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TESTE DE CONHECIMENTO DE CONCEITOS DE VETORES ELEMENTARES ENTRE ESTUDANTES DO PRIMEIRO SEMESTRE DE BACHARELADO EM ENGENHARIA E DE TECNOLOGIA

TEST OF KNOWLEDGE OF ELEMENTARY VECTORS CONCEPTS AMONG FIRST-SEMESTER BACHELOR OF ENGINEERING AND TECHNOLOGY STUDENTS

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RESUMO

A física, como disciplina acadêmica, possui uma linguagem matemática própria e os vetores são seu constituinte indispensável, mesmo no estágio pré-obrigatório. O objetivo desse estudo foi obter um entendimento claro e profundo sobre a percepção da cognição dos alunos sobre as facetas dos vetores que formam a base da mecânica newtoniana. Este estudo fornece não apenas o conhecimento fundamental sobre a cognição de construções fundamentais de vetores entre os alunos, mas também ajuda a melhorar as pedagogias existentes e, eventualmente, levando ao discernimento dos alunos. O autor desenvolveu um Teste de Conhecimento de Conceitos de Vetor Elementar (TKEVC) que foi entregue aos estudantes de engenharia do primeiro semestre que estavam iniciando o curso obrigatório de física de engenharia durante a 1ª semana de aula, antes de qualquer instrução sobre vetores. O TKEVC foi administrado em uma amostra de 476 alunos provenientes de 7 faculdades e universidades de engenharia diferentes da Índia. O resultado obtido com este teste enuncia que apenas 37% dos alunos matriculados no curso de física de engenharia possuem amplo conhecimento de vetores para avançar com tópicos de física, especificamente mecânica, enquanto quase 67% dos estudantes ingressam no curso sem uma compreensão completa dos princípios dos vetores em qualquer aspecto. Essas descobertas têm implicações fortes para as aulas de matemática e física. O pesquisador discutiu as perguntas feitas aos alunos e suas respostas, enfatizando os erros conceituais e metodológicos que eles cometeram. Os resultados têm fortes implicações no ensino de matemática e física de engenharia, no desenvolvimento profissional de professores e na preparação de equipes de professores para instituições de engenharia.

Palavras-chave: *Aprendizagem, Educação matemática, Adição de vetor, Direção de vetor, Magnitude de vetor.*

ABSTRACT

Physics, as an academic discipline, has its mathematical language, and vectors are its indispensable constituent, even at the pre-mandatory stage. This study aimed to obtain a clear and deep understanding of the perception of students' cognition about the facets of the vectors that form the basis of Newtonian mechanics. This study not only provides fundamental knowledge about the cognition of theoretical vector constructs among students, but also helps to improve existing pedagogies and, eventually, leading to students' discernment. The author developed a Test of Knowledge of Elementary Vector Concepts (TKEVC) that was delivered to engineering students in the first semester who were starting the required engineering physics course during the 1st week of class, before any instruction on vectors. TKEVC was administered to a sample of 476 students from 7 different engineering colleges and universities in India. The result obtained with this test states that only 37% of students enrolled in the engineering physics course have extensive knowledge of vectors to advance physics topics, specifically mechanics, while almost 67% of students enter the course without a complete understanding of the vector principles in any aspect. These findings have substantial implications for math and physics classes. The researcher discussed the questions asked to the students and their answers, emphasizing the conceptual and methodological errors they made. The results have substantial implications for the teaching of engineering mathematics and physics, the professional development of teachers, and the preparation of teams of teachers for engineering institutions.

Keywords: *Learning, Mathematics Education, Vector Addition, Vector Direction, Vector Magnitude.*

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1. INTRODUCTION:

Physics, as an academic discipline, has a mathematical language of its own, and vectors are its indispensable constituent, even at the introductory stage. The elemental conception of Newtonian mechanics is a *force* – a vector. Constraining the motion to one dimension, as is common at the commencing levels of physics education, cannot avoid the reality that the forces are by and large vectors in a plane. They must be added, by way of the rules of vector addition, to ascertain the net force along the axis of motion. Similarly, any treatment and discourse of electric forces or fields, beyond the exclusively descriptive “like charges repel”, must connect with the superposition of vectors (Deventer, 2008).

There has been extensive study, with astronomically large literature, of learners’ preconceived ideas and their incorrect excogitation of the concepts of motion and force. Pupil asperities with the superposition of forces and with drawing and making sense of free-body diagrams are long-familiar. Despite anything to the contrary, a good deal of research has riveted on exclusively one-dimensional illustrations where the vector facets of and of kinematic quantities are not instantly apparent. While it is assuredly a fact that learners confront an immediate obstacle with the basic concepts of force, it may well be that a lack of ability to argue accurately, about conceptions of vectors, portray an important, yet less studied, barrier (Barniol & Zavala, 2010, 2012).

The excessively used *Force Concept Inventory* of (Hestenes, Wells, & Swackhamer, 1992) does dig somewhat into learners’ instinctive knowing about how two forces combine. Their results suggest that learners oftentimes utilize non-vectorial reasoning. A common delusion is the “dominance principle”, by or through which, the larger of the two forces “wins out” and establishes the motion. Educatees, even after continued exposure to Newtonian mechanics, will, if asked to verbalize their reasoning, often prove such explanations as “Force A *overcomes* Force B” (J. Aguirre & Erickson, 1984).

The (Hestenes & Wells, 1992)’s *Mechanics Baseline Test* looks in more astuteness at the directional panoramas of the kinematic vectors and the superposition of force vectors. Some of the lowest accounted scores were fathered by those Baseline questions (particularly questions 5, 7, and 19), necessitating the discernment of vector properties (Flores, Kanim, & Kautz, 2004).

(J. M. Aguirre, 1988) looked into the opinions formed beforehand by high-school leaners regarding vector kinematics in projectile motion. He ascertained a large number of faulty preconceived notions, especially concerning the function and implication of the elements of the velocity vector. Perhaps this is not unexpected. After all, vectors are kind of abstract quantities demanding non-intuitive ideas. These results do, be that as it may, indicate that physics didactics needs to set out by giving denotative rumination to pupil conversance with and learning of vectors (Flores-García, Alfaro-Avena, Dena-Ornelas, & González-Quezada, 2008).

Just how much understanding about vectors and their properties do learners who start out the course bring with them into *engineering physics* class? Do all, or most, enter with a working knowledge of vectors? Anecdotal evidence, from casual conversations, indicates that most college-level physics instructors go forward with the hypothesis that pupils have acquired knowledge about vectors somewhere else and simply require a recapitulation to get them up to speed. Is such a premise reasonable and justified?

To find this out, the researcher has developed a *Test of Knowledge of Elementary Vector Concepts* and gave it to the first semester students who are beginning the *engineering physics* course. These were given to students from 7 different engineering colleges and universities of India. The test is reproduced here for other researchers to use if they wish. The elemental judgment reached out from this test enunciates that only a meager 37% of the educatees enrolled for the engineering physics course have ample cognition of vectors to go ahead with physics topics, specifically mechanics, while almost 67% of the students enter with no utile understanding of the principles of vectors in any respect. These findings have strong implications for both mathematics as well as physics instruction. The method used in this study is “*tests of open-ended problems”*.

With a strong and careful design, this study demonstrates a rather dramatic difference, in the generic mathematical context. While on some level, the findings may not come as a surprise to mathematics and physics teachers, it eventually conjure up several questions that are pretty crucial to the pedagogy of vector education: If a learner answers a problem correctly in the *ijk* format, can we still say that she may not have a good command over addition and subtraction of vectors? What do we actually have in mind when we say we expect from learners to cognize the underpinnings of addition and subtraction of vectors?

If a learner performs haplessly in the arrow format of vectors, with it, can we conclude their pitiful understanding of the fundamental physics, or can we call it the antiquity of the abstractionism that entrammels proper use of vector conceptions? In the teaching of vectors, by and large, the educators teach arrow format to the learners before the *ijk* format—but can we call it truly befitting? Can there be a more spontaneous order, i.e., can the introduction of *ijk* format before arrow format turn out to be a better idea vis-à-vis student's learning? Should there be a definitive pedagogical goal for the learners to efficiently and effectively formulate or explicate fundamental operations of vectors in both arrow and *ijk* formats, or is making students learn only one format good enough?

While few of the queries above distinctly command empirical investigations, which are to be exceedingly focused, the other interrogatives call for judgment that has to be highly professional, and community discussions can be apprised better with such probes. Consequently, to gain insight into responses to all of these queries, the researcher starts by reporting the results and analysis of the test, engineered to transcribe through empirical observations and effectively portray learners' achievement in the cognition of elementary vector operations in mathematical situations in both *ijk* as well as arrow formats.

Researcher's instructional experience with teaching of vectors has led him to conclude that the inadequate cognition exhibited by the learners with regards to ideas inherent in vector algebra presented in graph form demonstrates a peculiarly unmanageable obstruction to their triumph in becoming proficient in concepts inherently belonging to the discipline of physics. Vector ideas and their graphical and geometrical explanations imbue the totality of the curriculum of engineering physics course. Regardless of most learners' earlier encounter to the constructs that are fundamental to vectors, in their classes on maths or in physics which the learned in senior-secondary school (as has been pointed out in various researches), and the excessive stress educators have directed towards those vector constructs in their curriculum transaction, learners' relentless discombobulation about cardinal notions of vectors has befuddled their pedagogical endeavors.

The researcher, therefore, resolved to conduct an organized and directed examination of engineering students' cognition of fundamental conceptions of the addition of vectors, calculation of its magnitude, and determination of its direction during the first few weeks of their *engineering physics* course.

A study by (Randall D Knight, 1995) is reasonably related to this study, and his work colligates an amalgamation of questions from both arrow and *ijk*-format. Non-familiarity with *ijk* notation was showcased by nearly half of the participants. Constructs and calculation on vector algebra dwell at the core of the curriculum on engineering physics, implicit in almost all the themes covered up during compulsory courses at the higher level of engineering education.

As (Randall D Knight, 1995) has accentuated, the vectorial existence of forces, kinematical quantities, and fields commands that learners shall possess a handy and clear cognition of fundamental constructs about vector algebra if they wish to succeed in dominating even the most fundamental level of engineering physics. Knight has made a more or less disguised reference to the astounding inadequacy of documented investigations with regards to learners' cognition of constructs integral to vectors, and his "*Vector Knowledge Test"* allowed for a priceless primary quick look into the assessment of the knowledge of vectors among learners registered for the course on engineering physics, before the commencement of the course.

The current study focusses on dissecting learners' achievement on a test on vectors where the test has no direct bearing to the discipline of physics. The significant issue in this research is that the questions appearing in the test have focalized on the assessment of learners' cognition of the most basic constructs consociated to working with the vectors, like, for example, components and magnitude of vectors. Are there certain vital and even more all-encompassing facets of vectors in general and vector operations in particular that are inadequately expressed and staged, or even curbed, by the representation of vectors in the arrow format? Is it so that the arrow format complements the *ijk* format or contrarily inhibits learner cognition and functioning? This study focusses on gaining an in-depth, clear and deep understanding about the perception of learners' cognition, marked by careful evaluation and judgment on those facets of vectors, which form the base for Newtonian mechanics. Still, here it is done in generic math format by looking into learners' solutions to problems in both arrow as well as *ijk* formats. This study provides not only the crucial background knowledge about cognition of fundamental vector constructs among learners but also helps to ameliorate the existing pedagogies and eventually to lead to learners' discernment.

2. MOTIVATION:

Concepts in introductory physics, mostly covered in an engineering physics course, taught at the undergraduate level in the engineering colleges, are represented by vectors. Consequently, a comprehensive cognition of such physics constructs requires learners to have an effective hold over fundamental vector constructs. Certain specific issues that need special attention include the need for the development of a comprehensive and complete classification of the most frequently occurring errors that students of engineering colleges make. At the same time, they learn about concepts on vector in an engineering physics course, and this, as a consequence, concerns the availableness of MCQ type measurement instruments for testing learners' vector cognition.

Studies done previously on cognition of vector constructs among learners can be agglomerated into three groups: (1) those that didn't use concepts of physics (Barniol & Zavala, 2010, 2012; Deventer, 2008; Flores, Kanim, & Kautz, 2004; Hawkins, Thompson, & Wittmann, 2009; Hawkins, Thompson, Wittmann, Sayre, & Frank, 2010; Hinrichs, 2010; Randall D Knight, 1995; Nguyen & Meltzer, 2003; Wang & Sayre, 2010; Zavala & Barniol, 2010, 2013), (2) those that used concepts of physics (J. Aguirre & Erickson, 1984; J. M. Aguirre, 1988; J. M. Aguirre & Rankin, 1989; Barniol & Zavala, 2010; Barniol, Zavala, & Hinojosa, 2013; Deventer, 2008; Flores *et al*., 2004; Wang & Sayre, 2010; Zavala & Barniol, 2013), and (3) those that used the mix of both (Barniol & Zavala, 2010; Deventer, 2008; Shaffer & McDermott, 2005; Wang & Sayre, 2010; Zavala & Barniol, 2013). Studies belonging to the first group (Barniol & Zavala, 2009, 2010, 2012; Deventer, 2008; Flores *et al*., 2004; Hawkins *et al*., 2009; Hawkins *et al*., 2010; Hinrichs, 2010; Randall D Knight, 1995; Nguyen & Meltzer, 2003; Wang & Sayre, 2010; Zavala & Barniol, 2010, 2013) have a direct bearing to our inquiry, for they have analyzed learners' cognition of concepts of vectors in problems that had no context to physics.

Six pieces of research from this aggregation (Deventer, 2008; Flores *et al*., 2004; Randall D Knight, 1995; Nguyen & Meltzer, 2003; Wang & Sayre, 2010) identified some frequently occurring errors that students from engineering colleges make while they are learning vector constructs. The methods which they used in their investigations were either interviews from individuals or exercising open-ended tests or both.

Researches have done previously (J. M. Aguirre, 1988; J. M. Aguirre & Rankin, 1989; Barniol *et al*., 2013; Bollen, van Kampen, Baily, Kelly, & De Cock, 2017; Flores-García, Alfaro-Avena, Dena-Ornelas, & González-Quezada, 2008; Flores *et al*., 2004; Kanim, 1999) and researcher's didactic exposure depicts that the capability of comprehension of prominent themes in the calculus-based physics curriculum at the introductory level demands a firm grip over the fundamentals of vectors.

As being fully aware or cognizant of the essentiality and the requisiteness of laying the groundwork for vector noesis, (Randall D Knight, 1995) and (Nguyen & Meltzer, 2003) engineered tests to appraise the vector cognition, before the vector concept is taught to the engineering graduates in the calculus-based introductory physics course. Both the researches mentioned above established that out of all the learners enrolled for the course on introductory physics, almost 50% of them lacked the requisite understanding about vectors, even though 86% of them affirmed strongly of their familiarity with vectors, right from senior-secondary school (Randall D Knight, 1995).

With regards to learners' previous experience with vectors in senior-secondary schools, research by (Wutchana & Emarat, 2011) brought out that around 32% of students from senior-secondary schools can give right answers to problems on vector addition after they have been taught vectors in the class. In certain educational institutes of higher learning, the math requirement for the fundamental course in physics is Calculus I, and in some institutions, Calculus II is also a requirement, both of which lack treatment on vectors. The course which comes later on in the STEM (science, technology, engineering, and mathematics) course of study is Calculus III, which has time apportionment of approximately 3 weeks (reasonably close to four hours every week) for extended communication on basics of applications of vectors.

For the first time after the completion of school, engineering educatees are given exposure of vectors, in their course on mechanics, and they are given a maximum of 1 week (average of 2 hours and 30 minutes) on this very pertinent area of study. There have been made numerous attempts to address the issue as mentioned above, and to this front, many research studies have been conducted for the improvement of learners' cognition of vectors in a physics context. A course of study was formulated by (Shaffer & McDermott, 2005) with the idea that was based entirely on research findings and as a consequence they bring forth these two works: "*Tutorials in Introductory Physics"* and "*Physics by Inquiry"* to meliorate learners' cognition of the concepts of kinematics, viewed in terms of vectors.

Flores *et al*., 2004 and Barniol *et al*., 2013, researched on implementation of instructional alterations to raise learners' capability to give reasonable justifications with regards to vectors that typify kinematic quantities and forces. Colligated researches (Bollen *et al*., 2017; Kanim, 1999) also comprises deliberations applicability of vectors to varied themes, including that of electrostatics, electric circuits, and Gauss' law. But then, specific efforts have been made in the improvement of learners' cognition of vectors alone. (Mikula & Heckler, 2013) enquired certain vector problems with varied angular compositions by introducing pedagogy that made use of computers for practicing vector problems (Mikula & Heckler, 2013).

The research by (Flores-García *et al*., 2008) concludes that learners' inconvenience in cognizing physics constructs is engendered by a sheer deficiency of skills and comprehension with regards to math, specifically vectors. A majority of learners face difficulty with addition and subtraction of vector even when there is no context to physics in the problem. Learners confront complemental asperities in drawing associations with the conceptual framework when certain contexts on the discipline of physics is added to it (Barniol & Zavala, 2014; Barniol *et al*., 2013; Deventer, 2008).

An excessively limited number of studies have looked into the learner's cognition of subtraction and addition of vectors in the mathematical and/or fundamental physics constructs; the problems asked students in most of the cases employed vector arrow representation (Heckler & Scaife, 2015). (Heckler & Scaife, 2015) researched the learners' cognition of subtraction and addition of vectors in both arrow and algebraic notation (making use of unit vectors).

Quite a lot of investigators have transcribed an array of learner asperities with vector calculations in physical and mathematical environments (Barniol & Zavala, 2010, 2014; Deventer, 2008; Flores *et al*., 2004; Hawkins *et al*., 2009; Nguyen & Meltzer, 2003; Shaffer & McDermott, 2005). Nevertheless, in almost all of these researches, only the representations of vectors in their arrow form were reckoned. Although it is common knowledge that arrow representation of vectors is beyond doubt didactically, viscerally, conceptually, and technically effectual and efficacious, yet significant preconceptions are associated with it among the learners. Thus its small range and scope impede its viability in mathematical or logical computations in its capability and competence to individuate few significant attributes of vector algebra and in its acquirement of the representation of vector quantities (e.g., in 3 dimensions or above).

Comparisons could be drawn between the arrow representation and the algebraic representation with their respective attendant advantages and disadvantages making use of vectors in their unit form $(\hat{\imath}, \hat{\jmath}, \hat{k})$ in algebraic representation, which is frequently employed in didactics, specifically in the calculus-based introductory physics courses. While investigating learner cognition of vectors in the arrow format, (Nguyen & Meltzer, 2003) ascertained that only about 60%–70% of students with calculus background and 20%–40% of students with algebra background were able to accurately draw the vector sum presented in arrow format on a 2D grid, and only between 60% and 80% of all learners managed to precisely solve problems that asked to determine the direction and magnitude comparison problems related to arrow vectors on a grid.

Vectors represented by (Flores *et al*., 2004) in arrow format with given lengths and angles found that while calculating the vector sum for determining the net force, a meager 50% of the learners with calculus background managed to solve those problems. One other observation from the study was that only a very few learners could manage to solve those problems that were qualitative and involved finding the difference of vectors in a physical setting (e.g., in acceleration) where the vectors were in arrow format. (Shaffer & McDermott, 2005) also ascertained from his research that many learners were facing handicap with the subtraction of vector in which the vectors were expressed in arrow format on a grid with single dimensions, and learners did slightly better

(65% correct) in mathematics context in comparison to the physical setting of acceleration (where 45% came up with a correct solution).

Contrarily, (Deventer, 2008) determined that learners performed good in the physics (where 50% of the students correctly solved the problem) in comparison to the situation in mathematics (where 20% students found the correct answer) for subtraction in one dimension, reason being learners were inclined towards usage of physics setting to incur the right directions. (Barniol & Zavala, 2010) also established that learners' solution to test problems, where they were to draw the resultant vectors on a grid as arrows, can be marginally sensitive to the problems' context. The solutions given by learners to the problems on addition of vectors can also be reasonably sensitive to the vector's placement of those that are showcased as arrows, such as tail to tip, tail to tail, or separated (Barniol & Zavala, 2010; Hawkins *et al*., 2009), although no such effectuates were determined when the arrows were placed on a grid (Hawkins *et al*., 2009).

3. REVIEW OF LITERATURE:

Investigation on learners' vector cognition depicts that learners pretty frequently lack the mental faculty required for vector addition, which can eventually lead to learner deterrents (Deventer, 2008; Flores *et al*., 2004; Randall D Knight, 1995; Nguyen & Meltzer, 2003; Shaffer & McDermott, 2005). Investigators on the usage of varied visual representations of graphical vector addition problems have depicted substantial altercations in learners' method of solving the tasks on vectors. There are many mannerisms for asking learners to perform vector addition or subtraction, graphically. Learners often make use of varied methodologies for vector addition (Deventer, 2008). Head-to-tail and components are the only methodologies described in the fundamental textbooks used for this course (Randall Dewey Knight, 2008).

An extensive body of literature could be found on learners' cognition on the *mechanics* theme (McDermott & Redish, 1999). Mainly, learners' problems with the relationship between velocity and acceleration have been observed in a variety of researches (Trowbridge & McDermott, 1981). Learners' understanding of the relationship between force and motion has also been extensively researched and transcribed (Clement, 1982; Halloun & Hestenes, 1985; Viennot, 1979). Even though the context in most of the cases here is one-dimensional motion, few are on vector nature of acceleration as well. (Reif & Allen, 1992) inquired from learners enrolled in the fundamental course on physics and to physics faculty members to describe the direction of acceleration for the motion of objects along various paths.

They ascertained that both students (novices) and faculties (experts) found it challenging to answer qualitative questions about the vector nature of acceleration. Even though the answers that they gave to the problems were mostly right, an initial tendency was observed by the experts in the research to make inappropriate arguments and justifications on taking force as the basis rather than justifying solutions to the problems solely on the grounds of kinematics.

Shaffer (1993), in his research, found that only 6 out of a total of 48 graduate students managed to showcase the approximate direction of acceleration for all positions and the most frequently occurring errors committed were the inclusion of either only the radial or only the tangential components of the acceleration. (Randall D Knight, 1995) in his research found that only about 33% of the students "have sufficient vector knowledge to proceed with mechanics." (Nguyen & Meltzer, 2003) in their research concluded that just over 25% of the students finishing a calculus-based mechanics course and close to 50% completing the algebra-based course could somehow manage to add vectors in two dimensions.

In the inquiry of learner cognition of vectors in electrostatics, (Kanim, 1999) ascertained that most of the learners faced immaculate trouble in thinking coherently and logically about the net electric forces and fields from collections of point sources. That additional inconveniences are brought in when attempting to give a rational justification about the field and force vectors from continuous charge distributions. It has been deduced from nationally administered tests that problems that involve force or acceleration as vector quantities are not easy for most of the learners. For instance, on the Mechanics Baseline Test (Hestenes & Wells, 1992), one problem asks learners to draw a comparison between the magnitudes of four force vectors, acting on an object that is moving at a constant speed.

Another problem demands learners to determine the direction of acceleration for a block when it is at the lowest point on a curved ramp. Learners showcased poor performance on these problems after the standard instruction. About thirty-six percent managed to solve the problem on force correctly, and a meager eighteen percent solved the problem on acceleration accurately. Contrarily, the Force Concept Inventory (Hestenes *et al*., 1992) does not test for the cognition of vectors: On the revision of the test, the only question that required an understanding of vectors was removed from it. The aforementioned probes accentuate the essentiality of conceptual cognition of vectors for learners to immerse themselves in engineering physics course efficaciously.

(Kanim, 1999) researched learners' cognition of constructs of a vector in the context of electrical forces and fields. (J. M. Aguirre, 1988) and (J. M. Aguirre & Rankin, 1989) examined learners' conception concerning vector kinematics, their inquiry was cantered around the interrelationships among acceleration, velocity, and force instead of vectors properties intrinsically. (Ortiz, 2002) researched on pupil's inconvenience in cognizing fundamental vector operations commonly made use of in the beginning of physics courses.

4. MATERIALS AND METHODS:

4.1. Construction and goal of the *"test of knowledge of elementary vector concepts"*

The goal of the *Test of Knowledge of Elementary Vector Concepts* is neither to judge whether the learners are eloquent in vector mathematics nor that they can effectively utilize vectors in a refined way. Instead, it is to see if they have the minimalistic knowledge and skill required to work with vectors that could eventually make it possible for them to go forward with the study of qualitative and quantitative Newtonian mechanics. The *Test of Knowledge of Elementary Vector Concepts*, reproduced as an appendix to this article, is thus excogitated to assess pupils' knowledge of the most basic level of vector properties and operations. It probes a learners' ability to:

1. Identify and make appropriate use of the components of a vector,
2. Determine the vector's magnitude and direction,
3. Ascertain vector sum using graphs, and
4. Find the sum of two vectors using components.

These are essential skills for success in basic Newtonian mechanics. Also, the test looks, for completeness, at student ability with the more advanced capabilities of vector multiplication.

The test has two levels. It first asks students if they are familiar with a given vector topic. Those answering in affirmative are the asked one or more fundamental problems. The problems assess whether or not the student's self-rating is correct. There are a total of ten *questions* and ten *problems*. To minimize guessing, students are instructed to leave blank any issues they think they do not know how to work. The second question, asking students if they have studied vectors previously, instructs those answering "NO" to skip the rest of the test. This test structure, with the accompanying instructions, allows students to be classified as:

1. Having no previous experience with a particular topic (if they answer "NO" and leave the problem blank),
2. Having previous experience but having no working knowledge (if they answer "YES" but work the problem wrong), or
3. Having previous experience and working knowledge (if they answer "YES" and work the problem correctly).

The *Test of Knowledge of Elementary Vector Concepts* was given to all the first semester students (N = 476) enrolled in mid-July 2019 where they have a compulsory *engineering physics* course. Approvals were obtained from 11 local ethics committees. To shield the identity of the participants as well as of the engineering institutions, pseudonyms were rendered to them. Samples were drawn from seven engineering colleges and universities in India. It was during the 1st week of class, before any instruction about vectors, the test was administered. The students in the class were predominantly computer engineering majors, with sizable admixture of biotechnology, mechanical, electrical, civil, and architecture students.

All the students had cleared JEE Mains (some even had cleared JEE Advanced). Altogether, the students were above average in comparison to students taking up *engineering physics* classes at other engineering colleges and universities of India. Their performance on the *Test of Knowledge of Elementary Vector Concepts* is expected to be typical of engineering students beginning their course at most of the top-ranked engineering colleges and universities of the country.

5. RESULTS AND DISCUSSION:

Answer to the first question revealed that 91% of the students were taking the course for the first time while 9% (41 students) were repeating the course. It will be useful to break out the responses for this subgroup to see what light it sheds on why they did not succeed in passing the course previously.

Answer to question two establishes that 96%, i.e., 456 students beginning the *engineering physics* course have studied vectors before. This includes, of course, 100% of the students who were repeating the course as well as 87% of the new students. Their previous exposure to vectors was roughly evenly divided between math courses and prior physics courses, with a large fraction having seen vectors in both places. The response to this question seems to confirm the common assumption that these are not entirely new ideas for most students.

Table 1 presents the complete results of the *Test of Knowledge of Elementary Vector Concepts.* Responses as percentages are shown for three groups:

i. Students new to the course *and* who have studied vectors before (N_1=415)
ii. Students repeating the *engineering physics* course (N_2=41)
iii. All the students present in all the classes from 7 institutions who took part in the study (N_3=476)

Group I examines to what extent students who have studied vectors before can recall and use that knowledge. Group II provides information as to whether a lack of specific skills, in this case, vector mathematics, is a factor for students who fail to complete the course.

“Questions” and “Problems” from the test are indicated in the left column as “Q” or “P”. Since students were instructed to leave those questions blank, which they were not able to answer, rather than to guess, “blank” answers are tabulated separately from “wrong” answers. Here “wrong” means that the student provided an incorrect answer. The primary comparison to make with the Table 1 data is between the percentage of students answering “Yes, I know that,” and the rate who correctly worked a fundamental problem testing their knowledge of that concept. Except for Problems 2, 5, and 6, the percentage of learners giving the accurate solution to the problems is invariably less (often much less) than the percentage asserting that they possessed such knowledge.

Ideally, students answering “Yes” to a self-assessment question would then proceed to attempt the subsequent problem or problems while those answering “No” would leave it blank. In this case, the percentage of blank responses to the problem would equal the percentage responding “No” to the question. Sometimes, though, a student would answer “No” to the question and then proceed to attempt the problem anyway – occasionally getting the answer correct! However, the number of students who knew more than they thought was significantly fewer than the number who knew less than they thought.

Question 3 asks students to define a vector in their own words. Any answer mentioning *magnitude* and *direction*, or something equivalent (e.g., “length” rather than “magnitude”), was counted as correct. Answers that said at least something relevant to vectors were counted as partially correct. By far, the most common partially correct response equated vectors with specific physical quantities (e.g., “A vector is a force”). Even with very generous scoring, only 44% of the class provides an adequate definition, while 29% either left it blank or provided an answer having no connection at all to vectors. It is worth noting that many of the responses counted as fully correct seemed to view vectors as something used only in physics rather than as mathematical objects.

Questions 4 and 5 and Problems 1 and 2 are concerned with vector components. Two common errors of problem 1 were to omit the minus sign or to include the unit vector $\hat{\jmath}$ (i.e., $-\frac{21}{22\pi}\hat{\jmath}$) as part of the answer. The following error, curiously, was widespread among students repeating the course. Students making these errors fail to realize that the component of a vector is simply a scalar number, possibly signed. In Problem 2, answers are given as “(1, 3, 2)” were accepted, since this is how students often learn about vectors in math classes, but the reasonably common response of “x + 3y + 2z” was not accepted as correct.

Question 6, together with Problems 3 and 4 asks whether students can express a vector in direction and magnitude terms. The percentage of learners who found the magnitude correctly was significantly more than the percentage of learners who found the directions accurately, and these are shown in Table I as “Magnitude OK.” Even though 3-4-5 triangles were used, symbolic answers were accepted since not all students had scientific calculators.

Problem 5, on graphical vector addition, had more wrong attempts than any other problem from P1 through P10. The most common incorrect response, by far, was to draw a vector from the tip of $\vec{E}$ to the tip of $\vec{F}$. Pupils were found making use of the bisector method if they arranged the vectors tail-to-tail and drew the resulting vector between them. Educatees who made use of the bisector method, in particular, showed carelessness with regards to the magnitude of the resultant vector. The success rate of Problem 6, algebraic vector addition, was significantly higher. One possible explanation is suggested by looking at those students who answered Problem 2, when asked to write a vector in a component form, as (1, 3, 2). This group did significantly better than the sample average on Problems 2 and 6, dealing with components, but relatively poor than the sample average on Problem 5, i.e., graphical addition. While the number in this group (N=24) is too small to draw definite conclusions, their responses indicate that some fraction of the learners have learned about vectors exclusively as pairs of numbers, devoid of any geometric content.

Together, Problems 1 through 6 covers the basic vector properties and operational need for Newtonian mechanics. The average score for these six problems are shown at the bottom of Table 1. The sample average is only 36% correct responses. The final two questions and four problems are concerned with vector multiplication. These were for informational purposes since vector multiplication is not a prerequisite to a general understanding of mechanics. As anticipated, only a small fraction of students reported reasonable competence with vector multiplication, and even fewer (< 10%) could work a simple problem.

5.1. Educational implications

The findings from the *Test of Knowledge of Elementary Vector Concepts* are presented in Table 2. This table showcases the various abilities of students to use vectors (in approximate percentage) drawn from the first semester of engineering courses which have taken up *engineering physics* course. Taken as a whole, the *Test of Knowledge of Elementary Vector Concepts* suggests that roughly 20% of the class is sufficiently skilled with vectors, read the texts of the problem and solve typical problems without further practice. While this group still needs to acquire specific knowledge, such as finding vector directions, they can likely learn these skills with minimal instruction since their knowledge base is good. Roughly 12% of the students can work on the path of vectors as well. This means that this 12 % of students, i.e., a total of only 57 students out of 476 students enrolled for the course, are ready in the most real sense for the course, and the rest are not. Only these 12% of the entire sample have awareness about vector properties and are very much likely to incorporate vectors into their "working knowledge" of mathematics without additional instruction and practice.

This table presents the approximative percentage of learners who have taken an engineering physics course in their 1st semester of engineering program against the skill sets indicated. This leaves a full 88% of the students enrolled for this course in their engineering programs with no utile noesis of vectors in any respect. This does not, in and of itself, invalidate the assumption that most students have seen vectors in previous classes. Question 2 indicates that 96% of students had, indeed, "studied" vectors at some prior time. Further, high percentages of students gave affirmative self-assessments that they "knew" most of these basic ideas. The results of the *Test of Knowledge of Elementary Vector Concepts*, however, indicate otherwise. Since, as noted, the sample students enrolled for this course are relatively typical of a charitable institution, it is expected that similar results would be found at most other top engineering colleges and universities.

The responses of those students repeating the course, after withdrawing or failing on a prior try, are especially interesting. Their knowledge, after a full semester of college-level engineering physics, is only slightly better than that of new students who indicated a previous study of vectors. Only 43% of this group could correctly add two vectors graphically, although 86% thought they knew how, and only 45% gave a satisfactory definition of a vector. Combining force vectors, whether for free-body diagrams or for conceptual reasoning, is an essential skill for success in Newtonian mechanics. Although cause and effect are difficult to disentangle, the results suggest that a major reason for the failure of these students was their lack of understanding of vectors.

Like many other aspects of elementary physics, vectors seem "obvious" to those of us who have used them for many years. They are by no means evident to students, as these results show. Since the previous study, for most, has produced little usable knowledge, it is unlikely that a few quick sketches on the board, with expressions of “This is how you do it,” will suffice as instruction. From the earliest stages, understanding vectors are essential for progress in physics, so it behooves those who belong to the physics education community to think about how better to incorporate vectors into introductory physics.

It is tempting to think that the physics education community can let the mathematics department do this for them. Unfortunately, standard calculus texts do not treat vectors until shortly before multivariate calculus is introduced. At Indian universities and colleges, students do not see vectors in their “Engineering Mathematics” course until near the end of the first year. Providing learners with the necessary instruction is a task physics teachers themselves will have to take.

Arons & Holbrow, 1990, have discussed student difficulties with learning about vectors noting that “Many students would benefit from more exercises and drill in graphical handling of vectors than are usually available in texts”. That extended, on-going practice is needed seems apparent from the shallow scores of students who had studied vectors before but retained little. Spending one day on the standard vector chapter, found in mostly all texts, followed by one homework assignment is probably not sufficient for most students. There exists a variety of options for dealing with this problem. For example:

1. Instructors should make *explicit* mention of vector properties for several weeks as they proceed through projectile motion, forces, free-body diagrams, and so on. Don’t assume that students understand after one day; much reinforcement will be necessary.

2. Additional homework problems on vectors should be assigned for several weeks. Students need to keep practicing vector math and reasoning *after* they get feedback from their first vector homework.

3. A laboratory period early in the course could be devoted to a vector tutorial. Students would receive a hand-out containing a variety of problems on which to practice and then would get immediate feedback from their instructor. Working with groups would encourage discussion and active participation. This could be followed, the following week, with a force table or similar experiment in which students measure and add non-collinear forces.

4. Basic practice with vector math could be provided via computer-aided instruction. An endless variety of problems could be provided, with hints when needed and immediate feedback. Since such practice deals only with the mechanics of vector manipulation, it would need to be supplemented with a more general discussion of what vectors are and how they are recognized and used in physics.

6. CONCLUSIONS:

Irrespective of the method(s) put to use, pupils who take up the *engineering physics* course, need at the very beginning an explicit instruction, leaving nothing to opinions formed beforehand by the educators about their educatees, in and practice with the application of vectors. A bulk of learners join the course with no working knowledge of vectors. These students are not likely to be successful with the basic principles of Newtonian mechanics, such as the superposition of forces until their ability to reason with the use of vectors has been established. The researcher hopes that an investigation of students’ cognition of vectors to the general understanding of mechanics can render direction or helpful suggestions toward the designing of course modifications that will eventually beef up learners’ perspective of physics as a logically ordered branch of knowledge, instead of the aggregation of discrete facts, where the vector is mere a part. There is a need for heightening learners’ instinctual application of vectors in answering varied questions in the physics context. Nonetheless, researchers here also accentuate that improving learners’ cognition of vectors continues to be a significant challenge.

7. FUTURE RESEARCH:

Although many tests with their unique emphasis that integrate the recommendations, implications, and suggestions of investigators engaged in study of physics education (Beichner, 1994; Ding, Chabay, Sherwood, & Beichner, 2006; Paula V Engelhardt, 2009) have been developed (Aslanides & Savage, 2013; Beichner, 1994; Paula Vetter Engelhardt & Beichner, 2004; Hestenes *et al*., 1992; Maloney, O’Kuma, Hieggelke, & Van Heuvelen, 2001; McKagan, Perkins, & Wieman, 2010; Singh & Rosengrant, 2003; Thornton & Sokoloff, 1998; Tongchai, Sharma, Johnston, Arayathanitkul, & Soankwan, 2009; Wuttiprom, Sharma, Johnston, Chitaree, & Soankwan, 2009), one that assessed learners' cognition of concepts on vector algebra holistically is yet to be developed and consequently there arise the need for a large-scale longitudinal study at the level involving multiple universities from several countries that would dissect learners' cognition of concepts inherently enshrined in vectors, after finishing their introductory physics courses, is a pressing requirement.

8. CONFLICT OF INTEREST:

The author reported no potential conflict of interest in conducting this research.

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APPENDIX: TEST OF KNOWLEDGE OF ELEMENTARY VECTOR CONCEPTS

Question 1: Is this the first time you are taking this course, i.e., a course on *engineering physics*?

_______Yes.

_______No, I am repeating the course.

Question 2: Have you studied vectors before?

_______No. STOP! Do not answer any more questions on this test.

_______Yes. It was in:

Senior Secondary School Math _______

Senior Secondary School Physics _______

College Maths _______

College Physics _______

Question 3: Define Vector.

Question 4: Do you know how to write a vector in terms of its components?

_______Yes.

_______No.

Question 5: Are you familiar with the unit vectors $\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}, \, and \, \hat{\boldsymbol{k}}$?

_______Yes.

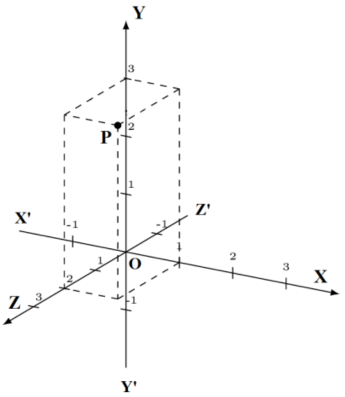
_______No.

Instructions: *For the problems that follow, answer only those that you can. If you don’t know how to solve the problem, leave a blank. Don’t make wild guesses.*

Problem 1: What is the y-component of the vector:

$$\vec{\boldsymbol{A}} = -\frac{\mathbf{3}}{\boldsymbol{\pi}} \frac{(\hat{\boldsymbol{\imath}} + \mathbf{7}\hat{\boldsymbol{\jmath}} - \mathbf{9}\hat{\boldsymbol{k}})}{\mathbf{22}} \text{ ?}$$

Problem 2: Write the vector $\overrightarrow{\boldsymbol{OP}}$, from the adjacent figure, in component form.

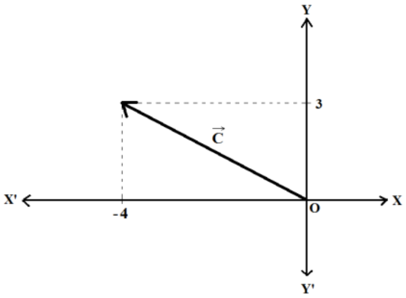


Question 6: Do you know how to express a vector as a magnitude and angle?

_______Yes.

_______No.

Problem 3: Write the vector $\vec{\boldsymbol{C}}$, shown in the figure, as a magnitude and an angle.



Problem 4: What is the magnitude and angle of the vector :

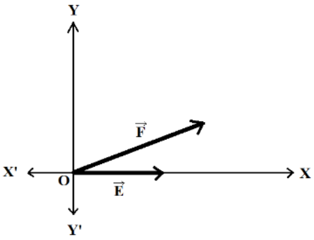
$$\vec{\boldsymbol{R}} = -\frac{\boldsymbol{3}}{\boldsymbol{4}}\,\hat{\boldsymbol{\imath}} - \frac{\boldsymbol{\pi\hat{\jmath}}}{\frac{\boldsymbol{9}}{\boldsymbol{8}}} \text{ ?}$$

Question 7: Do you know how to add two vectors graphically?

_______Yes.

_______No.

Problem 5: Working directly on this figure, add vectors $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{F}}$ graphically.



Question 8: Do you know how to add two vectors algebraically?

_______Yes.

_______No.

Problem 6: Given $\vec{\boldsymbol{P}} = -\frac{\boldsymbol{3\pi}}{\boldsymbol{29}}\,\hat{\boldsymbol{\imath}} + \frac{\boldsymbol{8}}{\boldsymbol{91}}\hat{\jmath}$ and $\vec{\boldsymbol{Q}} = \frac{\boldsymbol{-34\,\hat{\imath} - \frac{5}{3}\hat{\jmath} - 33\pi\,\hat{k}}}{\boldsymbol{\pi/6}}$,

what is the sum $\vec{\boldsymbol{P}} + \vec{\boldsymbol{Q}}$?

Question 9: Do you know how to evaluate the “dot product” (or “scalar product”) of two vectors?

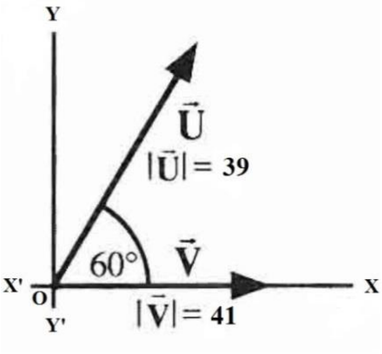
_______Yes.

_______No.

Problem 7: Given $\vec{\boldsymbol{P}} = -\frac{\boldsymbol{3\pi}}{\boldsymbol{29}}\,\hat{\imath} + \frac{\boldsymbol{8}}{\boldsymbol{91}}\hat{\jmath}$ and $\vec{\boldsymbol{Q}} = \frac{\boldsymbol{-34\,\hat{\imath} - \frac{5}{3}\hat{\jmath} - 33\pi\,\hat{k}}}{\boldsymbol{\pi/6}}$,

what is the dot product $\vec{\boldsymbol{P}}.\vec{\boldsymbol{Q}}$?

Problem 8: What is the dot product $\vec{\boldsymbol{U}}.\vec{\boldsymbol{V}}$ of the vectors $\vec{\boldsymbol{U}}$ and $\vec{\boldsymbol{V}}$ shown here?

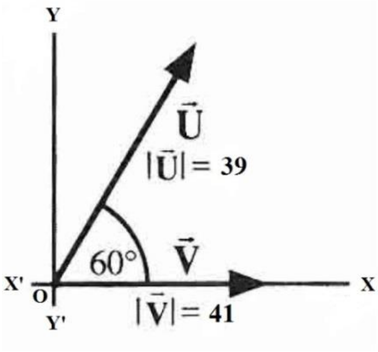


Question 10: Do you know how to evaluate the “cross product” of two vectors?

_______Yes.

_______No.

Problem 9: What is the cross product $\vec{\boldsymbol{U}} \times \vec{\boldsymbol{V}}$ for the two vectors shown here?



Problem 10: Given $\vec{\boldsymbol{U}} = -\frac{\mathbf{3\pi}}{\mathbf{29}}\,\hat{\boldsymbol{\imath}} + \frac{\mathbf{8}}{\mathbf{91}}\hat{\jmath} + 19\hat{k}$ and $\vec{\boldsymbol{V}} = \frac{-\mathbf{34}\,\hat{\boldsymbol{\imath}} - \frac{\mathbf{5}}{\mathbf{3}}\hat{\boldsymbol{\jmath}} - \mathbf{33\pi}\,\widehat{\boldsymbol{k}}}{\boldsymbol{\pi}/\mathbf{6}}$,

what is the cross product $\vec{\boldsymbol{U}} \times \vec{\boldsymbol{V}}$?

Table 1. *Results (in percentage) of the test to gauge the knowledge of elementary vector concepts*

Item	**Response**	**New Students (who have studied vectors before)** **N_1=415**	**Repeating Students** **N_2=41**	**Total students** **N=476**
Q1	Yes	100	0	91
Q2	Yes	100	100	96
Q3	Correct	46	45	44
	Partially Correct	28	31	27
	Wrong	17	19	16
	Blank	9	5	13
Q4	Yes	81	83	78
Q5	Yes	79	91	77
P1	Correct	38	41	36
	Wrong Sign	21	13	12
	Included $\hat{\jmath}$	25	43	31
	Completely wrong	14	2	20
	Blank	2	1	1
P2	Correct	71	83	76
	Wrong	15	14	14
	Blank	14	3	10
Q6	Yes	55	79	54
P3	Correct	17	24	16
	Magnitude OK	21	37	21
	Completely wrong	9	4	8
	Blank	53	35	55
P4	Correct	9	19	9
	Magnitude OK	18	35	18
	Completely wrong	4	8	4
	Blank	69	38	69
Q7	Yes	78	86	67

P5	Correct	43	43	37
	Wrong	35	48	34
	Blank	22	9	29
Q8	Yes	78	93	76
P6	Correct	49	73	46
	Wrong	18	15	17
	Blank	33	12	37
Q9	Yes	77	97	78
P7	Correct	11	6	9
	Wrong	15	27	14
	Blank	74	67	77
P8	Correct	4	6	4
	Wrong	6	7	5
	Blank	90	87	91
Q10	Yes	77	97	78
P9	Correct	3	2	3
	Wrong	5	2	4
	Blank	92	96	93
P10	Correct	0	2	1
	Wrong	4	3	3
	Blank	96	95	96
P1 - P6 average correct		38	47	36

Table 2. *Learner knowledge of elementary vector concepts*

Item	**Student's abilities**	**Percentage (%)**
i.	Have some idea what a vector is	71
ii.	Can recognize and use vector components efficiently	56
iii.	Can find vector magnitudes correctly	20
iv.	Can find vector directions	12
v.	Can effectively add vectors	41
vi.	Can evaluate dot products	6
vii.	Can evaluate cross products	3