INVESTIGAÇÃO DOS PROCESSOS DE DISTRIBUIÇÃO DO CALOR NA SUPERFÍCIE INTERNA DA ESTRUTURA DE ENTREGA, TENDO EM CONTA O MOVIMENTO DO AR NA ZONA DE LIMITE ENTRE O DISPOSITIVO E A VEDAÇÃO

INVESTIGATION OF HEAT DISTRIBUTION PROCESSES ON THE INNER SURFACE OF THE ENCLOSING STRUCTURE, TAKING INTO ACCOUNT THE MOVEMENT OF AIR IN THE BOUNDARY AREA BETWEEN THE DEVICE AND FENCING

ИССЛЕДОВАНИЕ ПРОЦЕССОВ РАСПРЕДЕЛЕНИЯ ТЕПЛОТЫ НА ВНУТРЕННЕЙ ПОВЕРХНОСТИ ОГРАЖДАЮЩЕЙ КОНСТРУКЦИИ С УЧЕТОМ ДВИЖЕНИЯ ВОЗДУХА В ПОГРАНИЧНОЙ ОБЛАСТИ МЕЖДУ ПРИБОРОМ И ОГРАЖДЕНИЕМ

UNASPEKOV, Berikbay A.¹; ZHUMADILOVA, Zhanar O.²; AUELBEKOV, Seilkhan S.³; TAUBALDIEVA, Axaule S.⁴; ALDABERGENOVA, Gaziza B.⁵

^{1,2} Kazakh National Research Technical University named after K.I.Satpayev, Department of Engineering Systems and Networks, 22 Satpayev Str., zip code 050013, Almaty – Republic of Kazakhstan (phone: +77016195111)

^{3,4,5} International educational corporation, Faculty of general construction, 28 Ruskulbekov Str., zip code 050048, Almaty – Republic of Kazakhstan

* Corresponding author
e-mail: unaspekov@yandex.kz

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RESUMO

Foi realizado um estudo sobre a natureza do movimento da massa de ar no espaço entre o dispositivo que aquece a sala e a parede externa confinante da sala. O movimento hidrodinâmico da massa de ar foi simulado com base nas equações de Navier-Stokes bidimensionais no espaço e na equação convectiva de condução de calor para encontrar uma imagem detalhada dos processos físicos que ocorrem. Também para alguns casos particulares, soluções analíticas foram obtidas para o problema do conjugado, onde o movimento do ar é causado por sua expansão térmica e a ação da gravidade em áreas com diferentes densidades (forças de Arquimedes). A simulação de tipos mais complexos de movimento de ar com a formação de fluxos de vórtices usando métodos numéricos e os códigos de programa correspondentes escritos em C++. Descobriu-se que o movimento da massa de ar no espaço entre o dispositivo e a parede depende fortemente não apenas da temperatura do dispositivo, da parede e do ar externo. Foi revelado que a hidrodinâmica e a transferência de calor são significativamente influenciadas por dois parâmetros geométricos: a distância entre o dispositivo e a parede, e a distância do piso ao fundo do dispositivo. Em particular, uma camada fina do fluxo descendente de ar frio ao longo da cerca e acima do chão é possível. A condição para a ocorrência de tal fluxo descendente foi encontrada, é determinada pela razão entre a temperatura da parede, o dispositivo e a temperatura média na sala.

Palavras-chave: *massa de ar, processo físico, modelagem, condutividade térmica, parâmetro geométrico.*

ABSTRACT

A study was made of the nature of the movement of air mass in the space between the device that warms the room and the confining outer wall of the room. The hydrodynamic motion of the air mass was simulated on the basis of the two-dimensional in space Navier-Stokes equations and the convective heat conduction equation to find out a detailed picture of the physical processes that occur. Also for some particular cases, analytical solutions were obtained for the conjugate problem, where the movement of air is caused by its thermal expansion and the action of gravity in areas with different densities (Archimedes' forces). The simulation of more complex types of air movement with the formation of vortex flows using numerical methods and the

corresponding program codes written in C++. It was found that the movement of air mass in the space between the device and the fence strongly depends not only on the temperature of the device, the wall, and the outside air. It was revealed that hydrodynamics and heat transfer are significantly influenced by two geometrical parameters: the distance between the device and the wall, and the distance from the flooring to the bottom of the device. In particular, a thin layer of the downward flow of cold air along the fence and above the floor is possible. The condition for the occurrence of such a downward flow has been found, it is determined by the ratio of the temperature of the wall, the device, and the average temperature in the room.

Keywords: *air mass, physical process, modeling, thermal conductivity, geometric parameter.*

АБСТРАКТ

Выполнено исследование характера движения воздушной массы в пространстве между обогревающим помещением прибором и ограничивающей наружной стены помещения. Проведено моделирование гидродинамического движения воздушной массы на основе двумерных по пространству уравнений Навье–Стокса и уравнения конвективной теплопроводности для выяснения детальной картины возникающих физических процессов. Также для некоторых частных случаев получены аналитические решения сопряженной задачи, где движение воздуха вызвано его тепловым расширением и действием силы тяжести на участках с различными плотностями (силы Архимеда). Выполнено моделирование более сложных видов движения воздуха с образованием вихревых потоков с помощью численных методов и соответствующих им программных кодов, написанных на языке C++. При этом установлено, что движение воздушной массы в пространстве между прибором и ограждением сильно зависит не только от температуры прибора, стены и наружного воздуха. Выявлено, что на гидродинамику и теплообмен существенное влияние оказывают два геометрических параметра: расстояние между прибором и стеной, и расстояние от полового покрытия до нижней части прибора. В частности, возможно возникновение тонкого слоя нисходящего потока холодного воздуха вдоль ограждения и над половым покрытием. Найдено условие возникновения такого нисходящего потока, оно определяется отношением температур стенки, прибора и средней температуры в помещении.

Ключевые слова: *воздушная масса, физический процесс, моделирование, теплопроводность, геометрический параметр.*

INTRODUCTION

The temperature field between the heater and the fence is highly dependent on the movement of air, and it can be caused by artificially or spontaneously. Of greatest interest to us is the second case, called free convection. The distribution of heat on the inner surface of the building envelope, i.e., inside the wall, the other side facing the street, is determined both by the state of the environment (air mass) outside the room, and by thermal physical and hydrodynamic processes in the boundary area between the device and the fence.

The formulated research direction is divided into parts, which are determined by the division of the physical regions of space into three parts:

- 1) including outdoor (street) environment;
- 2) the enclosing surface itself;
- 3) and the space between it and the heater.

The most complex processes occur in the third area, and as a matter of priority, consideration begins with it.

Modern numerical methods and schemes are widely used in the work since without them it is impossible to obtain solutions of nonlinear equations of hydrodynamics and convective heat conduction.

Program codes for the implementation of numerical methods and schemes are written in DevCpp language version 4.9.9.2/5.11. Graphic materials to illustrate the results obtained using software codes, created on the software package Origin Pro version 7.0.

MATERIALS AND METHODS

2.1. Description of the physical picture of the air flow between the device (heater) and the enclosing construction (wall)

In heated residential and industrial premises, they mainly deal with free convective movements of continuous media. Such free convective motions in the mechanics of liquids and gases arise due to a change in their density (thermal expansion) and the action of gravity. The change in density is caused by changes in temperature in space and in time. As a rule, temperature drops

ΔT arising here are very small if T_0 takes some average temperature as a reference point, then the normal speeds Δu and hydrodynamic pressure drops Δp will have orders of magnitude:

$$\Delta u \sim \frac{\Delta T}{T_0} \sqrt{gl}, \quad \Delta p \sim (\Delta u)^2 \sim \left(\frac{\Delta T}{T_0} \right)^2 gl,$$

where: g – acceleration of gravity; l – the characteristic length along the direction of gravity, and at which a temperature drop occurs.

If the relative temperature change is $\Delta T/T_0 = \varepsilon \ll 1$, then the characteristic values of the speed of the liquid or gas Δu turn out to be small values of the ε , and the characteristic changes in the hydrodynamic pressure Δp – are small values higher than the smallness order ε^2 . These data are further the main provisions to simplify the analysis of hydrodynamic and thermal-physical processes in a heated room.

The device for heating the room with characteristic dimensions a and b is located below the window and at some distance h from the building envelope, which in this case is the outer wall of the building (Figure 1). It also shows the selected coordinate system, which is necessary for further discussion. In the window, there may be a gap for refreshing the air in a heated room, it is placed in the lower or upper part. In addition, it is possible the occurrence of construction defects when installing window frames of unintended cracks through which cold outside air penetrates the room. The closer the gap is, the stronger its influence on the processes between the device and the wall.

To simplify the problem, the heating device will be considered to have a flat surface on the side facing the wall. This condition is well satisfied with a device made of aluminum alloys (Figure-2). It has a high temperature compared to the wall. Therefore, in the space between them, a spatially uneven temperature field is established. Outside of this space, the temperature is somewhat lower for several reasons, which leads to convective movement of air in the entire volume of the room due to the emerging Archimedes forces. This movement is the more significant, the higher the temperature difference between different points in space. Since the greatest temperature gradients are observed near a heating device located close to the cold window opening, here too there will be maximum air velocities.

2.1.1. Hydrodynamic processes and their

equations

To analyze free convective flow, it is necessary to simplify the Navier – Stokes equations; usually, this procedure begins with the representation of the density of the medium through a temperature gradient. Since in the case under consideration significant temperature changes are possible in both directions of the x and y coordinates, we perform the following expansion in a Taylor series in a neighborhood of a certain point (x_0, y_0) . In this case, you can write Equation 1.

The specificity of free convection lies in the relativity of such a motion of the medium; it arises only in the case of a density difference appearing at arbitrary points in space. For a point (x_0, y_0) and density in it usually choose a place where free convective motion is absent. In a heated room, it can be its central part.

Using the Equation of state of an ideal gas $p = \rho R_g T$, where R_g – the gas constant of air, we calculate the derivative

$$\frac{\partial \rho}{\partial T} = -\frac{P}{R_g T^2} = -\frac{\rho}{T} \approx -\frac{\rho_0}{T_0}$$

The use of the approximate equality sign here is justified by the smallness of the temperature change at room conditions and the leading role for the occurrence of convective motion of the temperature difference in height and width of the gap between the device and the wall. Thus, the Taylor series (Equation 1) will be:

$$\rho \approx \rho_0 - \frac{\rho_0}{T_0} \left(\frac{\partial T}{\partial x} \Delta x + \frac{\partial T}{\partial y} \Delta y \right)$$

Expressions in brackets are temperature changes, respectively, along with the x and y coordinates, they can be given the form:

$$\frac{\partial T}{\partial x} \Delta x = \Delta_x T, \quad \frac{\partial T}{\partial y} \Delta y = \Delta_y T$$

where the entries $\Delta_x T$ and $\Delta_y T$ mean temperature differences along with the x and y directions, their sum is that smallest quantity $\Delta T \sim \ll 1$. Then you can write Equation 2.

These temperature gradients $\Delta_x T$ and $\Delta_y T$ are scalar quantities and are included in the density decomposition as a sum, therefore, in some approximation, they can be combined into a general Equation 3.

With the transformations done, equation (2)

now takes the form Equation 4.

We now consider the question of simplifying the equations of motion of the air mass. Shown in Figure- 1, the dimensions are such that inequalities are well satisfied:

$$\frac{h}{a} \ll 1, \quad \frac{h}{b} \ll 1$$

As a rule, in practice $h = 0.03 \div 0.05$ m, $a = 0.5 \div 0.6$ m, $b = 1.0 \div 1.5$ m, so in order of magnitude $h/a \sim h/b \sim 0.1$ and less.

The second inequality means that to describe the physical processes between the device and the wall, we can neglect the dependences on the z coordinate, they are significant only in a relatively small region near the right and left edges of the device. Also, due to the symmetry of the transfer processes along the z coordinate, there is practically no movement of air in the region between the heating device and the wall in this direction, except there may be special conditions that are not of general interest. Thus, the first inequality allows us to apply the simplified two-dimensional Navier – Stokes equations [Loitsyansky, 2003] for stationary hydrodynamic air flow Equation 5.

where u , v – projections of the air velocity vector on the x and y -axes; p – pressure; ρ – air density; μ – dynamic air viscosity. The first two equations express the law of conservation of momentum (or, Newton's second law for continuous media), the third equation is the law of conservation of mass.

The above inequalities for the characteristic sizes h , a , b allow us to estimate the contribution of each term in the Navier – Stokes equations. The first inequality indicates the most significant changes in the absolute values of gas parameters in the vertical direction of the x coordinate, but on their weak gradients. At the same time, the changes in the direction of the y coordinate are opposite: the absolute values of the parameters of the gas very slightly along y , but the gradients in the direction of this coordinate are relatively large. By using the well-known standard techniques [Loitsyansky, 2003; Bennett, 1996] to simplify the Navier – Stokes equations, we can conclude that in order of magnitude:

$$u \sim \sqrt{gh}, \quad v \sim \frac{h}{a} \sqrt{gh},$$

$$\frac{\partial u}{\partial x} \sim \frac{\sqrt{gh}}{a}, \quad \frac{\partial u}{\partial y} \sim \frac{\sqrt{gh}}{h},$$

$$\frac{\partial v}{\partial x} \sim \frac{h}{a} \frac{\sqrt{gh}}{a} = h \frac{\sqrt{gh}}{a^2}, \quad \frac{\partial v}{\partial y} \sim \frac{h}{a} \frac{\sqrt{gh}}{h} = \frac{\sqrt{gh}}{a},$$

$$\frac{\partial^2 u}{\partial x^2} \sim \frac{\sqrt{gh}}{a^2}, \quad \frac{\partial^2 u}{\partial y^2} \sim \frac{\sqrt{gh}}{h^2}.$$

Using these qualitative representations of velocities and their derivatives, the following conclusions can be made:

1) in the first ratio from the system (Equation 5) the estimates are valid

$$v \ll u, \quad \frac{\partial u}{\partial x} \ll \frac{\partial u}{\partial y}, \quad \frac{\partial^2 u}{\partial x^2} \ll \frac{\partial^2 u}{\partial y^2};$$

2) in the second ratio, inequalities are approximately satisfied

$$\frac{\partial v}{\partial x} \ll \frac{\partial v}{\partial y}, \quad \frac{\partial^2 v}{\partial x^2} \ll \frac{\partial^2 v}{\partial y^2}.$$

As a result of applying these groups of inequalities instead of (Equation 5), we arrive at the following system of Equation 6.

But in this system of equations, there are still insignificant factors, and for further analysis, we will consider in more detail the region of air flow (Figure 3). Curves with arrows show approximate airflow directions.

On solid surfaces, $y = 0$ and $y = h$, both velocities u and v should vanish due to the gas “sticking”. The vanishing of the velocity v on these boundaries with a relatively small interval $h \ll a$ will lead to the fact that in the interval $[0, h]$ the velocity v does not have time to grow strongly in absolute value. Over the entire height of the device, this speed will always remain very small, and approximately any movement associated with this speed v can be ignored. In essence, in the language of mathematics, this means that one can take $v \approx 0$. Then completely discarding the second ratio in the system (Equation 6), and in the remaining equations all terms with factor v , we arrive at the following ratios in Equation 7.

Given the essential role of gravity (giving, ultimately, the power of Archimedes) and Equation 4 for steady processes, the product ρg can be written as in Equation 8.

Then instead of the system (Equation 7),

we get the equation Equation 9.

If you open the bracket in the first equation of the system (Equation 9), then a constant additive of $\rho_0 g$, appears, it leads to the appearance of the usual hydrostatic pressure in height [Loitsyansky, 2003; Bennett, 1996], which does not affect the hydrodynamic air flow. To eliminate the hydrostatic pressure from (Equation 9), we replace the pressure p in it according to the equality:

$$p = p_0 + \rho_0 g x + p',$$

where p_0 – normal atmospheric pressure, $p_0 = \text{const}$; p' – new hydrodynamic pressure, then we get the following system of ratio in Equation 10.

The main driving force here is the force of Archimedes F_A , up to an insignificant constant, it is represented in Equation 10 as a complex:

$$F_A \sim \frac{T - T_0}{T_0} g.$$

In practice $T_0 = 20 \div 25^\circ\text{C} = 293 \div 298 \text{ K}$ – This is the average comfortable temperature in heated premises. The maximum possible value of T is reached only in a very small vicinity of the device, where the air temperature is about 100°C , but in reality, and in most of the space, it is significantly less. Therefore, let $T \approx 70^\circ\text{C} = 343 \text{ K}$, then the relative temperature change:

$$\varepsilon = \frac{T - T_0}{T_0} \approx \frac{343 - 293}{293} \approx 0.17 \ll 1.$$

The small parameter ε also characterizes the relative change in the density of the air, and, where the density is less, the velocity u is higher. This means that the speed is $u \sim \varepsilon \ll 1$. Under such circumstances, the hydrodynamic pressure p' does not play a significant role, and its magnitude is of the order of $p' \sim u^2 \sim \varepsilon^2$ and the contribution can be neglected, assuming $p' \approx 0$.

Now, with this in mind, you can write out the final form of the equations of air velocity movement (Equation 11 and 12).

We make the following remark: the equation is approximate and can cause controversy. A weighty reason for this is the uncertainty of the point (x_0, y_0) , relative to which the Taylor series (Equation 2) is decomposed. This circumstance leaves the temperature T_0 accordingly uncertain. Therefore, solutions of

Equations 11 and 12 need to be rechecked using equality (2). Given this circumstance, the computational domain (Figure-3) is shown with an indication of the selected point A ($x_0 = 0, y_0 = h$) with the temperature T_0 in it. Moreover, the inequality $y_A \gg h$ must be fulfilled, and convective motion should not be observed at the point y_A .

So, the solution of Equations (11) and (12) is sought in the rectangular domain given by the inequalities:

$$0 < x < a; 0 < y < h.$$

The question of conditions at the boundaries, where the laws of conservation of energy, momentum, and mass should be reflected in mathematical form, is discussed below.

2.1.2. The equation to describe thermal processes

Gas properties such as thermal conductivity and heat capacity depending on temperature, but in the problem, under consideration, their changes can be neglected: temperature changes here are so small that with high precision heat transfer coefficient λ and heat capacity (in particular, at constant pressure) c_p can be taken as constant parameters. The fulfillment of the inequality and the presence of symmetry in the direction of the z coordinate for thermal processes means the possibility of neglecting the molecular transfer in the direction of the z coordinate in comparison with the directions of the x and y coordinates.

$$\frac{h}{b} \ll 1$$

Other inequality

$$\frac{h}{a} \ll 1$$

allows you to ignore the molecular transfer along the x coordinate compared to the direction along the y coordinate. This does not apply to the mechanisms of convective heat transfer, which must be analyzed separately. Therefore, with this in mind, we can accept the use of Equation 13.

The computational domain for Equation (13) is the same as for Equations 11, 12. The above justifications for neglecting changes in air density when writing Equation 13 remain valid for thermal-physical processes.

As can be seen, hydrodynamic and

thermal-physical processes have a strong influence on each other: the distribution of heat in the space between the device and the wall depends on the gas velocity u , which, in turn, is determined by the temperature field $T(x, y)$. But this field has the opposite effect of the velocity field.

2.1.3. Border conditions

The number of necessary boundary conditions is determined by the highest degree of derivatives in the differential equations. In Equations (11)–(13), describing hydrodynamic and thermal processes in the space between the device and the wall, there are second derivatives in the y coordinate and first derivatives in the x coordinate. Moreover, (11)–(13) contains the second derivatives with respect to y of the velocity u and the temperature T . Consequently, for the air velocity u , it is necessary to set two conditions at the boundaries $y = 0$ and $y = h$, the same picture will be for temperature T . The first pair of boundary conditions for speed expresses the “sticking” condition [Loitsyansky, 2003] and Equation 14.

To formulate the second pair of boundary conditions for temperature, we note that the device has a thin shell inside which hot water moves. The movement of water creates the effect of the presence inside the device heater. The air blowing around the surface of the device somewhat reduces the temperature of the contact surface $y=h$, but given the high thermal conductivity of the metal and the high heat capacity of water inside the device, compared to the heat capacity of air, the temperature of the contact surface can be considered constant and equal to the water temperature inside the device T_p and then the condition follows according to the Equation 15.

There are no internal heat sources on the other surface of the wall, and there is no, as in a heating device, moving mass flows with a given temperature. Therefore, on the surface $y = 0$, the condition of convective heat transfer should be specified [Bennett, 1966], Equation 16.

where α – heat transfer coefficient; T_w – average or effective temperature inside the machines. This temperature T_w can be determined theoretically by solving the problem of heat exchange between the wall and the outside air, or by direct experimental measurement. Below T_w is considered a known value.

The heat transfer coefficient α can be found

by the universal dependencies of the so-called Nusselt numbers Nu Reynolds numbers Re and Prandtl Pr , established by experimental and theoretical methods. When air moves near the wall, a very thin laminar boundary layer is formed, where significant changes in velocity in the direction of the y coordinate occur. When simplifying the hydrodynamic equations, this boundary layer “fell out”, since $v = 0$ was taken and with the intention to take into account processes in the entire coordinate interval from $y=0$ to $y=h$. In the solutions of Equations (11) – (13), although there is no boundary layer, the effect introduced by them is taken into account in the boundary conditions. We use the task of finding the heat transfer coefficient on the plate surface, where (for the average along the plate portion) the Nusselt number is expressed by the formula [Bennett, 1966]:

$$Nu = 0,664\sqrt{Re} \cdot Pr^{1/3}.$$

For the case considered here of the flow around a wall surface under the action of a device with height a , we can take $Nu = a\alpha / \lambda$ from Reynolds numbers $Re = u_* a / \nu$ and Prandtl $Pr = \nu / \kappa$. Here ν – kinematic viscosity of air; u_* – characteristic air velocity. Since for gases the number $Pr \approx 1$ [Bennett, 1966], then we can write

$$Nu = 0,664\sqrt{Re}.$$

Or in dimensional terms, this formula takes the form

$$\frac{a\alpha}{\lambda} = 0,664\sqrt{\frac{u_* a}{\nu}}.$$

From here, we find the heat transfer coefficient α :

$$\alpha = 0,664\frac{\lambda}{a}\sqrt{\frac{u_* a}{\nu}}.$$

Substituting it in Equation 16, we find the condition for the surface $y = 0$ in Equation 17.

Thus, two more necessary boundary conditions (15) and (17) are formulated. Here, the characteristic velocity u_* remains unknown. In a convective heat transfer on a plate, the rate of the blowing gas far from the plate, representing the maximum flow velocity, is assumed to be $u_* = u_\infty$. Here, with the free convective flow, there is no such a definite flow rate, therefore, the value u_*

will have to be established after finding the air velocity distribution in the space between the device and the wall.

Equations (11)-(13) contain the first derivative of the velocity u and the temperature T with respect to the variable x . This means that for a complete solution of the problem, one boundary condition is needed at the boundaries $x=0$ and (or) $x=h$, and if, for example, the condition for temperature is set at $x=0$, then the boundary condition is not needed at $x=h$, and vice versa, after setting the condition at $x=h$, it is not necessary to set the condition at $x=0$. The same applies to the velocity u .

The appearance of the boundary conditions along the x coordinate is a very difficult question since it is determined by the physical conditions near the boundaries $x=0$ and $x=h$. In particular, a difficult situation takes place in the upper part of the computational domain, where the complex hydrodynamic air flow is usually realized for the following reasons:

1) a window can have artificially created slots for the purpose of ventilating the room, and such ventilation slots can be intentionally created in the lower or upper part of the window opening (Figure- 4a) [Ryabova, 2015; Kalitkin, 1978; Tikhonov, 1966; Baker, 1981]. In some special and rare cases, the window may simply be slightly opened;

2) when installing the window, a defect may be made in the form of a loose fit of the window sill to the brickwork of the window opening (Figure-4b), or the walls in old buildings may have small cracks, they can also occur in places where earthquakes often occur. Here, the curved arrows indicate the possible movement of air masses.

Data on the gas velocity distribution at the boundaries $x=0$ and $x=h$ can be obtained by experimental measurements. But it is possible to solve the hydrodynamic and thermal problem, which includes together the space between the device and the wall, the area near the sill and the floor slab, but this problem is challenging in mathematical terms because of the complex geometry.

It should be noted that the authors carried out a preliminary mathematical analysis of the physical processes in the space between the heating device and the enclosing wall and on its basis obtained analytical solutions for the temperature distribution in Equation 18.

And airflow rates in Equation 19.

In the space between the heater and the enclosing wall. These solutions are approximate, but convenient for practical use because of their simplicity.

The sign and the numerical value of the ratio of two parameters T_{*1} and T_{*2} , which have the dimension of temperature, has a qualitatively strong effect on the velocity distribution $u(y)$:

$$\frac{T_{*1}}{T_{*2}} = \frac{T_p + \beta T_w - (1 + \beta)T_0}{\beta(T_p - T_w)}.$$

The denominator of the fraction here is always positive. The numerator can be either positive or negative, and it depends mainly on the numerical value of the parameter β . It can formally vary from 0 to ∞ . The condition for changing the sign of the relation T_{*1}/T_{*2} is given by the in Equation 20.

When it is executed, the ratio $T_{*1}/T_{*2} < 0$, which means the possibility of a thin layer of descending cold air flow along the wall, but the implementation of this inequality is still not enough. Downstream air flow appears if (besides the obvious and non-interesting roots $z=0$ and $z=1$), one more third root of the right-hand side (Equation 19) of the equation z_{cr} , equating to zero, will become positive:

$$z_{cr} = -1 - 3 \frac{T_{*1}}{T_{*2}} > 0.$$

In this case, the velocity u in the space adjacent to the wall $z < z_{cr}$ may become negative. Revealing inequality (Equation 21) and after simple transformations, we find the condition for the occurrence of downward airflow:

The condition for the occurrence of the downward flow of cold air can be given another convenient form for practical use. Note, according to Equation 18, the temperature on the surface of the wall T_s is equal to:

$$T_s = T(y=0) = \frac{T_p + \beta T_w}{1 + \beta}.$$

From here we express T_w in T_s :

$$T_w = \frac{(1 + \beta)T_s - T_p}{\beta}.$$

Easy to check that

$$T_p - T_w = \frac{1 + \beta}{\beta} (T_p - T_s),$$

and further

$$\frac{T_{*1}}{T_{*2}} = \frac{T_s - T_0}{T_p - T_s}.$$

Then the condition for the existence of a positive value z_{cr} is written in the form:

$$T_s < \frac{3T_0 - T_p}{2}.$$

This shows that the occurrence of the downward flow is determined only by the average temperature in the room T_0 and the temperature of the device T_p , and does not explicitly depend on the geometric parameters of the device, the gap with flowing air and the distance between the wall and the heating device.

Figure 5 shows the distribution of air velocity at various values of the parameter β . For calculations, the corresponding C++ program code was used.

Calculations (Figure-5) by the Equation 19 were carried out with the following values of the input parameters:

$\rho_0 = 1.1 \text{ kg/m}^3$, $g = 9.81 \text{ m/s}^2$, $h = 0.03 \text{ m}$, $\mu = 1.8 \cdot 10^{-5} \text{ Pa}\cdot\text{s}$,

$T_0 = 293 \text{ K}$ (20 °C), $T_w = 253 \text{ K}$ (–20 °C), $T_p = 343 \text{ K}$ (70 °C).

At selected temperatures T_0 , T_w and T_p , the critical value is $\beta_{cr} = 5.0$, curve 1 in Figure-5 was built at $\beta < \beta_{cr}$, therefore, no downward flow of cold air is observed, the other curves 2 and 3 are built at $\beta > \beta_{cr}$, here we can clearly see the downward movement of air. Since z varies from 0 to 1, the length z_{cr} gives as a percentage the part of width h occupied by the descending airflow, as in curve 3 in Figure-5, we determine that the downstream takes $z_{cr} = 0.2$ or $1/5$ of h . This stream is relatively weak, and the maximum speed reaches approximately modulo $|u| = 0.2 \text{ m/s}$, while the ascending part of the stream has a maximum speed of about 2.7 m/s .

Of interest is the distribution of the air flow rate at various temperatures "inside" the wall T_w . For fixed values of the other parameters, the variation of T_w would mean, for example, strong convective heat transfers on the outer surface of the wall, small thicknesses of the wall itself, many cracks in the wall, etc. Any factors that stimulate

heat loss from the outside of the building envelope lead to a decrease in the temperature T_w . In this connection on fig. 6 shows the results of calculations of the velocity field $u(y)$ for various values of T_w .

The arrows show the plot of speed for curve 3.

As the temperature T_w changes, each curve in Figure 6 corresponds to its critical value of the parameter β :

$T_w = 263 \text{ K}$ (–10 °C) $\rightarrow \beta_{cr} = 15.0$;

$T_w = 248 \text{ K}$ (–25 °C) $\rightarrow \beta_{cr} = 3.75$;

$T_w = 238 \text{ K}$ (–35 °C) $\rightarrow \beta_{cr} = 2.50$.

These data show a strong dependence of β_{cr} on temperature T_w , which means a high sensitivity of air flow in the area between the heater and the wall to heat loss through the outer surface of the building. If at $T_w = 248 \text{ K}$ (–25 °C) the downstream region occupies $z_{cr} \approx 0.25$ of the width h , about 7 mm, then with a relatively small temperature change $T_w = 238 \text{ K}$ (–35 °C), the $z_{cr} \approx 0.41$ of the width h , or 12 mm.

As the temperature T_w changes, each curve in fig. 6. corresponds to its critical value of the parameter β :

$T_w = 263 \text{ K}$ (–10 °C) $\rightarrow \beta_{cr} = 15.0$;

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The descending flows of cold air found here during modeling near the enclosing cold structure look weak enough to attract attention. But such flows can play a significant role in creating comfortable conditions when it comes to the heating of residential premises, wards of hospitals and clinics. Downward streams of cold air move along the surface of the floor, and slightly changing its temperature. Such streams lead to dramatic temperature fluctuations near the floor surface and can have a noticeable negative impact on people's health, or create thermal discomfort. In this regard, it should be noted that in the modern scientific literature on the design and optimization of heat supply [Tabunshchikov, 2002; Malyavina, 2007; Lysenko, 2015; Khurstalev, 2005], the above-mentioned near-wall cold currents do not receive due attention.

According to Equation 18, the temperature T_s on the surface of the enclosing wall facing the device is equal to:

$$T_s = T(y=0) = \frac{T_p + \beta T_w}{1 + \beta}.$$

The calculation using this equation with the above values of the parameter β and temperatures T_w , T_o , and T_p show changes in T_s approximately within 257 ... 271 K, or, in degrees Celsius, it will be from -24 °C to 2 °C. These are cases of very cold walls, but physically feasible in practice and they may well occur. This can be easily verified if T_s is calculated using known simple methods [Khrustalev, 2005; Baskakov, 1991; Roach, 1980; Lorenz, 1963; Schuster, 1985] under the assumption that on the side of the wall $x = -L$ facing the outside air with temperature T_c , the same temperature occurs. The temperature distribution in the wall $T_1(y)$ of thickness L and with a given outside temperature T_c is given by a linear law:

$$T_1(y) = T_p - \frac{T_p - T_c}{1 + \delta} \left(1 + \frac{y}{L} \right), \quad \delta = \frac{\lambda L}{\lambda_w h},$$

$$-L \leq y \leq 0,$$

where λ_w – coefficient of thermal conductivity of the building envelope material. In this case, in the space between the wall and the device, the temperature distribution is:

$$T(y) = T_p - \frac{T_p - T_c}{1 + \delta} (h - y), \quad 0 \leq y \leq h.$$

Hence, for the temperature T_s , we get Equation 22.

Using here the usual practical numerical data [Tabunshchikov, 2002; Malyavina, 2007; Lysenko, 2015; Khrustalev, 2005; Baskakov, 1991] $L=0.7$ m, $h=0.05$ m, $\lambda_w=0.4$ W/(m·K), $\lambda=0.024$ W/(m·K), $T_c = 253$ K (or, -20 °C), we obtain $T_s \approx 19$ °C. This temperature is not much different from the above values obtained in accordance with the data in fig. 5 and 6.

The results in the form of Equations 18, 19 and the conclusions that follow from them were obtained with rather severe restrictions on the dependence of the air flow velocity and temperature only on the coordinate y . This affected the absence of the necessary boundary conditions at $x=0$ and $x=h$. The observed downward flow of cold air along the wall is caused only by the relatively cold wall (enclosure). But such a flow of cold air can easily occur in the conditions of a construction defect and the presence of a ventilation gap in the windows, especially in its lower part.

Consideration of more complex phenomena becomes difficult or simply impossible with the use of simple mathematical equations and analytical techniques. Therefore, it is necessary here, firstly, to take into account partially or completely dropped terms in the Navier – Stokes equations; secondly, to involve numerical methods for solving partial differential equations. The use of numerical methods is a separate and very complex issue, and this is not only because of the many such methods but the complexity of the physical processes and phenomena being studied. Therefore, special attention is paid below to the mathematical aspects of the application of numerical methods to the solution of the Navier – Stokes equations and the heat equation.

2.2. Modeling results and their analysis

Thermal mode during normal operation of the heating system is well studied and described in modern scientific literature, for example, in books [Tabunshchikov, 2002; Malyavina, 2007; Khrustalev, 2005; Sabdenov and Baytasov *et al.*, 12/2014; Sabdenov and Baytasov *et al.*, 13/2014; Batchelor, 1973; Idelchik, 1992; Landau, 2000; Alyamovsky, 2005]. There are few cases where there is a gap near the device through which cold outside air enters the room. This flow is considered to be very narrow due to the small width of the gap. Since the indoor temperature differs from the outdoor temperature T_c , there is a complex movement of air masses in the gap between the external environment and the environment inside the room.

Initial conditions and initial physical parameters

Since the relative temperature changes are not too significant, the temperature change on the wall surface T_s on the interval $0 \leq x \leq a$ can be approximated by a linear dependence to Equation 23.

where T_{s0} - is the temperature at the bottom of the wall (at the level of the floor surface); $T_{c,in}$ – air temperature at the exit from the slot, it is believed that the outside air gets warm as it moves along the slot channel. The amount of warming remains unknown but can be readily determined empirically.

Using dependency (Equation 23), the first variants of modeling the processes of heat exchange and mass transfer in the space between the heating device and the enclosing outer wall of the heated room were carried out

with the following initial parameters:

- device height: $a = 0.7$ m;
- width of the gap between the device and the wall $h = 0.03$ m;
- air thermal conductivity coefficient $\lambda = 0.024$ W / (m·K);
- air heat capacity $c_p = 1000$ J / (kg·K);
- gravitational acceleration $g = 9.81$ m/s²;
- gas constant of air $R_g = 287$ J/(kg·K);
- atmospheric pressure $P_0 = 1.01 \cdot 10^5$ Pa;
- dynamic viscosity of air $\mu = 1.8 \cdot 10^{-5}$ kg/(m·s);
- Prandtl number $Pr = 0.75$;
- equilibrium room temperature $T_0 = 20^\circ\text{C}$;
- $T_{c,in} = 0^\circ\text{C}$;
- heater temperature $T_p = 70^\circ\text{C}$;
- wall surface temperature above the floor level $T_{s0} = 7^\circ\text{C}$;
- scale time $t_m = 167$ s.

Since the non-stationary problem is solved, they can be chosen arbitrarily - the final stationary distribution of temperature and air velocity does not depend on them. Therefore the initial conditions are taken as

$$T(t=0, x, y) = T_p - [T_p - T_w(x)] \cdot \left(1 - \frac{y}{h}\right)^4,$$

$$u(t=0, x, y) = 0, \quad v(t=0, x, y) = 0.$$

Regarding the dimensionless coordinates $x' = x/a$, $y' = y/h$ and time $\tau = t/t_m$, these initial conditions are rewritten as Equation 24.

2.2.1. Analysis of changes in the temperature field in the space between the device and the wall, depending on the variability in the height of the wall temperature.

The results of the obtained data on the temperature distribution between the device and the wall at different distances from the floor level, as well as the velocity distribution $u(y)$ between the device and the wall at different heights x from the floor level, are discussed below.

Figure 7 shows the temperature distribution in the transverse direction from the wall to the appliance at heights from the floor level.

Here in Figure-7 shows a noticeable deviation of the result of numerical simulation near the enclosing surface, where $y \rightarrow 0$. Changes in the parameters of air flow in height occurred due to the temperature variability of the wall along the x coordinate, in the discussed calculations $T_s = T_s(x) \neq \text{const}$.

The strongest spatial temperature changes are observed near the wall, where the temperature varies from -20°C to 3°C at the height of 14 cm. This is due to the weakening of the convective upward air flow near the wall, which is easily seen from the graphs of the velocity distribution $u(y)$ at different heights from the floor level (Figure-8).

A weak downward flow of cold air is observed near the wall, its maximum speed of about 0.25 m/s is reached at a distance of about 2.6 mm from the wall and at a distance of $x = 0.7$ m, i.e., at the level of the slot. But a significant area between the device and the wall is occupied by an ascending stream of warm air, the maximum value of which reaches almost 3 m/s at a distance of about 5.2 cm from the surface of the device.

RESULTS AND DISCUSSION

The graphs of the velocity distribution $u(y)$ at various heights from the floor level (Fig. 8) revealed that the strongest spatial temperature changes are observed near the wall, where the temperature changes from -20°C to 3°C at the height of 14 cm. This is due to the weakening of the convective upward air flow near the wall. Considering the location of the upward flow of warm air, it is clear that it occupies a significant area between the device and the wall (Fig. 8). However, its maximum value reaches almost 3 m/s at a distance of about 5.2 cm from the surface of the device.

CONCLUSIONS

1. A preliminary mathematical analysis of the physical processes in the space between the heating device and the enclosing wall. Analytical solutions (18) and (19) have been obtained for the distribution of temperature and air flow rate in the space between the heater and the enclosing wall. These solutions are approximate, but convenient for practical use because of their simplicity.

2. Based on a mathematical analysis of physical processes in the space between the heating device and the enclosing wall, it was established that the occurrence of a downward flow is determined only by the average temperature in the room T_0 and the temperature of the device T_p , and does not depend explicitly on the geometric parameters of the device, the slit with flowing air and the distance between the wall and the heating device.

3. The analysis of changes in the temperature field in the space between the device and the wall, depending on the variability in the height of the wall temperature. The temperature distribution in the transverse direction from the wall to the device was established at heights from the floor level (Figure-7). It was established that a noticeable deviation of the result of numerical simulation near the enclosing surface, where $y \rightarrow 0$. Changes in the parameters of air flow in height occurred due to the temperature variability of the wall along the x coordinate, in the discussed calculations $T_s = T_s(x) \neq \text{const}$.

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$$\rho \approx \rho_0 + \frac{\partial \rho}{\partial T} \left(\frac{\partial T}{\partial x} (x - x_0) + \frac{\partial T}{\partial y} (y - y_0) \right) = \rho_0 + \frac{\partial \rho}{\partial T} \left(\frac{\partial T}{\partial x} \Delta x + \frac{\partial T}{\partial y} \Delta y \right), \quad (\text{Eq.1})$$

$$\Delta x = x - x_0, \quad \Delta y = y - y_0.$$

$$\rho \approx \rho_0 - \rho_0 \frac{\Delta_x T + \Delta_y T}{T_0} = \rho_0 \left(1 - \frac{\Delta_x T + \Delta_y T}{T_0} \right). \quad (\text{Eq.2})$$

$$\Delta_x T + \Delta_y T \approx T - T_0. \quad (\text{Eq.3})$$

$$\rho \approx \rho_0 - \frac{\rho_0}{T_0} (T - T_0) = \rho_0 \left(1 - \frac{T - T_0}{T_0} \right). \quad (\text{Eq.4})$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2} - \rho g, \quad (\text{Eq.5})$$

$$\rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2} + \mu \frac{\partial^2 v}{\partial y^2},$$

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0.$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \rho g \quad (\text{Eq.6})$$

$$\rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial y^2},$$

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0.$$

$$\rho u \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \rho g, \quad (\text{Eq.7})$$

$$\frac{\partial \rho u}{\partial x} = 0.$$

$$\rho g = \rho_0 \left(1 - \frac{T - T_0}{T_0} \right) g. \quad (\text{Eq.8})$$

$$\rho u \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \rho_0 \left(1 - \frac{T - T_0}{T_0} \right) g, \quad (\text{Eq.9})$$

$$\frac{\partial \rho u}{\partial x} = 0.$$

$$\rho u \frac{\partial u}{\partial x} = -\frac{\partial p'}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} + \rho_0 \frac{T - T_0}{T_0} g, \quad (\text{Eq.10})$$

$$\frac{\partial \rho u}{\partial x} = 0.$$

$$\rho u \frac{\partial u}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} + \rho_0 \frac{T - T_0}{T_0} g, \quad (\text{Eq.11})$$

$$\frac{\partial}{\partial x}(\rho u) = 0 \quad (\text{Eq.12})$$

$$\rho u \frac{\partial T}{\partial x} = \frac{\lambda}{c_p} \frac{\partial^2 T}{\partial y^2} \quad (\text{Eq.13})$$

$$u(x, y = 0) = 0, \quad u(x, y = h) = 0. \quad (\text{Eq.14})$$

$$T(x, y = h) = T_p. \quad (\text{Eq.15})$$

$$\lambda \frac{\partial T}{\partial y} = \alpha(T - T_w), \quad (\text{Eq.16})$$

$$\frac{\partial T}{\partial y} = 0,664 \sqrt{\frac{u_*}{\alpha \nu}} (T - T_w). \quad (\text{Eq.17})$$

$$T(y) = \frac{T_p + \beta T_w}{1 + \beta} + \frac{\beta}{1 + \beta} (T_p - T_w) \frac{y}{h} \quad (\text{Eq.18})$$

$$u(y) = \frac{\rho_0 g h^2 T_{*2}}{6\mu T_0 (1 + \beta)} z(1-z) \left(z + 1 + 3 \frac{T_{*1}}{T_{*2}} \right), \quad z = \frac{y}{h} \quad (\text{Eq.19})$$

$$\beta > \frac{T_p - T_0}{T_0 - T_w}. \quad (\text{Eq.20})$$

$$\beta > \beta_{cr}, \quad \beta_{cr} = \frac{3(T_p - T_0)}{3T_0 - T_p - 2T_w} \quad (\text{Eq.21})$$

$$T_s = T_p - \frac{T_p - T_c}{1 + \delta}, \quad (\text{Eq.22})$$

$$T_s = T_{s0} - (T_{s0} - T_{c,in}) \frac{x}{a}, \quad (\text{Eq.23})$$

$$T(\tau = 0, x', y') = T_p - [T_p - T_w(x')] \cdot (1 - y')^4, \quad (\text{Eq.24})$$

$$T_w = T_s - (T_s - T_c)x', \quad u(\tau = 0, x', y') = 0, \quad v(\tau = 0, x', y') = 0$$

Figure 1. The layout and dimensions of the heater, windows and enclosing structures

Figure 2. Typical aluminum heating devices

Figure 3. The calculated area where the distribution of temperature and air mass velocity is mass

Figure 4. Location of the ventilation gap (a) and the gap as a construction defect (b)

Figure 5. Speed profiles $u(z)$ with different parameters β :
1 – $\beta = 4.5$; 2 – $\beta = 8.5$; 3 – $\beta = 10$.

Figure 6. The velocity profiles $u(z)$ at $\beta = 8.5$ and different temperatures T_w :
1 – $T_w = 263$ K (-10 °C); 2 – $T_w = 248$ K (-25 °C); 3 – $T_w = 238$ K (-35 °C).

Figure 7. Temperature distribution between the device and the wall at different distances from the floor level.

Figure 8. The velocity distribution $u(y)$ between the device and the wall at different heights x from the floor level.