ORIGEM E PROPAGAÇÃO DE SÓLITONS DE TEMPERATURA COM TRANSFERÊNCIA DE CALOR DE ONDA NA ÁREA LIMITADA DURANTE PROCESSOS TECNOLÓGICOS ADICIONAIS**ORIGINATION AND PROPAGATION OF TEMPERATURE SOLITONS WITH WAVE HEAT TRANSFER IN THE BOUNDED AREA DURING ADDITIVE TECHNOLOGICAL PROCESSES****ВОЗНИКНОВЕНИЕ И РАСПРОСТРАНЕНИЕ ТЕМПЕРАТУРНЫХ СОЛИТОНОВ С ВОЛНОВЫМ ТЕПЛООБМЕНЕМ В ОГРАНИЧЕННОЙ ОБЛАСТИ В ХОДЕ АДДИТИВНЫХ ТЕХНОЛОГИЧЕСКИХ ПРОЦЕССОВ**

FORMALEV, Vladimir F. ^{1*}; KOLESNIK, Sergey A. ²; KUZNETSOVA, Ekaterina L. ³; RABINSKIY, Lev N. ⁴;

^{1,2,3} Moscow Aviation Institute (National Research University), Institute No. 9 "General Engineering", Moscow – Russian Federation

⁴ Moscow Aviation Institute (National Research University), Department of Material Sciences, Moscow – Russian Federation

* Correspondence author
e-mail: formalev38@yandex.ru

Received 30 August 2019; received in revised form 24 October 2019; accepted 27 October 2019

RESUMO

Neste trabalho, com base em análises de problemas na transferência de calor por ondas em corpos delimitados, a teoria de ondas isoladas termicamente (solitons) foi desenvolvida para investigar os processos de transferência de calor na vizinhança inicial e nas proximidades do corpo delimitado. As escalas de tempo são proporcionais ao tempo de relaxamento (nanossegundos) e as escalas da variável espacial são medidas em nanômetros. Uma nova solução analítica para a transferência de calor por ondas foi obtida com base na equação do calor do tipo hiperbólico sob a ação de vários solitons, com base nos quais foram analisadas a interação dos solitons entre si, a absorção e a reflexão dos solitons a partir dos limites do corpo. Tendo analisado um grande volume de resultados, verificou-se que os solitons térmicos são refletidos diferentemente dos solitons mecânicos, uma vez que a energia térmica do soliton é primeiro absorvida pelo limite de isolamento térmico nas paredes com isolamento térmico e, em seguida, a energia é rejeitada pela condutividade térmica na direção oposta. Verificou-se que a diferença de temperatura no interior do soliton é negativa na direção de condução e positiva na direção refletida. Os resultados podem ser utilizados na interação térmica de radiação de alta potência com superfícies sólidas, bem como em problemas da mecânica quântica.

Palavras-chave: *sóliton térmico, velocidade da onda térmica, corpo limitado, condutividade térmica, equação de calor do tipo hiperbólico.*

ABSTRACT

Within this work, based on analyses of problems on wave heat transfer in bounded bodies, the theory of thermally isolated waves (solitons) is developed to investigate the heat transfer processes in the initial time vicinity and in the vicinity of the bounded body, that is the time scales are commensurate with the relaxation time (nanoseconds), and the scales of the spatial variable are measured in nanometers. A new analytical solution of the wave heat transfer based on the heat conduction equation of hyperbolic type under the action of a series of solitons was received, based on which the interaction of individual solitons with each other, absorption and reflection of the solitons from the body boundaries was analyzed. Analysis of a large number of results made clear that thermal solitons reflect not as mechanical ones, since first there is absorption of the soliton thermal energy by the heat-insulated boundary on the heat-insulated walls, and then the energy is rejected by the thermal conductivity in the opposite direction. It was found that the temperature gradient inside the soliton is negative in the forward direction and positive in the reflected direction. The results of the paper can

be used in thermal interaction of high-power radiation with solid surfaces, as well as in the problems of quantum mechanics.

Keywords: *thermal soliton, heatwave velocity, bounded body, thermal conductivity, hyperbolic type heat equation.*

АННОТАЦИЯ

В рамках данной работы, на основе анализа проблемы волнового теплообмена с ограничением тепла, была разработана теория термоизолированных волн (солитонов) для изучения процессов теплообмена около начального момента времени и вблизи ограниченного тела, т.е. процесс, когда временные масштабы соизмеримы со временем релаксации (наносекундами), и масштабы пространственной переменной измеряются в нанометрах. Новое аналитическое решение волнового теплообмена было получено на основе уравнения теплопроводности гиперболического типа под действием ряда солитонов, на базе которых было проанализировано взаимодействие солитонов друг с другом, поглощение и отражение солитонов от границ тела. Проанализировав большой объем результатов было установлено, что тепловые солитоны отражаются не так, как механические, ввиду того, что сначала происходит поглощение тепловой энергии солитона теплоизолирующей границей на теплоизолированных стенках, а затем энергия отбрасывается теплопроводностью в обратном направлении. Было обнаружено, что перепад температуры внутри солитона отрицательный в проводящем направлении и положительный в отраженном направлении. Результаты работы могут использоваться при тепловом взаимодействии мощного излучения с твердыми поверхностями, а также в задачах квантовой механики.

Ключевые слова: *тепловой солитон, скорость тепловой волны, ограниченное тело, теплопроводность, уравнение теплопроводности гиперболического типа.*

1. INTRODUCTION

It is well known that heat transfer with convection, thermal conductivity, and radiation at the micro-level occurs by individual quanta, the time intervals between which depend significantly on the free path length between the molecules, as well as on the degree of excitation of the degrees of freedom, mainly vibrational for the solids. Hence it immediately follows that there is a time lag of the heat flow from the temperature gradient. The lag time between the heat flux and the temperature gradient is called the relaxation time. It is very small for solids (Equation 1) (Lykov, 1967).

There are conditions for the occurrence of a heatwave in a solid body since the relaxation time accounted for in the heat transfer, leads to a finite velocity of the heat flow propagation, in contrast to the infinite velocity of heat propagation, determined by the classical Fourier's law. Such wave heat transfer is characterized by the presence of moving the first kind discontinuity of the temperature field with decreasing discontinuity amplitudes due to thermal energy dissipation. If you are considering not a continuous (over time) heat input into the solid, but a pulse one, which takes place in real heat exchange, then a series of isolated pulses (solitons) having front and rear discontinuity, at wave motion inside the body, should

hypothetically retain the basic geometric and thermal physical properties, taking into account the significant dissipation of heat (Belov *et al.*, 2014, Sokolov and Ryabinov, 2015).

Theoretically, the wave heat transfer is described by the heat equation of hyperbolic type with the wave term (the second temperature derivative in time) and the dissipative term (the first temperature derivative in time). Based on this equation the wave heat transfer was studied in papers (Zarubin and Kuvyrkin, 2000; Kartashov, 2001; Shashkov *et al.*, 2004; Zarubin *et al.*, 2018). In a paper (Kartashov, 2016) the analysis of wave heat transfer based on two relaxation times (by heat transfer and elasticity) was carried out.

It should be noted that wave heat transfer occurs not only because of the presence of the relaxation time but also because of the nonlinearity of the thermal physical properties of the medium, depending on temperature (Samarskiy *et al.*, 1987; Kolesnik, 2014; Kolesnik *et al.*, 2015; Gidasov and Severina, 2015; Gidasov *et al.*, 2016; Lurie *et al.*, 2017; Bulychev *et al.*, 2017; Bulychev *et al.*, 2018a; Lomakin *et al.*, 2018; Gidasov *et al.*, 2018; Bulychev *et al.*, 2018b). Paper (Formalev, 2012) contains the first theoretically obtained thermal shock waves in nonlinear medium and the conditions of their occurrence.

The purpose of this paper is a theoretical study of the propagation and interaction of isolated thermal pulses (solitons) in bounded bodies, determining of the properties of thermal solitons at reflection from the body boundaries, collision of solitons reflected from the right boundary and moving in a straight direction, their dissipation and the final formation of the temperature distribution under the action of a group of solitons.

2. MATERIALS AND METHODS

The problem is set to analyze the dynamics of motion and reflection of a series of isolated heat waves (solitons) of rectangular shape with the temperature amplitude T_0 , acting on the left boundary of a bounded rod, based on the wave equation of thermal conductivity. Such problems arise in quantum mechanics when the body is irradiated with powerful pulses of thermal energy of various origins.

Mathematically, the problem is formulated as follows (Equations 2 – 6), where τ_r – relaxation time – the time of the heat flux rate lag $q(x, t)$ from the temperature gradient in compliance with the law of heat conduction Vernotte-Cattaneo-Lykov, which is the Equation 7) in one-dimensional case. Based on Equation 7 the wave heat transfer Equation 2 was derived; $\eta(z)$ – Heaviside function ($\eta(z)=1$ at $z>0$, $\eta(z)=0$ at $z<0$); T_0 – absolute temperature value in a single heat pulse, acting during the time t_1 (this value is called the *pulse support*); d – time lag between the pulses; x, t – spatial and time variables; a – thermal conductivity m^2/s ; K – number of solitons.

Replacing Equation 8 the problem (Equations 2–6) can be reformulated in relation to function $T_1(x, t)$, but with zero initial condition $T_1(x, 0)=0$. Further replacing $T_1(x, t)$ by $T(x, t)$, the problem (Equations 2–6) can be cast without loss of generality with homogenous initial values (Equation 9). Herewith the final solution of Equation 1 and Equations 3–11 should be added with the value of T_e . As the relaxation time τ_r in solid bodies has order of $10^{-12} \div 10^{-14} s$, it is expedient (Equation 2, Equations 4–6 and Equation 8) to change to the non-dimensional variables, wherefore the Equation 1 is multiplied

by τ_r , and then divided by τ_r^2 . After that, authors derive the below non-dimensional variables (Equations 10–13). Embedding the non-dimensional variables Equation 9 in correlations (Equations 2, 4–6, 9) authors get the problem of wave heat transfer in a dimensionless form (replacing the non-dimensional variables $\bar{T}$, $\bar{t}$, $\bar{x}$, $\bar{l}$, $\bar{t}_1$, $\bar{d}$ with T , t , x , l , t_1 , d) (Equations 14 – 18).

As the velocity v of the heatwave is determined by Equations 19 (Lykov, 1967), from the equation, Equation 20 Equation 21 is obtained that is Equation 22 following that $\bar{t} = \bar{x}$. Thus, in the problem Equation 14–17 of the heat transfer the dimensionless special variable x coincides with the non-dimensional time variable t .

To solve the problem Equation 14–17 the source function method (Green) is applied. To get the Green function, $G(\bar{x}, \xi, \bar{t})$ it is needed to place the unit capacity source at a point of $\bar{x} = \xi$, and in compliance with the general theory formulate the problem to define this function, using the zero conditions for this function in the space-time boundary (Equations 23–27), where $\delta(\bar{x} - \xi)$ – the Dirac function. If the solution to the problem (Equations 23–27) for the function $G(x, \xi, t)$ is obtained, the solution to the problem (Equations 14–17) will be in time integral form $\tau < t$ (Equation 28) (Rabinskiy and Tushavina, 2019). By substitution Equation 29 Equation 23 is reduced to the parabolic type. Substituting Equation 29 in Equation 23 and putting to zero the ratio at $\partial V / \partial \bar{t}$, authors get $\mu = 1/2$, where the problem (Equations 23–27) turns into the problem (Equations 30–34). Problem (Equations 30–34) can be solved by the method of separation of variables with the presentation of Equation 35. Eigenvalues λ_n and their related proper functions $X_n(\bar{x})$ of the Sturm-Liouville problem is given by (Equation 36) and functions $\theta_n(\bar{t})$ equal to (Equations 37–38). Meeting Equation 38 the initial conditions Equation 33 and Equation 34, the ratios A_n , are obtained (Equation 39). Thus the function Equation (38) is given by Equation 40. Herewith the problem solution Equations 30–34 is the function (Equation 41), where Equation 42. Considering Equation 29 and Equation 41 the solution of Equations 23–25 for the source function

$G(\bar{x}, \bar{\xi}, \bar{t})$ in Equation 43. Substituting Equation 43 into Equation 28, authors get the solution of Equations 23–25 for the function $\bar{T}(x, t)$ given by Equation 44, where Equation 46. Equation 44 – is the solution of the original problem Equations 14–25 of the wave heat transfer for the group of K isolated heat waves (solitons).

3. RESULTS AND DISCUSSION:

Calculation of wave heat transfer in bounded bodies was done for a single thermal soliton with the pulse support in time $\bar{t}_1 = 0.5$, for two solitons with pulse supports $\bar{t}_1 = 0.5$ and period between them equal to Equation 47 and three solitons with pulse supports $\bar{t}_1 = 0.5$ and period between them equal to Equation 48. Results are presented in Figures 1-5 respectively, demonstrating the dynamics of isolated heat waves in bounded rod under conditions when the right boundary is heat isolated (Equation 49):

As the dimensionless time (Equation 50) and dimensionless spatial variable (Equation 51) equal each other, supports of the heat waves in time (time periods, in which the wave amplitude is above zero, and equals to zero outside these periods) are equal to supports with a spatial variable.

Input data for all results had the below values: time supports (as well as space supports) were taken equal to 0.5, dimensionless rod length (Equation 52) was taken equal to 3, maximum relative temperature (Equation 53) of solitons, entering the body on the left boundary, equals to 1 and the absolute temperature T_0 can be general.

Figures 1-2 presents dynamic motion of one soliton, having the front and the rear first kind discontinuity, herewith the temperature profile inside the support is decreasing $\left(\frac{\partial \bar{T}}{\partial \bar{x}} < 0\right)$ according to the heat power dissipation. Herewith the temperature before the front edge of the soliton equals the initial value (zero in this case), and behind the left edge, the temperature profile is different to the initial for some value (Figure 1).

Having reached the right end of the rod, the soliton energy is used to increase the temperature of the right thermally insulated boundary, and since the temperature to the left of the right boundary is approximately equal to the initial one, an isolated heatwave is formed in the

opposite direction (reverse wave). The edges in the reverse wave change their places – the rear edge becomes the front one, and the front edge is rear one.

Now the temperature on the new front edge is lower than that on the rear one, and between them, the temperature distribution is determined by Equation 34 with $\frac{\partial \bar{T}}{\partial \bar{x}} > 0$ (Figure 2). Having reached the left boundary of the rod, the temperature of the left boundary of the soliton takes a boundary value equal to zero (Figure 1, $\bar{t} = 6.5$).

Figure 3 shows the dynamics of motion, reflection, and interaction of two solitons with supports $\bar{t}_1 = 0.5$ and the period of time (and space) equal to 2.5. Temperature distribution, motion dynamics, and reflections are maintained in the same quality as for the variant in Figures 1-2 except for the interaction of the soliton reflected from the right boundary and moving in a straight direction from the left boundary to the right one (Figure 3, $\bar{t} = \{4; 4.5; 5\}$). In Figure 3 ($\bar{t} = 4.5$), the left and the right solitons converged into one, and its amplitude is the sum of the decreasing amplitudes of the "left" and the "right" solitons (Figure 3 ($\bar{t} = 4$)). The solitons diverge (Figure 3 ($\bar{t} = 5$)) preserving all qualitative and geometric characteristics except for the amplitudes, which go on to decrease (Figure 3 ($\bar{t} = 5$)) due to the dissipation of thermal energy.

Figures 4-5 shows the motion dynamics of three solitons with supports $t_1 = 0.5$ and periods of time (and the spatial variable) equal to $\bar{d} + \bar{t}_1 = 1$. It evidences all properties of single and double solitons. In addition, the interaction of a large number of solitons with the right body boundary shows that after reflection the number of "bursts" with the largest amplitudes is preserved (in this case there are three of them). Due to the smaller gap between the solitons compared to the options in Figures 1-3, the temperature at these intervals is much higher than the initial value, that is, a continuous temperature field is formed.

4. CONCLUSIONS:

On the basis of a new analytical solution of the problem of wave heat transfer in bounded bodies, a formation mechanism of an unsteady temperature field was defined using a series of

isolated thermal waves (solitons) at times proportional to several times of thermal relaxation, that is, at times of nanosecond durations with a heat depth of several nanometers. The mechanism of thermal solitons reflection from the opposite body boundary is set, according to which all soliton energy is absorbed by the heat -isolated wall, and then the inverse wave of the thermal soliton is formed, having the same geometric and kinematic properties as the direct wave taking into account the dissipation. In addition, the interaction mechanism between direct and reflected solitons is defined, according to which the total energy of the merged soliton (the temperature profile amplitude) is below the sum of soliton powers before their meeting. Herewith the temperature gradient inside the soliton is negative in the forward direction and positive in the reflected direction.

The merged solitons then diverge, continuing to move in the same directions as before the merger, preserving their geometric and kinematic characteristics, and taking into account the absorption (dissipation) of thermal energy.

5. ACKNOWLEDGMENTS:

The work was carried out with the financial support of the state project of the Ministry of Education and Science project code 2.9219.2017 / 8.9.

6. REFERENCES:

1. Belov, P.A., Kobets, L.P., Borodulin, A.S. *Inorganic Materials: Applied Research*, **2014**, 5(4): 403-406.
2. Bulychev, N.A., Kazaryan, M.A., Chaikov, L.L., Averyushkin, A.S., Krasovskii, V.I., Feofanov, I.N., Rasmagin, S.I., Zadorin, D.A., Zakharyan, R.A. *XIII International Conference on Pulsed Lasers and Laser Applications AMPL-2017*, **2017**, 12, 183-189.
3. Bulychev, N.A., Kazaryan, M.A., Kirichenko, M.N., Garibyan, B.A., Morozova, E.A., Chernov, A.A. *Proceedings – SPIE*, **2018a**, 10614(10614OZ), 2-6.
4. Bulychev, N.A., Kuznetsova, E.L., Bodryshev, V.V., Rabinskiy, L.N. *Nanoscience and Technology: an International Journal*, **2018b**, 9(2), 91-97. DOI: 10.1615/NanoSciTechnolIntJ.2018025703
5. Formalev, V.F. *High Temperature*, **2012**, 50(6), 744-748.
6. Gidaspov, V.Yu., Golubev, V.K., Severina, N.S. *Bulletin of the South Ural State University, Series: Mathematical Modelling, Programming and Computer Software*, **2016**, 9(3), 94-104.
7. Gidaspov, V.Yu., Moskalenko, O.A., Severina, N.S. *High Temperature*, **2018**, 56(5), 751-757.
8. Gidaspov, V.Yu., Severina, N.S. *High Temperature*, **2015**, 53(4), 526-530.
9. Kartashov, E.M. *Analytical Methods in Theory of Heat Conduction of Solid Bodies*, Moscow: Visshaya Shkola, **2001**.
10. Kartashov, E.M. *Journal of Engineering Physics and Thermophysics*, **2016**, 89(2), 346-356.
11. Kolesnik, S.A. *Mathematical Models and Computer Simulations*, **2014**, 6(5), 480-489.
12. Kolesnik, S.A., Formalev, V.F., Kuznetsova, E.L. *High Temperature*, **2015**, 53(1), 68-72.
13. Lomakin, E.V., Lurie, S.A., Rabinskiy, L.N., Solyaev, Y.O. *Reports of Physics*, **2018**, 63(4), 161-164.
14. Lurie, S.A., Solyaev, Y.O., Lizunova, D.V., Rabinskiy, L.N., Bouznik, V.M., Menshykov, O. *International Journal of Heat and Mass Transfer*, **2017**, 109, 511-519.
15. Lykov, A.V. *Theory of Heat Conduction*, Moscow: Vischaya Shkola, **1967**.
16. Rabinskiy, L.N., Tushavina, O.V. *INCAS Bulletin*, **2019**, 11(Special Issue), 203-211. DOI: 10.13111/2066-8201.2019.11.S.20.
17. Samarskiy, A.A., Galaktionov, V.A., Kurdyumov, S.P., Mihajlov, A.P. *Modes with aggravation in problems for quasilinear parabolic equations*, Moscow: Nauka, **1987**.
18. Sokolov, N.S., Ryabinov, V.M. *Soil Mechanics and Foundation Engineering*, **2015**, 52(1): 18-22.

19. Shashkov, A.G., Bubnov, A.V., Yanovskij, S.Yu. *Heat conduction wave phenomena*, Moscow: URSS, **2004**.

20. Zarubin, V.S., Kuvyrkin, G.N. *Mathematical modeling of*

thermomechanics, Moscow: Fizmatlit, **2000**.

21. Zarubin, V.S., Kuvyrkin, G.N., Savelyeva, I.Yu. *International Journal of Heat and Mass Transfer*, **2018**, 116, 833-839.

$$\left(10^{-12} - 10^{-14} \text{ s}\right), \quad (\text{Eq. 1})$$

$$\tau_r \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2}, \quad 0 < x < l, \quad t > 0; \quad (\text{Eq. 2})$$

$$T(x, 0) = T_e = \text{const}, \quad 0 \leq x \leq l; \quad (\text{Eq. 3})$$

$$\frac{\partial T(x, 0)}{\partial t} = 0, \quad 0 \leq x \leq l; \quad (\text{Eq. 4})$$

$$T(0, t) = T_0 \sum_{k=1}^K \left[\eta(t - (k \cdot d + (k-1)t_1) + d) - \eta(t - k(d + t_1) + d) \right] = \varphi(t), \quad t > 0; \quad (\text{Eq. 5})$$

$$\frac{\partial T(l, t)}{\partial x} = 0, \quad t > 0, \quad (\text{Eq. 6})$$

$$q(x, t) = -\frac{\partial T(x, t)}{\partial x} - \tau_r \frac{\partial q(x, t)}{\partial t}, \quad (\text{Eq. 7})$$

$$T(x, t) = T_1(x, y) + T_e \quad (\text{Eq. 8})$$

$$T(x, 0) = 0, \quad 0 \leq x \leq l. \quad (\text{Eq. 9})$$

$$\bar{T}(x, t) = T(x, t) / T_0; \quad (\text{Eq. 10})$$

$$\bar{t} = t / \tau_r; \quad \bar{x} = \frac{x}{\sqrt{a\tau_r}}; \quad (\text{Eq. 11})$$

$$\bar{l} = l / \sqrt{a\tau_r}, \quad \bar{t}_1 = t_1 / \tau_r; \quad (\text{Eq. 12})$$

$$\bar{d} = d / \sqrt{a\tau_r}. \quad (\text{Eq. 13})$$

$$\frac{\partial^2 \bar{T}}{\partial \bar{t}^2} + \frac{\partial \bar{T}}{\partial \bar{t}} = \frac{\partial^2 \bar{T}}{\partial \bar{x}^2}, \quad 0 < \bar{x} < \bar{l}, \quad \bar{t} > 0; \quad (\text{Eq. 14})$$

$$\bar{T}(\bar{x}, 0) = 0, \quad 0 \leq \bar{x} \leq \bar{l}; \quad (\text{Eq. 15})$$

$$\frac{\partial \bar{T}(\bar{x}, 0)}{\partial \bar{t}} = 0, \quad 0 \leq \bar{x} \leq \bar{l}; \quad (\text{Eq. 16})$$

$$\bar{T}(0, \bar{t}) = 1 \cdot \sum_{k=1}^K \left[\eta(\bar{t} - (k \cdot \bar{d} + (k-1)\bar{t}_1) + \bar{d}) - \eta(\bar{t} - k(\bar{d} + \bar{t}_1) + \bar{d}) \right] = \varphi(\bar{t}), \quad \bar{t} > 0; \quad (\text{Eq. 17})$$

$$\frac{\partial \bar{T}(\bar{l}, \bar{t})}{\partial \bar{x}} = 0, \quad \bar{t} > 0. / \quad (\text{Eq. 18})$$

$$v = \sqrt{a / \tau_r} \quad (\text{Eq. 19})$$

$$v = x / t \quad (\text{Eq. 20})$$

$$v = \sqrt{a \tau_r} \cdot \bar{x} / \tau_r \cdot \bar{t} = \sqrt{a / \tau_p} \bar{x} / \bar{t} = \sqrt{a / \tau_r}, \quad (\text{Eq. 21})$$

$$\sqrt{a / \tau_r} = \sqrt{a / \tau_r} \bar{x} / \bar{t} \quad (\text{Eq. 22})$$

$$\frac{\partial^2 G}{\partial \bar{t}^2} + \frac{\partial G}{\partial \bar{t}} = \frac{\partial^2 G}{\partial \bar{x}^2}, \quad 0 < \bar{x} < \bar{l}, \quad 0 < \xi < \bar{l}, \quad \bar{t} > 0; \quad (\text{Eq. 23})$$

$$G(\bar{x}, \xi, 0) = 0, \quad 0 \leq \bar{x} \leq \bar{l}, \quad 0 < \xi < \bar{l}; \quad (\text{Eq. 24})$$

$$\frac{\partial G(\bar{x}, \xi, 0)}{\partial \bar{t}} = 1 \cdot \delta(\bar{x} - \xi), \quad 0 \leq \bar{x} \leq \bar{l}, \quad 0 < \xi < \bar{l}; \quad (\text{Eq. 25})$$

$$G(0, \xi, \bar{t}) = 0, \quad \bar{t} > 0, \quad 0 < \xi < \bar{l}; \quad (\text{Eq. 26})$$

$$\frac{\partial G(\bar{l}, \xi, \bar{t})}{\partial \bar{x}} = 0, \quad \bar{t} > 0, \quad 0 < \xi < \bar{l}; \quad (\text{Eq. 27})$$

$$\bar{T}(\bar{x}, \bar{t}) = \int_0^{\bar{t}} \varphi(\tau) \left[\frac{\partial}{\partial \xi} G(\bar{x}, \xi, \bar{t} - \tau) \right]_{\xi=0} d\tau. \quad (\text{Eq. 28})$$

$$G(\bar{x}, \xi, \bar{t}) = V(\bar{x}, \xi, \bar{t}) \cdot \exp(\mu \bar{t}) \quad (\text{Eq. 29})$$

$$\frac{\partial^2 V}{\partial \bar{t}^2} - \frac{1}{4} V = \frac{\partial^2 V}{\partial \bar{x}^2}, \quad 0 < \bar{x} < \bar{l}, \quad 0 < \xi < \bar{l}, \quad \bar{t} > 0; \quad (\text{Eq. 30})$$

$$V(\bar{x}, \xi, 0) = 0, \quad 0 \leq \bar{x} \leq \bar{l}, \quad 0 < \xi < \bar{l}; \quad (\text{Eq. 31})$$

$$\frac{\partial V(\bar{x}, \xi, 0)}{\partial \bar{t}} = 1 \cdot \delta(\bar{x} - \xi), \quad 0 \leq \bar{x} \leq \bar{l}, \quad 0 < \xi < \bar{l}; \quad (\text{Eq. 32})$$

$$V(0, \xi, \bar{t}) = 0, \quad \bar{t} > 0, \quad 0 < \xi < \bar{l}; \quad (\text{Eq. 33})$$

$$\frac{\partial V(\bar{l}, \xi, \bar{t})}{\partial \bar{x}} = 0, \quad t > 0, \quad 0 < \xi < \bar{l}. \quad (\text{Eq. 34})$$

$$V = X(\bar{x}) \cdot \theta(\bar{t}). \quad (\text{Eq. 35})$$

$$\lambda_n = \frac{\pi(1/2 + n)}{l}, \quad X_n(\bar{x}) = \sin(\lambda_n \bar{x}), \quad n = 0, 1, 2, \dots, l \quad (\text{Eq. 36})$$

$$\theta_n(\bar{t}) = A_n \cdot \sin\left(\bar{t} \sqrt{\lambda_n^2 - 1/4}\right) + B_n \cdot \cos\left(\bar{t} \sqrt{\lambda_n^2 - 1/4}\right) \quad (\text{Eq. 37})$$

$$V(\bar{x}, \bar{t}) = \sum_{n=0}^{\infty} \theta_n(\bar{t}) \cdot X_n(\bar{x}) = \sum_{n=0}^{\infty} \left[A_n \sin\left(\bar{t} \sqrt{\lambda_n^2 - 1/4}\right) + B_n \cdot \cos\left(\bar{t} \sqrt{\lambda_n^2 - 1/4}\right) \right] \sin(\lambda_n \bar{x}). \quad (\text{Eq. 38})$$

$$B_n = 0, \quad A_n(\xi) = \frac{2}{\bar{l}} \frac{\sin(\lambda_n \xi)}{\sqrt{\lambda_n^2 - 1/4}}. \quad (\text{Eq. 39})$$

$$V(\bar{x}, \xi, \bar{t}) = \sum_{n=0}^{\infty} \frac{2}{\bar{l}} \frac{\sin(\lambda_n \xi)}{\sqrt{\lambda_n^2 - 1/4}} \sin\left(\bar{t} \sqrt{\lambda_n^2 - 1/4}\right) \cdot \sin(\lambda_n \bar{x}). \quad (\text{Eq. 40})$$

$$V(\bar{x}, \xi, \bar{t}) = \sum_{n=0}^N \frac{2}{\bar{l}} \frac{\sin(\lambda_n \xi)}{\sqrt{|\lambda_n^2 - 1/4|}} \text{sh}\left(\bar{t} \sqrt{|\lambda_n^2 - 1/4|}\right) \sin(\lambda_n \bar{x}) + \sum_{n=N+1}^{\infty} \frac{2}{\bar{l}} \frac{\sin(\lambda_n \xi)}{\sqrt{\lambda_n^2 - 1/4}} \sin\left(\bar{t} \sqrt{\lambda_n^2 - 1/4}\right) \sin(\lambda_n \bar{x}), \quad (\text{Eq. 41})$$

$$N < \frac{\bar{l}}{2\pi} - \frac{1}{2} \quad 42 \quad (\text{Eq. 42})$$

$$G(\bar{x}, \xi, \bar{t}) = \sum_{n=0}^N \left(\frac{2}{\bar{l}} \frac{e^{-\bar{t}/2}}{\sqrt{\lambda_n^2 - 1/4}} \text{sh}\left(\bar{t} \sqrt{\lambda_n^2 - 1/4}\right) \right) \sin(\lambda_n \bar{x}) \sin(\lambda_n \xi) + \sum_{n=N+1}^{\infty} \left(\frac{2}{\bar{l}} \frac{e^{-\bar{t}/2}}{\sqrt{\lambda_n^2 - 1/4}} \sin\left(\bar{t} \sqrt{\lambda_n^2 - 1/4}\right) \right) \sin(\lambda_n \bar{x}) \sin(\lambda_n \xi). \quad (\text{Eq. 43})$$

$$\begin{aligned} \bar{T}(\bar{x}, \bar{t}) &= \int_0^{\bar{t}} \varphi(\tau) \left(\frac{\partial}{\partial \xi} G(\bar{x}, \xi, \bar{t} - \tau) \right)_{\xi=0} d\tau = \\ &= \int_0^{\bar{t}} \varphi(\tau) \sum_{n=0}^N \left[\frac{2}{\bar{l}} \frac{e^{-(\bar{t}-\tau)/2}}{\sqrt{\lambda_n^2 - 1/4}} \text{sh}(\bar{t} - \tau) \sqrt{\lambda_n^2 - 1/4} \right] \times \lambda_n \cdot \sin(\lambda_n \bar{x}) d\tau + \int_0^{\bar{t}} \varphi(\tau) \sum_{n=N+1}^{\infty} \left[\frac{2}{\bar{l}} \frac{e^{-(\bar{t}-\tau)/2}}{\sqrt{\lambda_n^2 - 1/4}} \right. \\ &\quad \left. \sin\left((\bar{t} - \tau) \sqrt{\lambda_n^2 - 1/4}\right) \right] \lambda_n \cdot \sin(\lambda_n \bar{x}) d\tau, \end{aligned} \quad (\text{Eq. 44})$$

$$\lambda_n = \frac{\pi(1/2+n)}{l}, \quad n=0,1,2,\dots, \quad (\text{Eq. 45})$$

$$\varphi(\tau) = \sum_{k=1}^K \left[\eta \left(\tau - (k \cdot \bar{d} + (k-1)\bar{t}_1) + \bar{d} \right) - \eta \left(\tau - k(\bar{d} + \bar{t}_1) + \bar{d} \right) \right]. \quad (\text{Eq. 46})$$

$$\bar{d} + \bar{t}_1 = 2.5 \quad (\bar{d} = 2) \quad (\text{Eq. 47})$$

$$\bar{d} + \bar{t}_1 = 1 \quad (\bar{d} = 0.5). \quad (\text{Eq. 48})$$

$$\left(\frac{\partial \bar{T}(\bar{l}, \bar{t})}{\partial \bar{x}} = 0 \right). \quad (\text{Eq. 49})$$

$$\bar{t} = t / \tau_r \quad (\text{Eq. 50})$$

$$\bar{x} = x / \sqrt{a\tau_r} \quad (\text{Eq. 51})$$

$$\bar{l} = l / \sqrt{a\tau_r} \quad (\text{Eq. 52})$$

$$\bar{T} = T / T_0 \quad (\text{Eq. 53})$$

Figure 1. Motion dynamics and reflection from the right boundary of one temperature soliton with the support $\bar{t}_1 = 0.5$ in bounded space $\bar{t} = \{0.5; 1.0; 2.0; 3.0\}$

Figure 2. Motion dynamics and reflection from the right boundary of one temperature soliton with the support $\bar{t}_1 = 0.5$ in bounded space $\bar{t} = \{3.5; 4.0; 5.0; 6.0\}$

Figure 3. Motion, correlation dynamics, and reflection from the right boundary of two temperature solitons with the supports $\bar{t}_1 = 0.5$ and the period $\bar{d} + \bar{t}_1 = 2.5$ in bounded space:

$$\bar{t} = \{3; 3.5; 4.0; 4.5; 5.0\}$$

Figure 4. Motion, correlation dynamics, and reflection from the right boundary of three temperature solitons with the supports $\bar{t}_1 = 0.5$ and the period $\bar{d} + \bar{t}_1 = 1$ in bounded space: $\bar{t} = \{2.5; 3.0; 3.25; 3.5; 4.0\}$

Figure 5. Motion, correlation dynamics, and reflection from the right boundary of three temperature solitons with the supports $\bar{t}_1 = 0.5$ and the period $\bar{d} + \bar{t}_1 = 1$ in bounded space: $\bar{t} = \{4.5; 5.0; 5.5; 6.0\}$