ABORDAGEM ESPECTRAL DA AVALIAÇÃO DE UMA SÉRIE TEMPORÁRIA DE MUDANÇA DE POTÊNCIA NAS UNIDADES DE MEDIÇÃO DE SUBESTAÇÃO E SUA APLICAÇÃO NOS MODOS DE CONTROLE DOS SISTEMAS DE MEDIÇÃO DE ELETRICIDADE DE SUBESTAÇÃO**SPECTRAL APPROACH TO THE ESTIMATION OF THE TIME SERIES OF THE POWER CHANGE IN THE SUBSTATION METERING UNITS AND ITS APPLICATION IN THE CONTROL MODES OF SUBSTATION ELECTRICITY METERING SYSTEMS****СПЕКТРАЛЬНЫЙ ПОДХОД К ОЦЕНКЕ ВРЕМЕННОЙ СЕРИИ ИЗМЕНЕНИЯ МОЩНОСТИ В ПОДСТАНЦИОННЫХ УЧЕТНЫХ УСТРОЙСТВАХ И ЕГО ПРИМЕНЕНИЕ В РЕЖИМАХ УПРАВЛЕНИЯ ПОДСТАНЦИОННЫМИ СИСТЕМАМИ УЧЕТА ЭЛЕКТРОЭНЕРГИИ**

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RESUMO

Para criar os modelos de superfícies estendidas nas partes de ondas longas e ondas curtas do espectro de ondas, é necessário conhecer as características de dispersão dos obstáculos da fita, idealmente localizados tanto no espaço livre quanto em uma superfície estendida. Os métodos para o estudo remoto de superfícies estendidas, com base na análise de sinais gerados pela reflexão ou radiação de ondas eletromagnéticas que passam por neoplasias, permitem visualizar a topografia, características e tipo de vários locais, tamanhos de vários objetos. Um dos métodos de pesquisa eficazes é resolver o problema da difração de ondas eletromagnéticas por um objeto infinitamente estendido no espaço livre. O objetivo deste estudo é caracterizar ondas eletromagnéticas durante a difração de ondas eletromagnéticas em objetos que podem ser usados como sistemas de radar. Foram utilizados o método de processamento estatístico de dados experimentais de testes de campo e a descrição matemática da situação do radar eletromagnético, bem como sua simulação computacional. O resultado deste estudo pode ser formulado como o desenvolvimento de métodos analíticos e de software para analisar o padrão de difração de ondas eletromagnéticas no modo de recepção de ondas múltiplas em um objeto infinitamente estendido no espaço livre, bem como um método de software para calcular características de difração em frequências que excedem a frequência base, levando em consideração a falta de homogeneidade dos campos eletromagnéticos nessas condições. Os resultados da pesquisa utilizados no desenvolvimento da metodologia universal para o projeto e desenvolvimento de sistemas de radar capazes de funcionar adequadamente em um ambiente eletromagnético são de importância prática. Os resultados deste estudo vão permitir uma completa descrição do quadro eletrodinâmico da propagação de ondas e sua difração, ou seja, a consideração às propriedades de difração dos sistemas de radar nos permitirá desenvolver uma direção para sua produção e propagação mais amplas.

Palavras-chave: *sistemas de radar do tipo slot, ondas eletromagnéticas, funções espectrais, integral de Fourier, equações de onda.*

ABSTRACT

In order to create models of extended surfaces in the long and short wavelength parts of the wave spectrum, it is necessary to know the dispersion characteristics of tape-like obstacles perfectly located both in free space and on an extended surface. Methods of remote studying of extended surfaces, based on an

analysis of signals that are generated by reflection or radiation of electromagnetic waves passing neoplasms allows us to view the topography, characteristics, and type of the different locations, sizes of various objects. One of the effective methods of study is to solve the problem of diffraction of electromagnetic waves on an infinitely extended object in free space. The purpose of this study is characteristics of electromagnetic waves during diffraction of electromagnetic waves at objects that can be used as radar systems. The method of statistical processing of experimental data from field tests and mathematical description of electromagnetic radar situation, as well as its computer simulation were used. The result of this study can be formulated as the development of analytical and software methods for analyzing diffraction pattern of electromagnetic waves in multi-wave mode of reception on infinitely extended object in free space, as well as a software method for calculating diffraction characteristics at frequencies exceeding the basic frequency, taking into account the inhomogeneity of electromagnetic fields in these conditions. The practical significance is the research results used in the development of a universal technique for design and development of radar systems capable of working adequately in conditions of electromagnetic environment in question. The results of this research will allow a complete description of the electrodynamic picture of wave propagation and its diffraction, namely, taking into account the diffraction properties of radar systems will allow us to develop a direction for their wider production and distribution.

Keywords: *slot type radar systems, electromagnetic waves, spectral functions, Fourier integral, wave equations.*

АННОТАЦИЯ

Для создания моделей протяженных поверхностей в длинноволновой и коротковолновой частях спектра волны необходимо знать дисперсионные характеристики ленточных препятствий, идеально расположенных как в свободном пространстве, так и на протяженной поверхности. Методы дистанционного изучения протяженных поверхностей, основанные на анализе сигналов, генерируемых отражением или излучением электромагнитных волн, проходящих новообразования, позволяют нам просматривать топографию, характеристики и тип различных мест расположения, размеров различных объектов. Одним из эффективных методов исследования является решение проблемы дифракции электромагнитных волн на бесконечно протяженном объекте в свободном пространстве. Целью данного исследования является характеристика электромагнитных волн при дифракции электромагнитных волн на объектах, которые могут быть использованы в качестве радиолокационных систем. Использован метод статистической обработки экспериментальных данных полевых испытаний и математического описания электромагнитной радиолокационной обстановки, а также его компьютерное моделирование. Результат этого исследования может быть сформулирован как разработка аналитических и программных методов анализа дифракционной картины электромагнитных волн в многоволновом режиме приема на бесконечно вытянутом объекте в свободном пространстве, а также программный метод расчета дифракционных характеристик на частотах, превышение базовой частоты с учетом неоднородности электромагнитных полей в этих условиях. Практическое значение имеют результаты исследований, использованные при разработке универсальной методики проектирования и разработки радиолокационных систем, способных адекватно работать в условиях электромагнитной обстановки. Результаты этого исследования позволят полностью описать электродинамическую картину распространения волны и ее дифракции, а именно учет дифракционных свойств радиолокационных систем позволит нам разработать направление для их более широкого производства и распространения.

Ключевые слова: *радиолокационные системы щелевого типа, электромагнитные волны, спектральные функции, интеграл Фурье, волновые уравнения.*

1. INTRODUCTION

In a multi-location study of the earth's surface, a quantitative description of the phenomena caused by repeated structure and taking into account the influence of the underlying surface properties on the characteristics of the received signals are of great importance (Immoreev, 2009). An analysis of characteristics of scattering electromagnetic waves by an ideally conducted ribbon screen located on flat boundary of the conducting dielectric half-space allows us

to study problems associated with sharp change in electro physical properties of the earth's surface and the presence of well-reflecting objects located on it (Likhachev and Usov, 2010; Orlov *et al.*, 2003; Golovin, 2004).

There are some solutions for ideally conducting discs and bands that make it possible to obtain multi-location estimates of characteristics of wave scattering (Kurkchan *et al.*, 2016; He *et al.*, 2008; Özdenvar and McMechan, 1997). Nonetheless, these solutions are inapplicable in radiometric research, because

The parameters of thermal radiation of the medium are determined by active losses of electromagnetic waves in the medium, i.e., the possibility of radiometric observation of objects is conditioned by the difference in their absorption capacity. In these studies, the problem of diffraction by an infinitely extended tape in free space is considered. In Loshilov *et al.* (2010, Daus and Kharchenko, 2018), the problems were reduced to a system of pair integral equations with respect to spectral functions for surface current densities solved by the moment method. Similar problems for impedan ribbon (Figure 1) and for slit in the impedan shield can also be solved by the eigenfunction method (Semyonov and Loschilov, 2011; Loshilov *et al.*, 2012; Skianis, 2014; Gatignol *et al.*, 2010; Orofino and Pedersen, 1993).

2. MATERIALS AND METHODS

The infinite impedan ribbon and the infinite slit in impedan screen are represented as coordinate surfaces $\xi = 0$ in the elliptical coordinate system of Figure 2.

The diffraction field of an arbitrarily oriented slit in the screen in a Cartesian coordinate system independent of its orientation is determined by substituting the relations for field components in the elliptical system into formulas that relate vector functions to the indicated coordinate systems (Bashly *et al.*, 2017; Yao *et al.*, 2001; Inverarity, 2003). To do this, it is necessary to express the gap field in an elliptical coordinate system associated with gap for the case when the electric field vector $\vec{E}$ or magnetic field strength $\vec{H}$ of incident wave is oriented along the slit. The indicated expressions are obtained as a result of expressing a boundary problem by the method of dividing variable based on expansion of plane wave in series according to Mathieu function (Kultin, 2015; Pichugin *et al.*, 2013; Badylevich *et al.*, 2003).

Let us apply the Cartesian coordinate system XYZ so that the plane of the screen coincides with the plane XY , and the plane of incidence of electromagnetic wave coincided with the plane ZY . The direction of the normal to the front of incident plane wave forms an angle γ with axis OZ . Linear slit of infinite length and finite width $2h$ is angled τ relative to the axis OZ (Figure 2) (Pichugin and Bugrova, 2013; Li *et al.*, 2015; Morgunov, 2004). In general, the field of an incident plane electromagnetic wave in free space has the form Equations 1 and 2. The total field is the sum of the incident and diffracted by

ribbon fields (Equations 3, 4 and 5). Equation 6 is the solution for complex amplitudes satisfying the radiation conditions at infinity (Equation 7), where $(\pm)$ – are spectral densities for the amplitudes in the upper and lower half-spaces (Equations 8-16). Looking for solution of the wave inhomogeneous equation in the form of the Fourier expansion we get Equations 17, 18 and 19.

3. RESULTS AND DISCUSSION:

The spectral density depends on the distribution of external current in space. At that, the integral is taken only at those points in space where there are existing points. Then we need to solve the wave equation (Equations 20, 21). Let us see the expansion of the Fourier function (Equations 22, 23 and 24) where $g(x_1)$ - is spectral density, this expression can be interpreted as an infinite set of plane homogeneous waves located in the positive and negative direction of the axis x with phase speeds (Equation 25) where x_1 - plays the role of wave number, amplitude of these waves $g(x_1)$ does not depend on coordinate x . Therefore, the solution of the wave equation is in Equations 26, 27 and 28.

Solution for complex amplitudes satisfying the wave equation and depending on transverse coordinates x and z , and radiation conditions at infinity, can be recorded in the form of the Fourier integral. We shell use the variable separation method, the Fourier method to solve the wave equation (Equation 29). Then divide it by XYZ (Equations 30-33) minding the equality of the left and right sides of the equation and their independence, we assume that they are equal to some separation constant, let minding the equality of the left and right sides of the equation and their independence, we assume that they are equal to some separation constant, let (Equations 34-37), thus we reduce the first equation by Y and get a new system of equations (Equations 38, 39).

Let us consider the homogeneous equation (Equation 40). If we take into account (Equation 41) then (Equations 42, 43), where (Equations 44, 47). Let us separate the variables (Equation 48). Then we shell move on to dimensionless spatial variables (Equations 49, 50) then we get a system of equations in the form (Equation 51, 52).

To solve inhomogeneous wave equations, we use Fourier expansion. The function $X(u)$ is defined on the interval $-\infty \leq u \leq \infty$ therefore,

its expansion into the Fourier integral is (Equation 53). Analogically for function $Z(v)$ (Equation 54). Therefore, for the vector potential $\vec{A}(x, z)$ we have (Equations 55, 56) where (Equation 57).

Let us consider the internal integral (Equations 58, 59). By putting an expression for $\vec{A}(u, v)$ in the wave equation (Equations 38, 39 and 40) we get Equation 60 by comparing Equations 53, 54 and Equations 55-59. We can see that the spectral plane of the desired function $\vec{A}(u, v)$ differs from the spectral plane of external current decomposition $-\vec{j}^{st}$ only a factor $-\frac{1}{a^2} - (k^2 - p_0^2 - \varepsilon^2 - w^2)$. We can determine the spectral plane of the desired function $\vec{g}(\varepsilon)$. For this, we multiply Equations 55-59 by complex conjugate functions $\frac{1}{2\pi a^2} e^{-i\varepsilon' u - i w' v}$ where ε', w' are fixed values ε, w , then integrate over the entire infinite space (Equations 61, 62). Then we transfer the stroke from wave numbers ε' and w' spatial coordinates u and v (Equation 63).

This expression determines spectral density $g(u, v)$ in the expansion (Equations 53, 54). It depends on distribution of the external current in the space, and integral (Equation 63) is calculated only at those points in the space where currents exist (Equation 64). S denotes a cross section of volume of sources x by the plane $y = \text{const}$. Let us calculate integral w (Equations 65, 66 and 67). Transition to the plane of complex variable w and putting $k^2 < \varepsilon^2 + p_0^2$, which does not limit the generality of the results obtained, we note that in the integrand there are two singular points (bands): one in the upper half-plane $w = iw'$; another in the lower half-plane: $w = iw'$. For $(v - v') < 0$ integral L can be supplemented by vanishing integral in circle of infinitely large radius in the upper half-plane, where at $w \rightarrow i \infty$ the integrand tends to zero. Then the integral will be (Equations 68, 69). There is no special singular point in infinity (Equation 70). Thus, Green's function is equal to (Equation 71)

Let us substitute Equations 29-33 into Equations 42, 43 taking into account Equations 45-47, finite functions of surface electric densities and magnetic currents (Equation 72). Then we consider the Fourier transform to find the spectral density $\vec{a}_{\pm}(\varepsilon)$ (Equation 73).

We will take the inverse Fourier to transform from this system, for this we multiply the left side of the equation by complex conjugate functions $e^{-i\varepsilon' u}$ and integrate the resulting

expression over the space where currents exist, i.e. via variable and ranging from -1 to 1 (across the width of the cavity) (Equation 74). We'll use (Equation 75) and get (Equation 76). Let us consider the vector potential through the spectral density (Equation 77). We shall to solve the internal integral (Equation 78).

Each function $t(z)$ regular in circle $|z - a| < \rho$ decomposes into a power series converging in the circle (Equation 79) where the coefficient is determined by the formulas (Equation 80) or (Equation 81) where $\rho_1 < \rho$ this Taylor power series in vicinity of point $z = a$ (Equation 82). Let us multiply the first equation by vector $\vec{v}_0$ and replace $\vec{\varepsilon} \rightarrow \varepsilon$ (Equation 83). Then, combining the spectral densities for magnetic and electric currents, we obtain (Equation 84). We shall apply the mean value theorem (Equation 85). The average value $\frac{1}{b-a} \int_a^b f(x) dx$ on the interval $[-a; b]$ (Equation 86).

The spectral density depends on the distribution of external current. In its turn, the distribution of external current on the tape is associated with fulfillment of the boundary conditions of impedance type on the surface of tape above and below, i.e. on the coordinate plane $v=0$, for tangential components of the fields $\vec{E}_t$ and $\vec{H}_t$ the relation is satisfied (Equation 88) where H – are waves with capacitive nature, E – are waves with inductive nature.

The presence of the surface impedance can be considered as a condition for the existence of a surface wave in second medium. A plane impedance (dielectric structure) tape is virtually modeled as waveguide, waveguide-type waves can naturally exist in it. Outside of tape (layer) there is a surface wave above and below. Critical frequencies for a planar dielectric waveguide are (Equation 87). The condition $w = w_{kr}^{(n)}$ meets condition $\varphi = \varphi^*$ the Brewster angle, where φ – is angle of incidence of the wave. At $\varphi = \varphi^*$ outside the layer, an ordinary uniform wave propagates along the tape, and consequently, a longitudinal component at the boundary H_y or E_y is lost, which means that there is no dependence on the type of polarization, i.e. at $f \leq f_{kr} (\varphi \leq \varphi^*)$ waveguide type wave inside the tape (layer) is destroyed. Outside the tape on plane $v=0$, the following expressions are valid (Equation 89). For current systems outside tape $|u| > 1$, integral equations (Equation 89) are equal 0 (Equation 90).

The system of Equations 89, 90 is the initial point for solving asymptotic or numerical

methods. Then, it is solved by the method of moments; for this case, unknown functions of the current density are expanded in a series according to the complete system of functions orthogonal to width of the tape.

For numerical analysis of spectral functions by (Equations 89, 90, a software simulator was created in RAD Studio development environment (Pichugin *et al.*, 2015; Skvortsov *et al.*, 2012; Skvortsov and Karizin, 2017). The software simulator was designed to simulate spectral characteristics in long and short wavelengths of the spectrum (Figures 3-4). Performing a numerical analysis, we can notice that a jump occurs on the surface of the magnetic field and electric field: the jump in magnetic field is equal to the surface density of external electric current (Equation 91). The jump in electric field is equal to surface density of external magnetic current.

4. CONCLUSIONS:

During the research, corresponding algorithm was developed for asymptotic or numerical simulation of diffraction pattern of electromagnetic waves in multi-wave mode of reception on an infinitely extended object in free space, as well as a software method for calculating diffraction characteristics at frequencies exceeding the fundamental frequency, taking into account inhomogeneity of electromagnetic fields in these conditions. The developed methodology for researching characteristics of electromagnetic waves during diffraction of electromagnetic waves at objects can be used in design and development of radar systems capable of working adequately in the conditions of electromagnetic environment under consideration. Based on the provisions and conclusions, a software tool was developed. The results of the study can become the basis for development of means of electromagnetic compatibility of radar objects. Based on the recommendations developed in this study, new methods can be developed to improve the efficiency of radar systems.

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$$\dot{\vec{E}} = \dot{\vec{E}}_0 \left(\vec{x}_0 \cos(\alpha_1) + \vec{y}_0 \cos(\alpha_2) + \vec{z}_0 \cos(\alpha_3) \right) e^{-i\kappa \left(x \cos(\gamma_1) + y \cos(\gamma_2) + z \cos(\gamma_3) \right)} \quad (\text{Eq. 1})$$

$$\dot{\vec{H}} = \dot{H}_0 \left(\sqrt{\frac{\dot{\mu}\mu_0}{\dot{\varepsilon}\varepsilon_0}} \right)^{-1} \left(\vec{x}_0 \cos(\beta_1) + \vec{y}_0 \cos(\beta_2) + \vec{z}_0 \cos(\beta_3) \right) e^{-i\kappa \left(x \cos(\gamma_1) + y \cos(\gamma_2) + z \cos(\gamma_3) \right)} \quad (\text{Eq. 2})$$

$$\left. \begin{array}{l} \vec{H} = \vec{H}_{pad} + \vec{H}_{dif} \\ \vec{E} = \vec{E}_{pad} + \vec{E}_{dif} \end{array} \right\} \left\{ \begin{array}{l} \vec{E} = -i\omega\mu_a \vec{A}^3 + \frac{1}{i\omega\varepsilon_a} \text{grad div } \vec{A}^3 - \text{rot } \vec{A}^M \\ \vec{H} = -i\omega\varepsilon_a \vec{A}^3 + \frac{1}{i\omega\mu_a} \text{grad div } \vec{A}^M - \text{rot } \vec{A}^3 \end{array} \right. \quad (\text{Eq. 3})$$

$$\vec{A}(x, y, z) = \vec{A}(x, z) e^{-ip_0 y \frac{1}{a}} = \vec{A}(x, z) e^{-ip_0 \frac{y}{a}} = \vec{A}(x, z) e^{-ik_0 a \sin \theta \cos \varphi_0 y \frac{1}{a}} = \vec{A}(x, z) e^{-ik_0 \sin \theta \cos \varphi_0 y} \quad (\text{Eq. 4})$$

$$p_0 = k \sin \theta \cos \varphi_0 = k_0 a \sin \theta \cos \varphi_0 \quad (\text{Eq. 5})$$

$$\vec{A} \left(\frac{x}{a}; \frac{z}{a} \right) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \vec{a}_{\pm}(\varepsilon) e^{i(\varepsilon \frac{x}{a} \pm w \frac{z}{a})} d\varepsilon \quad (\text{Eq. 6})$$

$$\vec{a}_{\pm}(\varepsilon) = \frac{1}{2} \frac{z_0}{i\omega} (\vec{J}^m(\varepsilon) \mp \vec{M}^{-1}[\vec{V}_0] \vec{J}^e(\varepsilon)) \quad (\text{Eq. 7})$$

$$w = \sqrt{k^2 - \varepsilon^2 - p_0^2} \quad (\text{Eq. 8})$$

$$w = \sqrt{k^2 - \varepsilon^2 - (k \sin \theta \cos \varphi_0)^2} \quad (\text{Eq. 9})$$

$$\vec{A}(x, y, z) = \int_V \vec{J}^{ct}(x', y', z') G(x, y, z, x', y', z') dx' dy' dz' \quad (\text{Eq. 10})$$

$$G(x, y, z, x', y', z') = \frac{1}{(2\pi)^3} \iiint_{-00}^{00} \frac{e^{-i\gamma_1(x-x') - i\gamma_2(y-y') - i\gamma_3(z-z')}}{x_1^2 + x_2^2 + x_3^2 - k^2} dx_1 dx_2 dx_3 \quad (\text{Eq. 11})$$

$$\gamma = \sqrt{x_1^2 + x_2^2 - k^2} \quad (\text{Eq. 12})$$

$$w = \sqrt{k^2 - \varepsilon^2 - p_0^2} \quad (\text{Eq. 13})$$

$$-x_2 = \sqrt{x_1^2 + x_3^2 - k^2} = \gamma \quad (\text{Eq. 14})$$

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2} + k^2 A = \vec{j}^{st} \quad (\text{Eq. 15})$$

$$A(x, y, z) = X(x)Y(y)Z(z) \quad (\text{Eq. 16})$$

$$\vec{A}(x, y, z) = \frac{1}{(2\pi)^3} \iiint_{-00}^{00} g(x_1, x_2, x_3) e^{-ix_1x - ix_2y - ix_3z} dx_1 dx_2 dx_3 \quad (\text{Eq. 17})$$

$$Y(y), Z(z) = X(x) = \frac{1}{\sqrt{2\pi}} \int_{x_1=-00}^{00} g(x_1) e^{-ix_1x} dx_1 \quad (\text{Eq. 18})$$

$$g(x_1, x_2, x_3) = -\frac{1}{(2\pi)^3} \iiint_{-00}^{00} \frac{j_{(x,y,z)}^{ct} e^{-ix_1x - ix_2y - ix_3z}}{k^2 - x_1'^2 - x_2'^2 - x_3'^2} dx dy dz \quad (\text{Eq. 19})$$

$$\frac{\partial^2 \vec{A}}{\partial x^2} + \frac{\partial^2 \vec{A}}{\partial y^2} + \frac{\partial^2 \vec{A}}{\partial z^2} + k^2 \vec{A} = -\vec{j}^{st} \quad (\text{Eq. 20})$$

$$A(x, y, z) = X(x)Y(y)Z(z) \quad (\text{Eq. 21})$$

$$X(x) = \frac{1}{\sqrt{2\pi}} \int_{x_1=-00}^{00} g(x_1) e^{-ix_1x} dx_1 \quad (\text{Eq. 22})$$

$$Y(y) = \frac{1}{\sqrt{2\pi}} \int_{x_2=-00}^{00} g(x_2) e^{-ix_2y} dx_2 \quad (\text{Eq. 23})$$

$$Z(z) = \frac{1}{\sqrt{2\pi}} \int_{x_3=-00}^{00} g(x_3) e^{-ix_3z} dx_3 \quad (\text{Eq. 24})$$

$$v_{av} = \frac{\omega}{x_1} \quad (\text{Eq. 25})$$

$$\frac{\partial^2 \vec{A}}{\partial x^2} + \frac{\partial^2 \vec{A}}{\partial z^2} - \vec{A}(x, z) e^{-i\frac{p_0 y}{a}} \left(\frac{-ip_0}{a} \right)^2 + k_0^2 \vec{A}(x, z) - \vec{j}^{ct} \quad (\text{Eq. 26})$$

$$\frac{\partial^2 \vec{A}(x, y, z)}{\partial x^2} + \frac{\partial^2 \vec{A}(x, y, z)}{\partial z^2} - \vec{A}(x, z) e^{-i\frac{p_0 y}{a}} \left(k_0^2 - \left(\frac{p_0}{a} \right)^2 \right) - \vec{j}^{ct} \quad (\text{Eq. 27})$$

$$\frac{\partial^2 \vec{A}(x, y, z)}{\partial x^2} + \frac{\partial^2 \vec{A}(x, y, z)}{\partial z^2} + \vec{A}(x, z) \frac{1}{a} (k_0^2 - p_0^2) = -\vec{j}^{st} \quad (\text{Eq. 28})$$

$$Z \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_0^2 \right) XY + XY \frac{\partial^2}{\partial z^2} Z = 0 \quad (\text{Eq. 29})$$

$$\frac{1}{XY} \left(Y \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y^2} + k_0^2 XY \right) + \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} = 0 \quad (\text{Eq. 30})$$

$$Y = A(x, z) e^{-i\frac{p_0 y}{a}} \quad (\text{Eq. 31})$$

$$\frac{1}{XY} \left(Y \frac{\partial^2 X}{\partial x^2} + XY \left(k_0^2 - \left(\frac{p_0}{a} \right)^2 \right) \right) + \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} = 0 \quad (\text{Eq. 32})$$

$$\frac{1}{XY} \left(Y \frac{\partial^2 X}{\partial x^2} + XY \left(k_0^2 - \left(\frac{P_0}{a} \right)^2 \right) \right) = -\frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} \quad (\text{Eq. 33})$$

$$\varepsilon^2 = \Gamma^2 \quad (\text{Eq. 34})$$

$$\begin{cases} \frac{1}{XY} \left(Y \frac{\partial^2 X}{\partial x^2} + XY \left(k_0^2 - \left(\frac{P_0}{a} \right)^2 \right) \right) = \varepsilon^2 \\ -\frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} = \varepsilon^2 \end{cases} \quad (\text{Eq. 35})$$

$$\begin{cases} Y \frac{\partial^2 X}{\partial x^2} + XY \left(k_0^2 - \left(\frac{P_0}{a} \right)^2 \right) = XY \varepsilon_0^2 \\ \frac{\partial^2 Z}{\partial z^2} = -Z \varepsilon_0^2 \end{cases} \quad (\text{Eq. 36})$$

$$\begin{cases} Y \frac{\partial^2 X}{\partial x^2} + XY \left(k_0^2 - \varepsilon_0^2 - \left(\frac{P_0}{a} \right)^2 \right) = 0 \\ \frac{\partial^2 Z}{\partial z^2} + Z \varepsilon^2 = 0 \end{cases}; \quad (\text{Eq. 37})$$

$$\begin{cases} \frac{\partial^2 X}{\partial x^2} + X \left(k_0^2 - \varepsilon_0^2 - \left(\frac{P_0}{a} \right)^2 \right) = 0 \\ \frac{\partial^2 Z}{\partial z^2} + Z \varepsilon_0^2 = 0 \end{cases} \quad (\text{Eq. 38})$$

$$\begin{cases} w_0^2 = k_0^2 - \varepsilon_0^2 - \left(\frac{P_0}{a} \right)^2 \\ w = (k_0 a)^2 - (\varepsilon_0 a)^2 - (P_0)^2 \\ w = k^2 - \varepsilon^2 - P_0^2 \end{cases} \quad (\text{Eq. 39})$$

$$\frac{\partial^2 \vec{A}(x, y, z)}{\partial x^2} + \frac{\partial^2 \vec{A}(x, y, z)}{\partial y^2} + \frac{\partial^2 \vec{A}(x, y, z)}{\partial z^2} + k_0^2 \vec{A}(x, y, z) = 0 \quad (\text{Eq. 40})$$

$$\vec{A}(x, y, z) = \vec{A}(x, z) e^{-\frac{ip_0 y}{a}} \quad (\text{Eq. 41})$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \vec{A}(x, z) e^{-\frac{ip_0 y}{a}} - \vec{A}(x, z) e^{-\frac{ip_0 y}{a}} \left(\frac{p_0}{a} \right)^2 + k_0^2 \vec{A}(x, z) e^{-\frac{ip_0 y}{a}} = 0 \quad (\text{Eq. 42})$$

$$\left(\frac{\partial}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \vec{A}(x, z) + \vec{A}(x, z) \frac{1}{a^2} (k^2 - p_0^2) = -j^{\text{ct}} \quad (\text{Eq. 43})$$

$$k = k_0 a \quad (\text{Eq. 44})$$

$$\vec{A}(x, z) = \vec{X}(x) \vec{Z}(z) \quad (\text{Eq. 45})$$

$$\vec{Z}(z) \frac{\partial^2 \vec{X}(x)}{\partial x^2} + \vec{X}(x) \frac{\partial^2 \vec{Z}(z)}{\partial z^2} + \vec{X}(x) \vec{Z}(z) \frac{1}{a^2} (k^2 - p_0^2) = -j^{\text{st}} \quad (\text{Eq. 46})$$

$$Z \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Z}{\partial z^2} + XZ \frac{1}{a^2} (k^2 - p_0^2) = 0 \quad (\text{Eq. 47})$$

$$\begin{cases} \frac{\partial^2 Z}{\partial z^2} + Z \frac{1}{a^2} (k^2 - \varepsilon^2 - p_0^2) = 0 \\ \frac{\partial^2 X}{\partial x^2} + X \frac{1}{a^2} \varepsilon^2 = 0 \end{cases} \quad (\text{Eq. 48})$$

$$u = \frac{x}{a} \quad (\text{Eq. 49})$$

$$v = \frac{z}{a} \quad (\text{Eq. 50})$$

$$\begin{cases} \frac{\partial^2 Z}{\partial v^2} + Z w^2 = -\vec{j}^{st} \\ \frac{\partial^2 X}{\partial u^2} + X \varepsilon^2 = -\vec{j}^{st} \end{cases} \quad (\text{Eq. 51})$$

$$w = \sqrt{k^2 - \varepsilon^2 - p_0^2}, \quad (\text{Eq. 52})$$

$$\vec{X}(u) = \frac{1}{\sqrt{2\pi}} \int_{\varepsilon=-\infty}^{\infty} \vec{g}(\varepsilon) e^{-i\varepsilon u} d\varepsilon \quad (\text{Eq. 53})$$

$$\vec{Z}(v) = \frac{1}{\sqrt{2\pi}} \int_{w=-\infty}^{\infty} \vec{g}(w) e^{-i w v} dw \quad (\text{Eq. 54})$$

$$\vec{A}(u, v) = \frac{1}{2\pi} \int_{w=-\infty}^{\infty} \int_{\varepsilon=-\infty}^{\infty} \vec{g}(\varepsilon, w) e^{-i\varepsilon u - i w v} d w d \varepsilon \quad (iu) \quad (\text{Eq. 55})$$

$$k^2 = \varepsilon^2 + p_0^2 + \gamma^2 \quad (\text{Eq. 56})$$

$$\gamma = \gamma_0, -\gamma^2 = w^2, w_1 = -i\gamma, w_2 = i\gamma \quad (\text{Eq. 57})$$

$$L = \int_{w=-\infty}^{\infty} e^{-i w v} \vec{g}(w) dw \quad (\text{Eq. 58})$$

$$\vec{A}(x, z) = \int_s \vec{j}^{ct}(x', z') G(x, z; x', z') dx' dz' \quad (\text{Eq. 59})$$

$$\vec{A}(x, z) = \int_s \vec{j}^{ct}(x', z') G(x, z; x', z') dx' dz' \quad (\text{Eq. 60})$$

$$-\frac{1}{2\pi} \iint_{-\infty}^{\infty} \vec{j}^{st}(u, v) e^{i\varepsilon' u + i w' v} du dv = \iint_{-\infty}^{\infty} (k^2 - p_0^2 - \varepsilon^2 - w^2) g(\varepsilon, w) \delta(\varepsilon' - \varepsilon) \delta(w' - w) d\varepsilon dw \quad (\text{Eq. 61})$$

$$-\frac{1}{2\pi} \iint_{-\infty}^{\infty} \vec{j}^{st}(u, v) e^{i\varepsilon' u + i w' v} du dv = (k^2 - p_0^2 - \varepsilon^2 - w^2) g(\varepsilon', w') \quad (\text{Eq. 62})$$

$$(k^2 - \varepsilon^2 - p_0^2 - w^2) g(\varepsilon, w) = -\frac{1}{2\pi} \iint_{-\infty}^{\infty} \vec{j}^{st}(u', v') e^{i\varepsilon u' + i w v'} du' dv' \quad (\text{Eq. 63})$$

$$\begin{aligned} \vec{A}(u, v) &= \int_s \vec{j}^{st}(u', v') G(u, v, u', v') du' dv' = \\ &= \int_s \vec{j}^{st}(u', v') \left(\frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} \frac{e^{i(u-u')\varepsilon - i(v-v')w}}{k^2 - \varepsilon^2 - p_0^2 - w^2} d\varepsilon dw \right) du' dv' \end{aligned} \quad (\text{Eq. 64})$$

$$L = - \int_{-\infty}^{\infty} \frac{e^{-i(v-v')w}}{(w-iw')(w+iw')} dw \quad (\text{Eq. 65})$$

$$w' = \sqrt{k^2 - \varepsilon^2 - p_0^2} \quad (\text{Eq. 66})$$

$$w' = \sqrt{\varepsilon^2 + p_0^2 - k^2} \quad (\text{Eq. 67})$$

$$L_{(z-z')<0} = \int_{\Gamma} t(w) dw = \sum_{j=1}^n \int_{\gamma_j} f(w) dw + \int_{\gamma_R} f(w) dw \quad (\text{Eq. 68})$$

$$\int_F f(w) dw = 2\pi i \left(\sum_{j=1}^n \overset{res}{w=w_j} t(w) + \overset{res}{w=\infty} t(w) \right) \quad (\text{Eq. 69})$$

$$L_{(v-v')<0} = 2\pi i \overset{res}{w=-i w'} f(w) = \frac{\pi e^{-w'(v-v')}}{w'} \quad (\text{Eq. 70})$$

$$G(u, v; u', v') = \frac{1}{4\pi} \int_{-\infty}^{\infty} \frac{e^{-i(u-u')\varepsilon \pm w'(v-v')}}{w'} d\varepsilon \quad (\text{Eq. 71})$$

$$\begin{cases} \vec{j}^e(u) = \frac{iw}{2\pi z_0} \tilde{M} \left[\int_{-\infty}^{\infty} [\vec{v}_0 \vec{a}_+(\varepsilon)] e^{i\varepsilon u} d\varepsilon - \int_{-\infty}^{\infty} [\vec{v}_0 \vec{a}_-(\varepsilon)] e^{i\varepsilon u} d\varepsilon \right] \\ \vec{j}^m(u) = (-) \frac{iw}{2\pi z_0} \int_{-\infty}^{\infty} [\vec{v}_0 [\vec{v}_0 \vec{a}_+(\varepsilon)]] e^{i\varepsilon u} d\varepsilon - \frac{iw}{2\pi z_0} \int_{-\infty}^{\infty} [\vec{v}_0 [\vec{v}_0 \vec{a}_-(\varepsilon)]] e^{i\varepsilon u} d\varepsilon \end{cases} \quad (\text{Eq. 72})$$

$$\begin{cases} (\pm) \frac{z_0}{iw} \tilde{M}^{-1} \vec{j}^e(u)_{\pm} = \frac{1}{2\pi} \int_{-\infty}^{\infty} [\vec{v}_0 \vec{a}_{\pm}(\varepsilon)] e^{i\varepsilon u} d\varepsilon \\ (-) \frac{z_0}{iw} \vec{j}^m(u)_{\pm} = \frac{1}{2\pi} \int_{-\infty}^{\infty} [\vec{v}_0 [\vec{v}_0 \vec{a}_{\pm}(\varepsilon)]] e^{i\varepsilon u} d\varepsilon \end{cases} \quad (\text{Eq. 73})$$

$$\begin{cases} (\pm) \frac{z_0}{iw} \tilde{M}^{-1} \int_{-1}^1 \vec{j}^e(u) e^{-i\varepsilon' u} du = \int_{-1}^1 \frac{1}{2\pi} \int_{-\infty}^{\infty} [\vec{v}_0 \vec{a}_{\pm}(\varepsilon)] e^{i(\varepsilon-\varepsilon')u} du d\varepsilon \\ (-) \frac{z_0}{iw} \int_{-1}^1 \vec{j}^m(u) e^{-i\varepsilon' u} du = \int_{-1}^1 \frac{1}{2\pi} \int_{-\infty}^{\infty} [\vec{v}_0 [\vec{v}_0 \vec{a}_{\pm}(\varepsilon)]] e^{i(\varepsilon-\varepsilon')u} du d\varepsilon \end{cases} \quad (\text{Eq. 74})$$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i(\varepsilon-\varepsilon')u} du d\varepsilon = \delta(\varepsilon - \varepsilon') = \begin{cases} \infty; \text{at } \varepsilon' = \varepsilon \\ 0; \text{at } \varepsilon' \neq \varepsilon \end{cases} \quad (\text{Eq. 75})$$

$$\begin{cases} (\pm) \frac{z_0}{iw} \tilde{M}^{-1} \int_{-1}^1 \vec{j}^e(u) e^{-i\varepsilon' u} du = \int_{-1}^1 [\vec{v}_0 \vec{a}_{\pm}(\varepsilon)] \delta(\varepsilon - \varepsilon') d\varepsilon \\ (-) \frac{z_0}{iw} \int_{-1}^1 \vec{j}^m(u) e^{-i\varepsilon' u} du = \int_{-1}^1 [\vec{v}_0 [\vec{v}_0 \vec{a}_{\pm}(\varepsilon)]] \delta(\varepsilon - \varepsilon') d\varepsilon \end{cases} \quad (\text{Eq. 76})$$

$$\vec{A}(u, v) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(\varepsilon, w) e^{-iu\varepsilon - iwv} dw d\varepsilon \quad (\text{Eq. 77})$$

$$L = \int_{-\infty}^{\infty} \frac{e^{-iwv} dw}{k^2 - \varepsilon^2 - p_0^2 - w^2} = \int_{-\infty}^{\infty} \frac{e^{-iwv} dw}{(-)(w-iw')(w+iw')} \quad (\text{Eq. 78})$$

$$t(z) = \sum_{n=0}^{\infty} C_n (z-a)^n \quad (\text{Eq. 79})$$

$$C_n = \frac{1}{n!} t^n(a) \quad (\text{Eq. 80})$$

$$C_n = \frac{1}{2\pi i} \int_{|\varepsilon-a|=\rho_1} \frac{t(\varepsilon) d\varepsilon}{(\varepsilon-a)^{n+1}} \quad (\text{Eq. 81})$$

$$\left. \begin{aligned} & \sum_{n=0}^{\infty} C_n (z-a)^n \\ & \sum_{n=-1}^{-\infty} C_n (z-a)^n = \sum_{n=1}^{\infty} \frac{C_{-n}}{(z-a)^n} \end{aligned} \right\} \quad (\text{Eq. 82})$$

$$\begin{cases} (\mp) \frac{z_0}{2iw} \tilde{M}^{-1} \left[\vec{v}_0 \left(\int_{-1}^1 \vec{j}^e(u) e^{-i\varepsilon u} du \right) \right] = a_{\pm}(\varepsilon) \\ \frac{z_0}{2iw} \int_{-1}^1 \vec{j}^m(u) e^{-i\varepsilon u} du = a_{\pm}(\varepsilon) \end{cases} \quad (\text{Eq. 83})$$

$$a_{\pm}(\varepsilon) = \frac{z_0}{2iw} \left[\vec{j}^m(\varepsilon) \mp \tilde{M}^{-1} [\vec{v}_0 \vec{j}^e(\varepsilon)] \right] \quad (\text{Eq. 84})$$

$$a_{\pm}(\varepsilon) = \left(\vec{a}^m_{\pm}(\varepsilon) + \vec{a}^e_{\pm}(\varepsilon) \right) \frac{1}{2} \quad (\text{Eq. 85})$$

$$\vec{A}(u, v) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \vec{a}_{\pm}(\varepsilon) e^{i\varepsilon u \pm iwv} d\varepsilon \quad (\text{Eq. 86})$$

$$w_{\text{kp}}^{(n)} = \frac{n\pi}{d} \frac{c}{\sqrt{\varepsilon_1 \mu_1 - G_2 \mu_2}} = 2\pi f \quad (\text{Eq. 87})$$

$$\left. \begin{aligned} \vec{E}_t(x, y, +0) &= z z_0 [\vec{v}_0, \vec{H}_t(x, y, +0)] \\ \vec{E}_t(x, y, -0) &= z z_0 [\vec{v}_0, \vec{H}_t(x, y, -0)] \end{aligned} \right\} \text{as } |x| < a \text{ or } |a| < 1 \quad (\text{Eq. 88})$$

$$\left\{ \begin{aligned} \frac{1}{4\pi} \int_{-\infty}^{\infty} \{ (Z\tilde{M} + \tilde{I}) \vec{j}^m(\varepsilon) \}^{i\varepsilon u} d\varepsilon &= (-) 2z \vec{H}_t^u \\ \frac{1}{4\pi} \int_{-\infty}^{\infty} \{ (\tilde{M} + Z\tilde{I}) \vec{j}^l(\varepsilon) \}^{i\varepsilon u} d\varepsilon &= -2[\vec{n} \vec{H}_t^n] \end{aligned} \right. \quad (\text{Eq. 89})$$

$$\left\{ \begin{aligned} \frac{1}{4\pi} \int_{-\infty}^{\infty} \vec{j}^m(\varepsilon) l^{i\varepsilon u} du &= 0 \\ \frac{1}{4\pi} \int_{-\infty}^{\infty} \vec{j}^l(\varepsilon) l^{i\varepsilon u} du &= 0 \end{aligned} \right. \quad (\text{Eq. 90})$$

$$\left[V_0 \left(H_t^{pad}(u_2 + 0) - H_t^{pad}(u_2 - 0) \right) \right] = j_i(u) \quad (\text{Eq. 91})$$

Figure 1. Waveguide-slot type of radar system

Figure 2. The model of the antenna system (diffuser)

Figure 3. Spectral Characterization Analysis Results

Figure 4. The study of spectral characteristics in a software simulator