such models do not allow describing the known effects during the conductor deformation (Dubois *et al.*, 1991; Costello, 1997; Cardou and Jolicoeur, 1997; Papailiou, 1997; Pilkey, 2002; Hong *et al.*, 2005; Feyrer, 2007; Goudreau *et al.*, 2010; Foti and Martinelli, 2011), and their use in calculations can lead to unacceptable errors (Papailiou, 1997; Orlov *et al.*, 2003a; Orlov *et al.*, 2003b; Sokolov and Ryabinov, 2015).

Two assumptions about the joint deformation of wire spirals are used to calculate the bending or torsional conductor stiffness. They can be interpreted as "opposite" in meaning and define the lower and upper bounds of the theoretical estimate. Indeed, the first one assumes the independent wire deforming while the second one assumes their coupled deforming, i.e. wire spirals are rigidly connected into one, forming a kind of monolithic rod.

However, comparative calculations show (Vinogradov, 2005) that the upper and lower stiffness values calculated by both methods for the selected conductor or cable can differ by more than 70 times. Uncertainty in the knowledge of stiffness or damping conductor parameters leads to errors and inefficiency of the existing calculation algorithms. Wrong physical and mechanical ideas about the deformation of wires and similar systems destroy the integrity of the mathematical model is accurate in many other aspects. It is, of course, possible to use the "detailed" finite element simulation of wire structures (Fekr *et al.*, 1999, Lévesque *et al.*, 2011); however, this way is extremely time-consuming. On the other hand, the results strongly depend on the geometry of contact surfaces formed by the solving algorithm. Thus, the "direct" finite element simulation approach is practically unacceptable when investigating the dynamic behavior of wire structures or optimizing their structures.

An "intermediate" simulation strategy is based on the averaging of the mechanical properties of wire spirals in each wire layer of a conductor. For instance, one can consider the deformation of an arbitrary spiral wire and then use the assumption of coupling all wires of a layer or the whole conductor (Costello, 1997; Cardou and Jolicoeur, 1997; Hong *et al.*, 2005;

Rawlins, 2005a; Rawlins, 2005b; Feyrer, 2007; Rawlins, 2009; Goudreau *et al.*, 2010; Foti and Martinelli, 2011; Shejko *et al.*, 2016). However, it is difficult to introduce a correct model of layer interaction, taking into account the forces of pressure and friction between adjacent layers.

A method of energy averaging treating each wire layer as an equivalent anisotropic cylindrical shell has been proposed in (Shalashilin *et al.*, 2005; Danilin *et al.*, 2011a; Danilin *et al.*, 2011b). As a result, conductors are considered as systems of cylindrical shells embedded in each other and interacting by means of pressure and friction forces. The further development of this approach is presented below.

In the engineering practice, a new class of problems related to the design and manufacture of spiral-type clamps for conductors suspending, tensioning, connecting, protecting, and repairing appeared recently (Vinogradov *et al.*, 1994; Ryzhov, 1998; Vinogradov *et al.*, 1998; Ryzhov and Tsvetkov, 2005; Preformed Line Products..., 2007; Anosov *et al.*, 2015; Catalog APRESA..., 2019). One or more wire layers of limited length form the typical spiral clamp that could be well combined with conductors due to its flexibility. Moreover, a spiral clamp is integrated after installation with the conductor into almost monolithic structure, and the layers of a spiral clamp mounted on a conductor could be considered as additional external layers of limited length.

The clamp design requires the load-bearing capacity as well as the optimal design parameters, i.e., the length of the clamping, direction, and pitch (angle of rising) of the spirals, to be determined accurately, otherwise the clamping could be inefficient and even lead to damaging of the core of the structure. The appropriate estimate becomes possible only considering spiral structures as systems of wire layers interacting by means of pressure and friction forces, as shown below.

2. MATERIALS AND METHODS

Let us consider the equilibrium of a helical rod in the osculating plane (Figure 1), and let (Equation 1) be the radius of curvature, where α is the angle of ascent and (Equation 2). The equilibrium equations of forces referenced to the axis and normal to the rod can be then formulated as follows (Equation 3). while the equilibrium of momenta gives the known relation (Equation 4). Let S be the helix length along its

axis, and L be the length of a turning pitch along the conductor axis (Equation 5). Let us consider hence the deformation of the helix rod under the loads inducing internal forces being constant along the rod length.

First, let us apply the force P along the conductor axis, as shown in Figure 2a. The resulting momentum in the rod cross-section may be decomposed into the bending momentum $M_b(P)$ and torque $M_t(P)$ (Equation 6). Also, the longitudinal force arises in the wire cross-section (Equation 7). The effect of shearing forces on the rod deformation is neglected. Let us consider hence the loading of the helical rod in the plane normal to the x -axis by the external torque M . The bending momentum M_b and torque M_t arise in the cross-sections of the rod (Figure 2b) (Equation 8).

Let the transversal load q directed along the curvature radius of the rod, and let the points of the rod axis translate strictly along the curvature radii; this displacement referred hereinafter as w is constant along the rod under the given load q . Then, only the longitudinal force arises in the rod cross-sections (Equation 9). Where (Equation 1) is the curvature radius of the rod in the osculating plane. In general, the overall bending, torsional, and longitudinal reactions appear in the rod cross-section under the simultaneous action of three external loads (Equation 10).

As a result, the potential energy of the helix strain can be represented as follows (Equation 11). Where EJ_z , GJ_t , EF are bending, torsional and longitudinal stiffness of the helical rod corresponding to the momenta M_b , M_t , and the force N . Substituting (Equation 10) into (Equation 11) and accounting for (Equation 5), we obtain finally (Equation 12).

Let us define the elongation Δ of the helical rod along the axis x , the angle φ of rod twisting around the axis x , and the area Sw as generalized displacements, and let introduce the conjugated generalized forces (Equation 13). The generalized displacement Sw corresponds to the force intensity q . Indeed, the work of the force on the displacement w is determined as follows (Equation 14). Assuming that w remains constant within a coil of length S , we have hence (Equation 15) which determines the required conjugated "force-displacement" pair. Substituting the forces P and M expressed through P^* , M^*

, and q accordingly to (Equation 13) into (Equation 12) we obtain the following strain energy formulation (Equation 16). As provided by the Castigliano theorem, the generalized displacements are defined as (Equation 17). The corresponding matrix formulation is represented as (Equation 18). Where the elements of the matrix (Equation 19) are introduced below (Equation 20). Where (Equation 21) is the dimensionless factor.

Let us consider hence the cylindrical shell of radius r and length L elastically equivalent to the system of helical rods (wires). Let the shell be loaded at its ends by the longitudinal force T , the torque H , and the internal pressure p (Figure 3). The loads P_1 , M_1 , q_1 acting on the separate rod of the system, and the loads acting on the shell are interrelated as follows (Equation 22). Where n is the number of rods. Introduction of the relative elongation and the relative twist angle for the cylindrical shell and accounting for (Equation 22) leads to the matrix relation (Equation 23). The elements of the matrix (Equation 24) are determined as follows (Equation 25).

As we treat the conductor as a rod with specific mechanical properties, we could establish the relationships between the forces acting on the rod, and its strains within Hookean law (Equation 26). Where ε , θ , and κ are the relative elongation, the relative twist angle, and the curvature of the rod axis, respectively; N , H , M are longitudinal forces, twisting and bending moments in the rod; B and R are the flexibility and rigidity matrices, i.e. $R = B^{-1}$. The traditional definition of tensile, torsional, and bending stiffness follows directly from (Equation 26) if the matrices B and R are diagonal (i.e., for the isotropic rod considered in the principal central axes).

The conductor is a system consisting of a core (central wire) and wire layers modeled as anisotropic cylindrical membranes accordingly to (Equation 23), (Equation 25). Let us assume the conductor layers laid without gap and interference fit. The layers are numbered from 1 to n , and the index "0" is assigned to the core. Thus, for the i -th layer, Equation 23 takes the form (Equation 27). Here $T^{(i)}$ denotes the longitudinal force in the shell mid surface, which is related to the force $N^{(i)}$ acting on the shell as usually (Equation 28). And $r^{(i)}$ is the radius of the mid surface of the i -th layer.

Eliminating $p^{(i)}$ from the Equation (27), we reduce them to the following form (Equations 29-30). So that (Equation 31). For the whole wire, the longitudinal force N and the torque H are computed by summation of the forces and momenta acting on the core and the layers (Equation 32). Let us assume the contact between wires being ideal with no relative sliding; as a result, their deformations are equal (Equation 33). The following matrix formulation of the equations (Equation 32) can be obtained (Equation 34).

Substituting hence (Equation 31) into (Equation 34), accounting for (Equation 31), we obtain (Equation 35). Where $E^{(0)}$ and $G^{(0)}$ are moduli for tension and shear of the core, (Equation 36) is the area (Equation 37) is the polar moment of inertia and $d^{(0)}$ is the diameter of the core wire.

The resulting stiffness matrix R is defined for the stretched and twisted wire. It is expected that the elements $R_{12} = R_{21}$ are small as compared to the diagonal R_{11} and R_{22} due to the counter winding of the conductor layers. In any case, the angles of winding layers can be selected so that its elements out of the diagonal vanish, then R_{11} and R_{22} can be interpreted as classic stiffness of the conductor on stretching and torsion. The Equations 29-30 allow determining the bending and torsional stiffness of the wire structure. First, let us consider the torsion. At $T = 0$ it follows from (Equation 29) (Equation 38). When the wire is twisted by an angle θ , all layers are twisted equally (Equation 39).

At the same time, the torque H is defined by the summation of the torques of layers (Equation 40). Therefore, taking into account (Equation 38)-(Equation 40), we have (Equation 41). Let denote the torsional conductor stiffness as GJ_c , where G is some shear modulus. Thus, (Equation 42), and on the basis of the (Equation 41), we obtain (Equation 43). For the core and the i -th layer we have (Equation 44). Where $E^{(i)}$, $\mu^{(i)}$ are the moduli of elasticity and the Poisson's ratio of the layer material. Thus (Equation 45).

Let us consider hence the conductor bending stiffness. The curvature κ , the curvature radius, ρ and the bending moment are interrelated by the following formula (Equation

46). The bending moment M_b is defined as follows (Equation 47). When the flat cross-sections hypothesis is fulfilled, the fiber elongation is introduced as (Equation 48). If the i -th bend is treated as a shell consisting of longitudinal fibers in which the stresses (Equation 49) act, then Hooke's law for a longitudinal fiber in the form (Equation 50) follows from (Equation 29), (Equation 30). Accounting for (Equation 46)-(Equation 48), we obtain (Equation 51).

In particular, for the circular cross-section of radius $r^{(i)}$ and thickness $d^{(i)}$ we have (Equation 52). Thus, the bending stiffness of the i -th layer can be defined as follows (Equation 53). As the axes curvature of all the layers and the conductor is assumed to equal, we have (Equation 54). Therefore, considering (Equation 53), we define the bending stiffness of a conductor as (Equation 55).

3. RESULTS AND DISCUSSION:

The torsional and bending stiffness of aluminum-steel conductors (AS series, Russia) were calculated (Boshnyakovich, 1971; Glazunov, 1956). The typical cross-section of an AS conductor is shown in Figure 4. The aluminum conductive layers are referred hereinafter as 1 and 2, and the steel core is formed by the layer 3 and the central wire 4. The used data are presented below in Tables 1 and 2. The first columns of the Tables contain the conductors' marks in the form of the ratio of nominal cross-section areas of aluminum and steel parts (mm^2). In the second column, the sums of the number of wires in layers are given; the value of each term of the sum gives the number of wires in the layer while the number of terms corresponds to the number of layers. The nominal wire diameters and the lay ratios defined by the formula (Equation 56) are presented in the third and fourth columns, respectively. Here L is the twist step, r is the radius of the layer (i.e. radius of the circle on which the centers of cross-sections of wires lie), and d is the diameter of wires. Thus, (Equation 56), and finally (Equation 57) accordingly to the second formula (Equation 5).

The calculation results are presented below in Table 3. The first column contains the wire mark; the second is the outer wire's diameter (i.e. the diameter of the corresponding circumcircle). The third and fourth columns contain bending and torsional stiffness obtained using (Equation 55) and (Equation 45),

respectively. The fifth column contains the values of the torsional stiffness obtained as (Equation 58). Where d is the outer diameter of the wire (in millimeters), and the numerical coefficient dimension is $\text{N}\times\text{m}^2/\text{mm}^4$ (Koch, 2001). This formula was obtained at the Montefiore Institute (Belgium, University of Liège) (Dubois *et al.*, 1991) as a result of extensive analysis of the experimental data. Comparison of the 4th and 5th columns of Table 3 indicates a good correlation of the theoretical prediction given by the proposed model and the known experimental data.

A typical spiral type fittings is a spiral tension clamp (Figure 5) consisting of one or more layers formed by wire strands using special friction coatings (Catalog APRESA..., 2019; Preformed Line Products..., 2007). Spiral clamps are used for suspending and tensioning OPL conductors and cables. Its installation can be carried out manually by sequential coiling of the layers. The main problem of the clamp design is to find optimal values for its parameters (the clamp length, direction, and pitch (winding angle) of the spirals, and the coefficients of friction). Otherwise, the clamp may not function effectively and even cause damage to the core.

The spiral clamp can be reduced to an equivalent cylindrical shell as well as to a conductor layer. To solve the problem of interaction of the clamp with the conductor, we could consider the equilibrium of the elementary ring of the shell with the length dx equivalent to the clamp. The shell element is loaded by the longitudinal force T , the circumferential force (Equation 59), and the friction force f with longitudinal and circumferential components referred hereinafter as f_x and f_y . The ring element of length dy in the circumferential direction is shown in Figure 6, while the equilibrium equations are written as follows (Equation 60).

Let us assume the frictional force be distributed over the contact surface between the clamp and the cable as (Equation 61), where p is the contact pressure and k_T is the friction factor. Let also the force f be directed along the resultant shearing force vector formed by the forces T and h (Equation 62). The contact pressure p for given forces T , h , and the radial displacement w_0 corresponding to the interference fit of the clamp is determined from

the third equation (Equation 23) as (Equation 63).

The displacement w_0 is given by the difference between the initial radii of the conductor and the clamp if the reduction in the diameter of the cable due to its interaction with the clamp is neglected. The displacement w_0 is assumed positive if the initial radius of the clamp is less than the radius of the conductor. If the initial values T_0 and H_0 are given by the fastening conditions of the clamp on the support at $x=0$, then the equations (Equation 60), (Equation 62) allow one to formulate the Cauchy problem (Equation 64). Where the pressure p is given by (Equation 63). The problem statement (Equation 64) assumes that the clamp is so lengthy that the mutual interference of its ends could be neglected, i.e. the semi-infinite shell is considered.

Let us consider the condition of the fastening of the clamp on the support with unconstrained rotation, i.e. $H_0=0$, as a first approximation. Then the second equation (Equation 64) has the solution $H(x) \equiv 0$, and the corresponding Cauchy problem for the first equation (Equation 64) takes the form (Equation 65). The solution of the Cauchy problem (Equation 65) can be written as follows (Equation 66). Let us note that the solution (Equation 66) has the asymptotic component (Equation 67) (see Figure 7). The minimum length $l_{\min}$ of the clamp corresponding to the force T_0 can be obtained from the solution (Equation 66) as (Equation 68).

Let us consider hence the problem of interaction for the system shown in Figure 8, consisting of a clamp, the outer layer of a conductor and the inner part of it, which we will call the core. The equilibrium equations of the clamp and the outer layer of the conductor have the form (Equation 69). And the force $N^{(3)}$ in the conductor core is determined by the equilibrium condition as follows (Equation 70). If we assume that the frictional forces $f^{(1)}$ and $f^{(2)}$ arising between the layers are proportional to the pressure forces $p^{(1)}$ and $p^{(2)}$ between the layers, then (Equation 71).

As before, we assume that the friction forces are directed along with the resultant shearing force between the layers. Then (Equation 72). The pressure $p^{(1)}$ between the clamp and the conductor is determined by the

ratio (37) (Equation 73). The pressure $p^{(2)}$ between the external layer of the conductor and its core is determined by the same ratio, but written for the external layer, taking its deflection equal to zero (Equation 74).

As before, we assume that the clamp on the support can freely rotate. Then for Equation 40, taking into account the notation (Equation 71)–(Equation 74), we obtain the following Cauchy problem (Equations 75-78). In the case of rigid fixing of the clamp on the support, when $\theta^{(1)}=0$, from the second equation, taking into account the expression for $p^{(1)}$ (Equation 73), it follows that (Equation 79).

Then the initial conditions for equations (Equation 75) are written in the form (Equation 80). Where $H_0^{(1)}$ is given by formula (Equation 79). As an example, let's consider the problem of the interaction of the tension clamp NS-17.1-01 (according to the ESSP catalog, XI.2004) and the conductor AS 150/24. Clamp parameters: radius of the middle surface $r^{(1)}=10.45\text{ mm}$; diameter of the wires $d^{(1)}=3.8\text{ mm}$; tensile modulus $E^{(1)}=200\text{ kN/mm}^2$; Poisson's ratio $\mu^{(1)}=0.29$; angle of inclination of the wires relative to the conductor cross-section $\alpha^{(1)}=67^\circ$; the number of wires $n^{(1)}=14$; interference fit $w_0^{(1)}=0.9\text{ mm}$. The friction coefficient between the shells simulating the layers of the clamp and the conductor was assumed to be equal to $k_T^{(1)}=0.6$.

The parameters of the conductor outer layer: radius of the middle surface $r^{(2)}=7.2\text{ mm}$; diameter of wires $d^{(2)}=2.7\text{ mm}$; tensile modulus $E^{(2)}=63\text{ kN/mm}^2$; Poisson's ratio $\mu^{(2)}=0.34$; the angle of inclination of the wires relative to the conductor cross-section $\alpha^{(2)}=103.6^\circ$; the number of wires $n^{(2)}=16$; technological interference $w_0^{(2)}=0.2\text{ mm}$. The coefficient of friction between the shell modeling the outer conductor layer and the cylindrical surface of the conductor core (the inner part of the conductor) was taken to be equal to $k_T^{(2)}=0.15$.

Figure 9 shows, respectively, the dependence of the forces $T^{(i)}$ and the torques $H^{(i)}$, acting in the clamp ($i=1$) and the outer conductor layer ($i=2$) from the longitudinal coordinate x . Changing the tensile force in the inner conductor part (core) along the x -coordinate

is shown in Figure 10a. Figure 10b shows the distribution of pressure $p^{(i)}$ between the clamp and the outer conductor layer, as well as between the outer layer of the conductor and its internal part. Figure 11 shows the distributions of rotation angles of the clamp cross-sections ($i = 1$) and the outer conductor layer ($i = 2$) relative to the axis x .

In Figures 9-11, solid lines represent the solution of the initial value problem (Equation 75), (Equation 80), when the turn of the clamp is forbidden (the case of rigid fastening). The dashed lines correspond to the initial value problem (Equation 75), (Equation 78) when the clamp on the support can turn freely. The initial force in the clamp was calculated as (Equation 81), where the force $N_0^{(1)}$ was taken equal 23 kN

4. CONCLUSIONS:

1. An approach to the modeling of multilayer wire structures of a regular structure has been developed, taking into account the interaction of wire layers by the forces of pressure and friction. Each layer is represented from the position of energy averaging as an anisotropic cylindrical shell, and the wire structure itself is considered as a system of cylindrical shells enclosed into each other, between which slip is permitted, taking into account the forces of pressure and friction.

2. On the basis of the developed approach, formulas for calculating the stiffness and compliance matrices are obtained, which allow evaluating the torsion and bending stiffness of the conductor taking into account its internal structure and the interaction of the layers between each other.

3. On the example of the problem of the interaction of the OPL conductor with the tension spiral clamp, a method of analysis of the bearing capacity and the structural efficiency of the spiral clamps is shown.

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$$\rho = r / \cos^2 \alpha \quad (\text{Eq. 1})$$

$$d\theta = ds / \rho \quad (\text{Eq. 2})$$

$$\frac{dN}{ds} - \frac{Q}{\rho} + t = 0; \quad \frac{dQ}{ds} + \frac{N}{\rho} - q = 0, \quad (\text{Eq. 3})$$

$$dM/ds = Q \quad (\text{Eq. 4})$$

$$S = \frac{2\pi r}{\cos \alpha}; \quad L = S |\sin \alpha| = 2\pi r \frac{|\sin \alpha|}{\cos \alpha}. \quad (\text{Eq. 5})$$

$$M_b(P) = Pr \sin \alpha, \quad M_t(P) = Pr \cos \alpha. \quad (\text{Eq. 6})$$

$$N(P) = P \sin \alpha. \quad (\text{Eq. 7})$$

$$M_b(M) = -M \cos \alpha, \quad M_t(M) = M \sin \alpha. \quad (\text{Eq. 8})$$

$$N(q) = q\rho = \frac{qr}{\cos^2 \alpha} \quad (\text{Eq. 9})$$

$$\begin{aligned} M_b &= M_b(P) + M_b(M) = Pr \sin \alpha - M \cos \alpha, \\ M_t &= M_t(P) + M_t(M) = Pr \cos \alpha + M \sin \alpha, \\ N &= N(P) + N(q) = P \sin \alpha + qr / \cos^2 \alpha. \end{aligned} \quad (\text{Eq. 10})$$

$$U = \frac{1}{2} \int_0^S \left(\frac{M_b^2}{EJ_z} + \frac{M_t^2}{GJ_t} + \frac{N^2}{EF} \right) ds \quad (\text{Eq. 11})$$

$$U = \frac{1}{2} S \left[\frac{(Pr \sin \alpha - M \cos \alpha)^2}{EJ_z} + \frac{(Pr \cos \alpha + M \sin \alpha)^2}{GJ_t} + \frac{(P \sin \alpha + qr / \cos^2 \alpha)^2}{EF} \right]. \quad (\text{Eq. 12})$$

$$\begin{aligned} P^* &= P + N(q) \sin \alpha = P + qr \sin \alpha / \cos^2 \alpha, \\ M^* &= M + N(q) r \cos \alpha = M + qr^2 / \cos \alpha. \end{aligned} \quad (\text{Eq. 13})$$

$$A_q = \int_0^S w q ds \quad (\text{Eq. 14})$$

$$A_q = q S w \quad (\text{Eq. 15})$$

$$\begin{aligned} U &= \frac{1}{2} S \left\{ \frac{1}{EJ_z} \left[P^* r \sin \alpha - M^* \cos \alpha - qr^2 (\text{tg}^2 \alpha - 1) \right]^2 + \right. \\ &\left. + \frac{1}{GJ_t} \left(P^* r \cos \alpha + M^* \sin \alpha - 2qr^2 \text{tg} \alpha \right)^2 + \frac{1}{EF} \left(P^* \sin \alpha + qr \right)^2 \right\}. \end{aligned} \quad (\text{Eq. 16})$$

$$\Delta = \frac{dU}{dP^*}; \quad \varphi = \frac{dU}{dM^*}; \quad Sw = \frac{dU}{dq}. \quad (\text{Eq. 17})$$

$$[\Delta \quad \varphi \quad Sw]^T = A [P^* \quad M^* \quad q]^T \quad (\text{Eq. 18})$$

$$A = \|a_{ij}\| \quad (i, j = 1, 2, 3) \quad (\text{Eq. 19})$$

$$\begin{aligned} a_{11} &= Sr^2 \left(\frac{1+\psi}{EJ_z} \sin^2 \alpha + \frac{\cos^2 \alpha}{GJ_t} \right); \quad a_{12} = a_{21} = Sr \left(\frac{1}{GJ_t} - \frac{1}{EJ_z} \right) \sin \alpha \cos \alpha; \\ a_{22} &= S \left(\frac{\sin^2 \alpha}{GJ_t} + \frac{\cos^2 \alpha}{EJ_z} \right); \quad a_{13} = a_{31} = -Sr^3 \left(\frac{\text{tg}^2 \alpha - (1+\psi)}{EJ_z} + \frac{2}{GJ_t} \right) \sin \alpha; \\ a_{23} &= a_{32} = -Sr^2 \left(\frac{2\text{tg}^2 \alpha}{GJ_t} + \frac{1-\text{tg}^2 \alpha}{EJ_z} \right) \cos \alpha; \quad a_{33} = Sr^4 \left[\frac{(1-\text{tg}^2 \alpha)^2 + \psi}{EJ_z} + \frac{4\text{tg}^2 \alpha}{GJ_t} \right]. \end{aligned} \quad (\text{Eq. 20})$$

$$\psi = J_z / Fr^2 \quad (\text{Eq. 21})$$

$$P_1 = T 2\pi r / n, \quad M_1 = H / n, \quad q_1 = p 2\pi r L / (nS), \quad (\text{Eq. 22})$$

$$\begin{bmatrix} \varepsilon & \theta & w \end{bmatrix}^T = B \begin{bmatrix} T & H & p \end{bmatrix}^T, \quad (\text{Eq. 23})$$

$$B = \|b_{ij}\| \quad (i, j = 1, 2, 3) \quad (\text{Eq. 24})$$

$$\begin{aligned} b_{11} &= \frac{2\pi r^3}{n|\sin\alpha|} \left(\frac{1+\psi}{EJ_z} \sin^2\alpha^{(i)} + \frac{\cos^2\alpha}{GJ_t} \right), & b_{12} &= \frac{r}{n} \text{sign}\alpha \cos\alpha \left(\frac{1}{GJ_t} - \frac{1}{EJ_z} \right), \\ b_{13} &= -\frac{2\pi r^4}{n} \sin\alpha \left(\frac{\text{tg}^2\alpha - 1 - \psi}{EJ_z} + \frac{2}{GJ_t} \right), & b_{22} &= \frac{1}{n|\sin\alpha|} \left(\frac{\sin^2\alpha}{GJ_t} + \frac{\cos^2\alpha}{EJ_z} \right), \\ b_{21} &= 2\pi r b_{12}, & b_{23} &= -\frac{2\pi r^3}{n} \cos\alpha \left(\frac{2\text{tg}^2\alpha}{GJ_t} + \frac{1 - \text{tg}^2\alpha}{EJ_z} \right), \\ b_{31} &= b_{13}, & b_{32} &= \frac{b_{23}}{2\pi r}, & b_{33} &= \frac{2\pi r^5}{n} |\sin\alpha| \left[\frac{(1 - \text{tg}^2\alpha)^2 + \psi}{EJ_z} + \frac{4\text{tg}^2\alpha}{GJ_t} \right]. \end{aligned} \quad (\text{Eq. 25})$$

$$\begin{pmatrix} \varepsilon & \theta & \kappa \end{pmatrix}^T = B \begin{pmatrix} N & H & M \end{pmatrix}^T \quad \text{or} \quad \begin{pmatrix} N & H & M \end{pmatrix}^T = R \begin{pmatrix} \varepsilon & \theta & \kappa \end{pmatrix}^T. \quad (\text{Eq. 26})$$

$$\begin{pmatrix} \varepsilon^{(i)} & \theta^{(i)} & 0 \end{pmatrix}^T = B^{(i)} \begin{pmatrix} T^{(i)} & H^{(i)} & p^{(i)} \end{pmatrix}^T \quad (\text{Eq. 27})$$

$$N^{(i)} = 2\pi r^{(i)} T^{(i)} \quad (\text{Eq. 28})$$

$$\begin{pmatrix} \varepsilon^{(i)} & \theta^{(i)} \end{pmatrix}^T = \bar{B}^{(i)} \begin{pmatrix} T^{(i)} & H^{(i)} \end{pmatrix}^T; \quad (\text{Eq. 29})$$

$$\bar{B}^{(i)} = \|\bar{b}_{kl}\| = \begin{pmatrix} b_{11}^{(i)} - \frac{b_{13}^{(i)}b_{31}^{(i)}}{b_{33}^{(i)}} & b_{12}^{(i)} - \frac{b_{13}^{(i)}b_{32}^{(i)}}{b_{33}^{(i)}} \\ b_{21}^{(i)} - \frac{b_{23}^{(i)}b_{31}^{(i)}}{b_{33}^{(i)}} & b_{22}^{(i)} - \frac{b_{23}^{(i)}b_{32}^{(i)}}{b_{33}^{(i)}} \end{pmatrix}; \quad k, l = 1, 2, \quad (\text{Eq. 30})$$

$$\begin{pmatrix} T^{(i)} & H^{(i)} \end{pmatrix}^T = C^{(i)} \begin{pmatrix} \varepsilon^{(i)} & \theta^{(i)} \end{pmatrix}^T; \quad C^{(i)} = \|c_{pq}^{(i)}\| = (\bar{B}^{(i)})^{-1}; \quad p, q = 1, 2. \quad (\text{Eq. 31})$$

$$N = N^{(0)} + \sum_{i=1}^n N^{(i)} = N^{(0)} + 2\pi \sum_{i=1}^n r^{(i)} T^{(i)}; \quad H = H^{(0)} + \sum_{i=1}^n H^{(i)}. \quad (\text{Eq. 32})$$

$$\varepsilon^{(i)} = \varepsilon, \quad \theta^{(i)} = \theta; \quad i = 0, 1, \dots, n. \quad (\text{Eq. 33})$$

$$\begin{pmatrix} N \\ H \end{pmatrix} = \begin{pmatrix} N^{(0)} \\ H^{(0)} \end{pmatrix} + \sum_{i=1}^n \begin{pmatrix} 2\pi r^{(i)} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} T^{(i)} \\ H^{(i)} \end{pmatrix}. \quad (\text{Eq. 34})$$

$$\begin{pmatrix} N \\ H \end{pmatrix}^T = R \begin{pmatrix} \varepsilon \\ \theta \end{pmatrix}^T, \quad R = \begin{pmatrix} E^{(0)} F^{(0)} & 0 \\ 0 & G^{(0)} J_t^{(0)} \end{pmatrix} + \sum_{i=1}^n \begin{pmatrix} 2\pi r^{(i)} & 0 \\ 0 & 1 \end{pmatrix} C^{(i)} \quad (\text{Eq. 35})$$

$$F^{(0)} = \pi d^{(0)2} / 4 \quad (\text{Eq. 36})$$

$$J_t^{(0)} = \pi d^{(0)4} / 32 \quad (\text{Eq. 37})$$

$$\theta^{(i)} = \bar{b}_{22}^{(i)} H^{(i)}. \quad (\text{Eq. 38})$$

$$\theta^{(i)} = \theta; \quad i = 0, 1, \dots, n. \quad (\text{Eq. 39})$$

$$H = \sum_{i=0}^n H^{(i)}. \quad (\text{Eq. 40})$$

$$H = \sum_{i=0}^n \theta^{(i)} c_{22}^{(i)} = \theta \sum_{i=0}^n c_{22}^{(i)}. \quad (\text{Eq. 41})$$

$$\theta = H / (GJ_t) \quad (\text{Eq. 42})$$

$$GJ_t = \sum_{i=0}^n c_{22}^{(i)}. \quad (\text{Eq. 43})$$

$$b_{22}^{(0)} = \frac{1}{G^{(0)} J_t^{(0)}} \approx \frac{10}{G^{(0)} d^{(0)4}}; \quad J_t^{(i)} = 0.1 d^{(i)4}; \quad J_b^{(i)} = 0.05 d^{(i)4}; \quad G^{(i)} = \frac{E^{(i)}}{2(1 + \mu^{(i)})}; \quad (\text{Eq. 44})$$

$$GJ_t = 0.1 G^{(0)} J_t^{(0)} + \sum_{i=1}^n c_{22}^{(i)}. \quad (\text{Eq. 45})$$

$$\kappa = \rho^{-1} = M_b / EJ_b \quad (\text{Eq. 46})$$

$$M_b = \int_F y \sigma dF \quad (\text{Eq. 47})$$

$$\varepsilon = y \rho^{-1} \quad (\text{Eq. 48})$$

$$\sigma^{(i)} = T^{(i)} / d^{(i)} \quad (\text{Eq. 49})$$

$$\varepsilon^{(i)} = \bar{b}_{11}^{(i)} T^{(i)} = \bar{b}_{11}^{(i)} d^{(i)} \sigma^{(i)} \quad (\text{Eq. 50})$$

$$E^{(i)} J_b^{(i)} = M_b^{(i)} \rho^{(i)} = \rho \int_F y \frac{\varepsilon^{(i)}}{\bar{b}_{11}^{(i)} d^{(i)}} dF = \frac{1}{\bar{b}_{11}^{(i)} d^{(i)}} \int_{F^{(i)}} y^2 dF^{(i)}. \quad (\text{Eq. 51})$$

$$\int_{F^{(i)}} y^2 dF^{(i)} = \pi r^{(i)3} d^{(i)}. \quad (\text{Eq. 52})$$

$$E^{(i)} J_b^{(i)} = \frac{\pi r^{(i)3}}{\bar{b}_{11}^{(i)}}. \quad (\text{Eq. 53})$$

$$M_b = \sum_{i=0}^n M_b^{(i)}. \quad (\text{Eq. 54})$$

$$EJ_b = M_b \rho = \rho \sum_{i=0}^n M_b^{(i)} = \rho \sum_{i=0}^n \frac{E^{(i)} J_b^{(i)}}{\rho} = 0.05 d^{(0)4} E^{(0)} + \pi \sum_{i=1}^n r^{(i)3} c_{11}^{(i)}. \quad (\text{Eq. 55})$$

$$m = L/(2r + d) \quad (\text{Eq. 56})$$

$$\alpha = \text{Arctg}(L/2\pi r) \quad (\text{Eq. 57})$$

$$GJ_t = 0.00027 d^4, \quad (\text{Eq. 58})$$

$$h = H/(2\pi r^2) \quad (\text{Eq. 59})$$

$$\frac{dT}{dx} = -f_x, \quad \frac{dH}{dx} = -2\pi r^2 f_y. \quad (\text{Eq. 60})$$

$$f = k_T p \quad (\text{Eq. 61})$$

$$f_x = f \sin \beta, \quad f_y = f \cos \beta; \\ \sin \beta = \frac{|T|}{\sqrt{T^2 + h^2}}, \quad \cos \beta = \frac{|h|}{\sqrt{T^2 + h^2}}; \quad h = \frac{H}{2\pi r^2}. \quad (\text{Eq. 62})$$

$$p = \frac{w_0 - b_{31}T - b_{32}H}{b_{33}}. \quad (\text{Eq. 63})$$

$$\frac{dT}{dx} = -k_T p \sin \beta, \quad \frac{dH}{dx} = -2\pi r^2 k_T p \cos \beta; \\ T(0) = T_0, \quad H(0) = H_0, \quad (\text{Eq. 64})$$

$$\frac{dT}{dx} = -k_T \frac{w_0 - b_{31}T}{b_{33}} = -k_T (a_0 + a_1 T); \quad T(0) = T_0 \quad \left(a_0 = \frac{w_0}{b_{33}}, \quad a_1 = -\frac{b_{31}}{b_{33}} \right). \quad (\text{Eq. 65})$$

$$T(x) = T_\infty + (T_0 - T_\infty) e^{-k_T a_1 x}, \quad T_\infty = -\frac{a_0}{a_1}. \quad (\text{Eq. 66})$$

$$T_\infty = \lim_{x \rightarrow \infty} T = -\frac{w_0}{|b_{31}|}. \quad (\text{Eq. 67})$$

$$l_{\min} = \frac{1}{k_T a_1} \ln \left(1 - \frac{T_0}{T_\infty} \right). \quad (\text{Eq. 68})$$

$$2\pi r^{(1)} \frac{dT^{(1)}}{dx} = -f_x^{(1)} 2\pi \left(r^{(1)} - \frac{d^{(1)}}{2} \right), \quad \frac{dH^{(1)}}{dx} = -f_y^{(1)} 2\pi \left(r^{(1)} - \frac{d^{(1)}}{2} \right)^2,$$

$$2\pi r^{(2)} \frac{dT^{(2)}}{dx} = f_x^{(1)} 2\pi \left(r^{(2)} + \frac{d^{(2)}}{2} \right) - f_x^{(2)} 2\pi \left(r^{(2)} - \frac{d^{(2)}}{2} \right), \quad (\text{Eq. 69})$$

$$\frac{dH^{(2)}}{dx} = f_y^{(1)} 2\pi \left(r^{(2)} + \frac{d^{(2)}}{2} \right)^2 - f_y^{(2)} 2\pi \left(r^{(2)} - \frac{d^{(2)}}{2} \right)^2,$$

$$N^{(3)} + T^{(1)} 2\pi r^{(1)} + T^{(2)} 2\pi r^{(2)} = T_0^{(1)} 2\pi r^{(1)} \quad (\text{Eq. 70})$$

$$f^{(1)} = k_T^{(1)} p^{(1)}, \quad f^{(2)} = k_T^{(2)} p^{(2)} \quad (\text{Eq. 71})$$

$$f_x^{(1)} = f^{(1)} \sin \beta^{(1)}, \quad f_y^{(1)} = f^{(1)} \cos \beta^{(1)},$$

$$\cos \beta^{(1)} = |h^{(1)}| / \sqrt{T^{(1)2} + h^{(1)2}}, \quad \sin \beta^{(1)} = |T^{(1)}| / \sqrt{T^{(1)2} + h^{(1)2}},$$

$$h^{(1)} = H^{(1)} / (2\pi r^{(1)2}),$$

$$f_x^{(2)} = f^{(2)} \sin \beta^{(2)}, \quad f_y^{(2)} = f^{(2)} \cos \beta^{(2)}, \quad (\text{Eq. 72})$$

$$\cos \beta^{(2)} = |h_{12}| / \sqrt{T_{12}^2 + h_{12}^2}, \quad \sin \beta^{(2)} = |T_{12}| / \sqrt{T_{12}^2 + h_{12}^2},$$

$$h^{(2)} = H^{(2)} / (2\pi r^{(2)2}),$$

$$T_{12} = T^{(1)} (r^{(1)} / r^{(2)}) + T^{(2)}, \quad h_{12} = h^{(1)} (r^{(1)} / r^{(2)}) + h^{(2)}.$$

$$p^{(1)} = (w_0^{(1)} - b_{31}^{(1)} T^{(1)} - b_{32}^{(1)} H^{(1)}) / b_{33}^{(1)} \quad (\text{Eq. 73})$$

$$p^{(2)} = p^{(1)} (r^{(1)} / r^{(2)}) - (b_{31}^{(2)} T^{(2)} + b_{32}^{(2)} H^{(2)}) / b_{33}^{(2)} \quad (\text{Eq. 74})$$

$$\frac{dT^{(1)}}{dx} = -f_x^{(1)} \left(1 - \frac{d^{(1)}}{2r^{(1)}} \right), \quad \frac{dH^{(1)}}{dx} = -f_y^{(1)} 2\pi r^{(1)2} \left(1 - \frac{d^{(1)}}{2r^{(1)}} \right)^2,$$

$$\frac{dT^{(2)}}{dx} = f_x^{(1)} \left(1 + \frac{d^{(2)}}{2r^{(2)}} \right) - f_x^{(2)} \left(1 - \frac{d^{(2)}}{2r^{(2)}} \right), \quad (\text{Eq. 75})$$

$$\frac{dH^{(2)}}{dx} = f_y^{(1)} 2\pi r^{(2)2} \left(1 + \frac{d^{(2)}}{2r^{(2)}} \right)^2 - f_y^{(2)} 2\pi r^{(2)2} \left(1 - \frac{d^{(2)}}{2r^{(2)}} \right)^2;$$

$$f^{(1)} = k_T^{(1)} p^{(1)}, \quad f^{(2)} = k_T^{(2)} p^{(2)};$$

$$p^{(1)} = (w_0^{(1)} - b_{31}^{(1)} T^{(1)} - b_{32}^{(1)} H^{(1)}) / b_{33}^{(1)}, \quad p^{(2)} = p^{(1)} (r^{(1)} / r^{(2)}) - (b_{31}^{(2)} T^{(2)} + b_{32}^{(2)} H^{(2)}) / b_{33}^{(2)}; \quad (\text{Eq. 76})$$

$$f_x^{(1)} = f^{(1)} \sin \beta^{(1)}, \quad f_y^{(1)} = f^{(1)} \cos \beta^{(1)}, \quad f_x^{(2)} = f^{(2)} \sin \beta^{(2)}, \quad f_y^{(2)} = f^{(2)} \cos \beta^{(2)}; \quad (\text{Eq. 77})$$

$$T^{(1)}(0) = T_0^{(1)}, \quad T^{(2)}(0) = 0, \quad H^{(1)}(0) = H^{(2)}(0) = 0 \quad (\text{Eq. 78})$$

$$H_0^{(1)} = - \left(b_{22}^{(1)} - b_{23}^{(1)} \frac{b_{32}^{(1)}}{b_{33}^{(1)}} \right)^{-1} \left[\frac{b_{23}^{(1)}}{b_{33}^{(1)}} w_0^{(1)} + \left(b_{21}^{(1)} - b_{23}^{(1)} \frac{b_{31}^{(1)}}{b_{33}^{(1)}} \right) T_0^{(1)} \right] \quad (\text{Eq. 79})$$

$$T_0^{(1)}(0) = T_0^{(1)}, T_0^{(2)}(0) = 0, H_0^{(1)}(0) = H_0^{(1)}, H_0^{(2)}(0) = 0 \quad (\text{Eq. 80})$$

$$T_0^{(1)} = N_0^{(1)} / (2\pi r^{(1)}) \quad (\text{Eq. 81})$$

Table 1. Parameters of conductive (aluminum) part of AS conductors

AS conductors	Number of wires in the layers	Wire diameter, mm	Lay ratio
1	2	3	4
120/19	10+16	2.40	15, 12
120/27	12+18	2.20	15, 12
150/19	9+15	2.80	15, 12
150/34	12+18	2.50	15, 12
185/24	9+15	3.15	15, 12
240/39	10+16	3.40	15, 12
185/128	24+30	2.10	15, 12
300/39	9+15	4.00	15, 12
300/66	12+18	3.50	15, 12
330/30	10+16+22	2.98	15, 12, 10
330/43	12+18+24	2.80	15, 12, 10
400/22	10+16+22+28	2.57	18, 15, 13, 11
400/64	10+16	4.37	15, 12
400/93	12+18	4.15	15, 12
300/204	24+30	2.65	15, 12
500/26	8+14+20	3.90	15, 12, 10
500/64	12+18+24	3.40	15, 12, 10
550/71	12+18+24	3.60	15, 12, 10
600/72	12+18+24	3.70	15, 12, 10
650/79	15+21+27+33	2.90	18, 15, 12, 10

Table 2. Parameters of cores – steel part of AS conductors

AS conductor	Number of wires in the layers	Wire diameter, mm	Lay ratio
1	2	3	4
120/19	1+6	1.85	20
120/27	1+6	2.20	20
150/19	1+6	1.85	20
150/34	1+6	2.50	20
185/24	1+6	2.10	20
240/39	1+6	2.65	20
185/128	1+6+12+18	2.10	25, 20, 15
300/39	1+6	2.65	20
300/66	1+6+12	2.10	25, 20
330/30	1+6	2.30	20
330/43	1+6	2.80	20
400/22	1+6	2.00	20
400/64	1+6	3.40	20
400/93	1+6+12	2.50	25, 20
300/204	1+6+12+18	2.65	25, 20, 18
500/26	1+6	2.20	20
500/64	1+6	3.40	20
550/71	1+6	3.60	20
600/72	1+6+12	2.20	25, 20
650/79	1+6+12	2.30	25, 20

Table 3. Stiffnesses of AS conductors

AS conductors	d , mm	EJ_b , N·m ²	GJ_t , N·m ²	GJ_t^{exp} , N·m ²
1	2	3	4	5
120/19	15.15	113.6	15.37	14.22
120/27	15.40	125.3	16.50	15.19
150/19	16.75	166.8	23.03	21.25
150/34	17.50	208.9	27.50	25.32
185/24	18.90	269.5	37.18	34.45
240/39	21.55	463.1	62.60	58.23
185/128	23.10	835.9	82.66	76.88
300/39	23.95	695.4	96.14	88.84
300/66	24.50	812.2	101.48	97.28
330/30	24.78	807.5	103.77	101.81
330/43	25.20	867.2	120.92	108.88
400/22	26.56	1041.6	163.73	134.36
400/64	27.68	1261.3	170.52	158.50
400/93	29.10	1615.0	200.79	193.61
300/204	29.15	2119.1	209.61	194.95
500/26	30.00	1684.2	244.93	218.70
500/64	30.60	1885.4	262.89	236.73
550/71	32.40	2369.7	330.42	297.54
600/72	33.20	2632.8	360.17	328.03
650/79	34.70	3134.4	468.10	391.45

Figure 1. The equilibrium of the differential element of the helical rod in the osculating plane

Figure 2. Momenta in the cross section of the helical rod from the action of external force factors: a) Momenta from the action of longitudinal force; b) Momenta from the action of torque)

Figure 3. The reduction of the conductor layer to an equivalent shell

Figure 4. AS conductor construction

Figure 5. Spiral tension clamp on the conductor

Figure 6. Equilibrium of an element of an equivalent shell ring

Figure 7. Type of function (Equation 51)

Figure 8. The interaction of the clamp, the outer layer of the conductor and its internal part

Figure 9. The variation of the longitudinal force (a) and torque (b) in the clamp and outer conductor layer along with the coordinates

Figure 10. The variation of the tensile force $N^{(3)}$ (a) in the inner conductor part and the internal pressure $p^{(i)}$ (b) between the clamp and the outer conductor layer ($i = 1$), and between the outer layer of the conductor and its inner part ($i = 2$) along the coordinate x

Figure 11. The variation of rotation angles of clamp cross sections $\theta^{(1)}$ (a) and cross-sections of outer conductor layer $\theta^{(2)}$ (b) along the coordinate x