

USO DE REDES NEURAIAS ARTIFICIAIS PARA A DIFERENÇA DE TEMPO NA LOCALIZAÇÃO DO ALVO DE CHEGADA COM BASE NA TRANSFORMADA DE COSSENO DISCRETA REDUZIDA**USING ARTIFICIAL NEURAL NETWORKS FOR TIME DIFFERENCE OF ARRIVAL TARGET LOCALIZATION BASED ON REDUCED DISCRETE COSINE TRANSFORM****ИСПОЛЬЗОВАНИЕ ИСКУССТВЕННЫХ НЕЙРОННЫХ СЕТЕЙ ДЛЯ РАЗНИЦЫ ВО ВРЕМЕНИ ЛОКАЛИЗАЦИИ ЦЕЛИ ПРИХОДА НА ОСНОВЕ УМЕНЬШЕННОГО ДИСКРЕТНОГО КОСИНУСНОГО ПРЕОБРАЗОВАНИЯ**SHEVGUNOV, Timofey ¹;¹ Moscow Aviation Institute (National Research University), Department of Theoretical Radio Engineering, Moscow – Russian Federation* Correspondence author
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RESUMO

As coordenadas angulares da fonte de pulso são determinadas pela comparação dos sinais recebidos simultaneamente por vários canais. Para resolver esse problema, o uso de redes neurais é importante. Este artigo discute a aplicação da abordagem de rede neural artificial (RNA) na tarefa de localização de destino. O estudo foi realizado com base em uma técnica de extração de características realizada por uma transformada discreta de cosseno que permitiu obter uma representação compacta da energia do sinal submetido ao processamento digital. O autor define um esquema para estimar o ângulo de chegada do sinal com base na diferença de tempo entre as estimativas de chegada e os problemas estreitos de estimativa de atrasos constantes como parâmetros informativos incorporados nos sinais recebidos que são cópias ruidosas e fracas do sinal de referência. A estrutura do dispositivo adaptativo foi usada para sintetizar a conexão direta da RNA, que é equipada com um conjunto abreviado dos coeficientes de transformada discreta de cosseno (TDC) mais sensíveis e que fornecem uma representação exaustiva do processo aleatório cíclico estacionário de primeira ordem. Um estudo da precisão da estimativa do atraso foi realizado para avaliar o desempenho de RNAs com diferentes tamanhos de sua camada oculta e diferentes números de coeficientes de TDC em suas entradas. Foi provado que cinco coeficientes de TDC são suficientes para distinguir a mudança de fase em toda a faixa. Por sua vez, isso leva a uma estimativa confiável do atraso produzido pela RNA treinada, que contém oito neurônios em sua camada oculta.

Palavras-chave: *estimativa de atraso, ciclo-estacionariedade, perceptron multicamada, sinal de referência, estrutura de elementos adaptativos.*

ABSTRACT

The angular coordinates of the pulse source are determined by comparing the signals received simultaneously on several channels. To solve this issue, the application of neural networks is highly important. In this article, the application of the artificial neural network (ANN) approach to the task of target localization is discussed. The research was performed on the basis of a feature extraction technique executed by a discrete cosine transform, which allowed to obtain a compact representation of the signal energy subjected to digital processing. The author defines the angle-of-arrival estimation scheme based on time difference of arrival estimators and the particular problem of estimating constant delays as informative parameters embedded into received signals that are noisy and damped copies of the reference signal. The adaptive element framework is used for synthesis of the feedforward ANN which is fed with the reduced set of the most sensitive discrete cosine transform (DCT) coefficients, which provide a concise representation of the first-order cyclostationary random process. The investigation on the delay estimation accuracy has been carried out to evaluate the performance of the ANNs with different size of their hidden layer and various numbers of the DCT coefficients at their input. It has been proved that five DCT coefficients are enough for the discrimination of the phase shift in the whole range. In turn, it results in the reliable delay estimation produced by the trained ANN that contains

eight neurons in its hidden layer.

Keywords: *delay estimation, cyclostationarity, multilayer perceptron, reference signal, adaptive element framework.*

АННОТАЦИЯ

Угловые координаты импульсного источника определяются путем сравнения сигналов, принятых одновременно несколькими каналам. Для решения данной проблемы важное значение имеет применение нейронных сетей. В данной статье обсуждается применение подхода искусственной нейронной сети (ИНС) к задаче целевой локализации. Исследование проводилось на основе методики выделения признаков, выполняемой дискретным косинусным преобразованием, что позволило получить компактное представление энергии сигнала, подвергнутого цифровой обработке. Автор определяет схему оценки угла прихода сигнала, исходя из разницы во времени оценок прихода и узкую проблематику оценки постоянных задержек в качестве информативных параметров, внедренных в принимаемые сигналы, которые являются шумными и затухающими копиями опорного сигнала. Структура адаптивного устройства используется для синтеза прямой связи ИНС, которая снабжается сокращенным набором наиболее чувствительных коэффициентов дискретного косинусного преобразования (ДКП), что обеспечивает исчерпывающее представление циклоstationарного случайного процесса первого порядка. Исследование точности оценки задержки было проведено для оценки производительности ИНС с различным размером их скрытого слоя и различным числом коэффициентов ДКП на их входе. Доказано, что пяти коэффициентов ДКП достаточно для различения фазового сдвига во всем диапазоне. В свою очередь, это приводит к достоверной оценке задержки, производимой обученной ИНС, которая содержит восемь нейронов в своем скрытом слое.

Ключевые слова: *оценка задержки, циклоstationарность, многоуровневый перцептрон, опорный сигнал, структура адаптивных элементов.*

1. INTRODUCTION

The problem of angle of arrival estimation consists in determining the bearing, which is the angle between the signal source, where the signal originates, and the direction chosen to be the reference of the passive radar system, i.e., the direction to the North. It is known that the optimal solution to this problem can be obtained via the maximum likelihood estimator (MLE) that maximizes the likelihood function over all bearings with a given measurement obtained by the set of sensors. Although the great advantage of this method is thought to be the highest potential accuracy, it suffers from the disadvantage that is that the direct implementation has no close-form expression and, therefore, will require high computational efforts for obtaining the numerical solution of the underlying optimization problem with required accuracy. This paper introduces an alternative approach based on multi-layer perceptron type ANNs which is based on the alternative representation of the received signal via discrete cosine transformation Discrete Cosine Transform (DCT) rather than the canonical Fourier series basis.

It was shown in (Efimov *et al.*, 2014) that a low-cost, high-speed, compact solutions for a number of avionics-related tasks are available via

the approach based on artificial neural networks (ANN) (Hassoun, 1995). Being aligned to that approach, this paper presents the results of research focused on the tasks of the target detection problem (Rihaczek and Hershkowitz, 2000). One of the issues raised in the target detection is known as the position location that determines one's ability to estimate the coordinates of the particular target using its radio emission. There are several techniques to solve the positioning but angle of arrival (AOA) and time difference of arrival (TDOA) are the only which are vital enough to be implemented for the avionics. Since one can easily realize that there is nothing but time delay estimation (Dubrovin *et al.*, 2014; Shevgunov *et al.*, 2014) is in the key point of both techniques. This makes important the developing of a robust algorithm of time delay estimation based on feature extraction. The direct time delay evaluation originated on those features nonlinear combining can be successfully performed in the ANN-basis. Although the optimal solution based on maximum likelihood approach can be naturally extended for ANN (Shevgunov and Efimov, 2019a), it will require too much computational resources.

The offered approach uses DCT to obtain a set of the DCT coefficients from the received signal. Then, selection is used to reduce number of coefficients in the selection, hence getting

Reduced Cosine Transform Coefficients (RDCTC) set. The phase shift embedded in the original signal will also be encoded into RDCTC set. Since the form of the reference signal is known a series of RDCTC sets can be synthesized in advance and used as training data to the Neural Network. Reduced size of the DCT coefficient set guarantees relatively small size of the network itself.

Using these synthesized RDCTC sets to train network one can obtain the neural network designed specifically for the known reference signal; since the neural network is featured with the ability to handle noisy and incomplete data this solution becomes both accurate and efficient in terms of calculation cost. The ANN trained once can be easily replicated to perform within other systems which resemble the same architecture. It opens the road to fast processing of the signal and further developing algorithms which can be implemented in small-size computers carried onboard as well as low-cost mass produced systems working autonomously or being asked with low rate.

2. MATERIALS AND METHODS

The single-station model being considered in this paper contains a passive radar system and the signal source. The passive radar system is equipped with antenna system of at least two sensors. The local coordinates of the sensors within the radar system are known with high accuracy. The signal source which bearing is to be estimated emits the unknown signal which can be considered as a realization of first-order stationary process with a known power spectrum spectral density (PSD). This model fits well the real signals belonging to the class of man-made amplitude modulated signals used in modern radar and communication systems.

One of the important problems within position location task is calculating delay between two signals received at the specially separated points. The prime concept of angle-of-arrival (AOA) estimation technique is shown in Figure 1.

The signal $s(t)$ radiated by a radio source (RS) is received at two spatially separated points where signals can be expressed in the following forms (Equations 1, 2), where K_1 and K_2 are dimensionless attenuation ratios, T_1 and T_2 are time delays and n_1 and n_2 are noises for the first and second points correspondingly.

Considering the instance of the first-order cyclostationary signal (Shevgunov *et al.*, 2018a)

$s(t)$ with the circular frequency ω , the points separated by the distance d being not farther than the half of its wavelength and noises uncorrelated with each other as well as with the signal, one can obtain the following expressions for the received signals (Equations 3, 4), where amplitude A is assumed to be random (Shevgunov, 2019) while the phases φ_1 and φ_2 can be used for evaluation of angle of arrival (Equation 5), where c is a phase speed e.g. the speed of light.

There are few approaches to determine phase shift between two harmonics. The most widely used technic is cross-correlation. One of its efficient applications is shown in (Chan *et al.*, 1999). But not all approaches involve the cross-correlation, the algorithm of direct phase estimation is based on neural networks and was originally introduced in (Shaltaf and Mohammad, 2009).

In order to implement successfully this group of algorithms, one needs at first to determine such features of the signal which will most effectively embed the information about the delay or phase shift. Those features are obviously expected to be very sensitive to the changes in the delay. It was shown in (Efimov and Shevgunov, 2014), that a subset of Direct Cosine Transform coefficients can be used for that purpose. This subset can be used to decode the value of phase shift which allows one to calculate time delay.

The Discrete Cosine Transform is a transform with even harmonic functions basis. Although there are 8 types of DCT due to the symmetry property; in this paper, the author uses the most thoroughly researched type described in (Oppenheim *et al.*, 1999) as the normalized DCT-II and defined by the following Equations 6, 7, 8.

That transform was chosen among the others due to its property known as "energy compression": the coefficients of the sequence are concentrated at some range of indices with higher density rather than in Discrete Fourier Transform (DFT) or any other types of DCT. This property is illustrated by 3D-plot shown in Figure 2 where the actual values of DCT coefficients are plotted against coefficient number and values of the time delay measured in the part of harmonic period.

The results depicted in Figure 2 indicate that some of coefficients are less sensitive to the phase shift; hence, the usage of the whole set of DCT coefficients is redundant. The criteria of how to choose the most sensitive of the phase shift

coefficients must be defined. In paper (Shaltaf and Mohammad, 2009), the criteria of class L_1 based on the sum of absolute difference (SAD) is proposed (Equation 9), where R_m is the m -th DCT coefficient under analysis, φ_n are possible phase values of the received signal defined with some predefined steps. Alternatively to the SAD, L_2 criteria can be used, in which the sequential selection of the coefficients containing the major part of the energy (Equation 10).

Generally speaking, both criteria led to the same results in selecting the most sensitive DCT coefficients (R) in order to form RDCTC sets. These RDCTC sets are to be used later as training data for the artificial neural network.

3. RESULTS AND DISCUSSION:

3.1. ANN synthesis

The solution investigated in this paper is based on the application of ANN that is considered to be a versatile mathematical model inspired by biological neural networks formed by the neural cells of animals. For the problem under consideration, an ANN could be mainly considered as a computational system that performs some transformation upon the input data vector (Equation 11), where \mathbf{p} is the input data vector, α is the output value of the bearing to be estimated, H stands for the functional transform performed over \mathbf{p} by the neural network, \mathbf{Q} is the network topology descriptor, Θ is the arranged set neural network inner parameters.

Even if the mapping between input and output data cannot be expressed analytically in closed form, in case of neural network it is possible (according to the universal approximation theorem (Haykin, 1994)) to increase the number of the training samples causing network transform function to converge to the desired function. Moreover, trained neural network is believed to be able to perform the desired functional transform with a given accuracy. That can be achieved via its training which goal is to determine the vector of inner network parameters Θ defined for a given topology \mathbf{Q} using a series of S training patterns that are input-output data sample pairs $[(p_1, \alpha_1), (p_2, \alpha_2), \dots, (p_S, \alpha_S)]$.

The series of training patterns is calculated deterministically based on a single station mathematical model for the whole value range of the bearing parameter a . The direct

transform from bearing a_i to input data matrix X_i is thought to be known (Equation 12). The structural graph of feed-forward neural network known as multi-layer perceptron (MPL) is shown in Figure 3.

The network can solve the task of estimation of the constant delay be easily built using adaptive elements (Efimov and Shevgunov, 2012a). This network is built from neurons united into two layers traditionally called hidden and output ones. Each neuron embeds the summing element and the functional transform; for the hidden layer hyperbolic tangent function is used. The output layer is chosen to be relevant to its functional transform element and contains only summation element.

Using adaptive elements as a starting point makes synthesizing the most classes of neurons way easier (Efimov and Shevgunov, 2012b). Such approach provides network structure to be plain and simple yet allows creating complex transfer functions. This implements the paradigm known as network's point of view which means all elements are quite identical in terms of signal propagation that also allows introducing new types of elements easily while any new elements would be able to represent various systems considering their parts as masked sub-networks (Hagan *et al.*, 2002).

A regular or linear separating neuron can be synthesized by means of a set of amplifying elements, a summing element and a functionally transforming element performing the required transformation function. Each amplifier is connected to a certain summing element input thus implementing synoptic weight.

The structural scheme of a regular neuron with 3 inputs is shown in Figure 4. Its synaptic weights are represented by the amplifiers in the left-hand part of the scheme, where the weighted signals are summed together and then transmitted to the functionally transforming element that embeds the necessary activation function.

The overall transform functions of regular neuron define as follow (Equation 13). According to the back-propagation characteristics of the underlying elements the back-propagation function and the structural scheme in case of back-propagation for regular neuron define as shown in the Figure 5.

The transform function in that case has the formula defined in the expression below

(Equation 14), where g is the first derivative of the activation function (Equation 15).

One can easily see that the transformation of the graph shown in Figure 4 into the graph shown in Figure 5 is straightforward. Thus, each amplifier has been transformed into the amplifier of the same weight, the combiner has turned into the hub. However, the non-linear element implementing the activation function is replaced by the linear amplifier which weight is equal to its derivative evaluated at the same argument.

3.2. Numerical simulation

As input signal $s(t)$ for the numerical modeling the estimated cyclostationary signal (Shevgunov *et al.*, 2018b) of the frequency Ω_0 . The sampling period was chosen to be T in accordance with the sampling theorem. We assumed that, for the sake of the simulation, the signal was delayed with constant delay D . However, it is important to note that this delay will vary from one experiment to another in order to estimate how this change in its value will affect the coefficient of DCT coefficients.

A series of DCT transforms were performed on each input signal related to various delay values. The statistics has been collected and those results are depicted in the single plot which is shown in Figure 6. There one can see the difference in the behavior of DCT coefficients of different number. Thus, those which belong to small values vary significantly while other, with greater index value, take small values.

Figure 6 clarifies an important point which consists in the issue that, despite the fact that all DCT coefficients are sensitive to the value of the delay (Knapp and Carter, 1976) in the signal, the most sensitive to that change are only a few coefficients marked there as R_1 , R_2 and R_3 . These results also indicate that no matter how sensitive any coefficient to the value of the delay is, it is sensitive only for the delay changes in some range, for example R_1 coefficients is only sensitive to the change in ranges $[0.18 - 0.4]$ and $[0.6 - 0.86]$. That means R_1 will improve quality of the estimation only for the values within these ranges; outside of them the coefficient will be almost useless. In order to cover all possible values of the time delay changes, one needs to pick a few DCT coefficients so their ranges of sensitivity will overlap. In this particular case, we need at least 3 coefficients to obtain appropriate estimation procedure. As a matter of fact, the problem of how to combine the information of

three coefficients and use it in an optimal way rises.

The whole outlook of the SAD ability could be presented by the dependency of SAD, written in Equation 8, from the number of DCT coefficient as it shown in Figure 7 where the SAD is depicted against the number of the coefficient in DCT representation. One can easily see that coefficient R_2 is the most sensitive to the information about the angle, which is stored in the signal. The coefficient R_1 has the close value of the SAD measure, though it is smaller. The SAD value for R_3 is half as much as for R_2 . The other coefficient, which R_5 is the greatest one among, do not exceed a third of what it takes for R_3 .

After the indices of the most sensitive DCT coefficients have been determined, the next step is selecting optimal neural network topology (Efimov and Shevgunov, 2013) in order to make it efficient in the estimation of phase-shift value. The numerical modeling was performed to choose the topology leading to lower value of Mean Square Error (MSE). The parameter of the topology to be optimized is the size of the hidden layer. A series of 40 experiments was performed for each size of hidden layer in order to rule out the effect of the random initial synaptic weights. In each experiment the value of MSE for neural network with the hidden layer of given size was estimated, taken values were averaged afterwards. The obtained dependency between MSE and the hidden layer size is shown in Figure 8.

The curve shown in Figure 8 means that the more neurons is in the hidden layer, the more accurate estimation will be achieved. However, the excessive accuracy has no sense in practice since there are other source of errors which have to be taken into account. That is why it is so important to strike a balance between the size of the network and computational cost. The other factor considered above is the size of RDCTC, and it can be reasonable to assess these factors simultaneously.

In fact, the set of experiments were carried out to estimate the influence of both the size of the hidden layer and the number of the most sensitive DCT coefficients. The results are compiled into the diagram shown in Figure 9; the value of MSE are depicted with intensity of grayscale. The plot is divided into square cells which is for pair value (*Number of neurons*, *Number of DCT coefficients*). The lighter the cell is, the greater the value of the error is.

The analysis above shows that it is enough to take 5 most sensitive DCT coefficients while the ANN contains 8 neurons in the hidden layer. The results of the numerical simulation indicate that MSE generally decreases as the number of DCT coefficients in the RDCTC sets or the number of the neurons in the hidden layer increases (Haykin, 1994; Shevgunov and Efimov, 2019b). It is important to note that the drawback of the increase of either size of hidden layer or the size of RDCTC sets is the increase of the calculation cost.

4. CONCLUSIONS:

The present paper reflects the current advances in the ANN-based signal processing related to implementation in modern onboard systems. The concept of adaptive elements has been used as a clear yet effective tool forming the framework for the synthesis of the artificial neural networks which are able to solve the estimation of the angle of arrival which is the crucial part of the target localization problem. The adaptive element approach consists in building up a graph for the backpropagation where the graph that is used at the feed forward stage is given.

It was shown that the neural networks provide a researcher the strong framework for effective time delay estimating algorithm. The estimation procedure relies on the feature extraction technique carried out by discrete cosine transform which appropriate type allows obtaining the compact energy representation of signal undergone to the digital processing. It has been shown that there is a small subset of discrete cosine transform coefficients which are extremely sensitive to the changes in the delay. Since each of those coefficients in the subset is sensitive only in a range of the delay variation, the effective non-linear combination must be employed instead of the individual coefficients processed separately. It was also shown that DCT coefficient with smaller indices are more valuable since they vary greater than those of higher indices.

The feed-forward neural network, which was chosen in the class of multilayer perceptron, appears to be the most computationally effective architecture which allows one to build up the best combination via the training process. The DCT coefficients chosen to be included into the RDCTC set are used as the input of the ANN while the value of the delay is the output parameter, which the ANN is trained to fit. The

optimization problem for the number of features enough for signal representation and for the number of elements in the hidden layer was also solved as the illustrative example in the current research. It was shown that the set containing 5 DCT coefficients could achieve the error rate less than 10⁻⁹ if the ANN containing 8 neurons in its hidden layer is applied. Since time delay estimation is the core of commonly used position location techniques, i.e. angle of arrival and time difference of arrival, ANN provides the reliable foundation for the robust integrated procedure.

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$$s_1(t) = K_1 \cdot s(t - T_1) + n_1(t), \quad (\text{Eq. 1})$$

$$s_2(t) = K_2 \cdot s(t - T_2) + n_2(t), \quad (\text{Eq. 2})$$

$$x_1(t) = A \cdot \cos(\omega \cdot t + \varphi_1), \quad (\text{Eq. 3})$$

$$x_2(t) = A \cdot \cos(\omega \cdot t + \varphi_2), \quad (\text{Eq. 4})$$

$$\cos \alpha = \frac{c(\varphi_2 - \varphi_1)}{\omega d} \quad (\text{Eq. 5})$$

$$X[k] = \sqrt{\frac{2}{N}} \cdot \beta[k] \cdot \sum_{n=0}^{N-1} \Psi_{n,k}, 0 \leq k \leq N-1 \quad (\text{Eq. 6})$$

$$\Psi_{n,k} = x[n] \cdot \cos\left(\frac{\pi \cdot k \cdot (2n+1)}{2N}\right), \quad (\text{Eq. 7})$$

$$\beta[k] = \begin{cases} \frac{1}{\sqrt{2}}, & k = 0; \\ 1, & 1 \leq k \leq N-1. \end{cases} \quad (\text{Eq. 8})$$

$$SAD(m) = \sum_{n=0}^{N-1} |R_m(\varphi_{n+1}) - R_m(\varphi_n)| \quad (\text{Eq. 9})$$

$$ME(m) = \sum_{n=0}^{N-1} |R_m(\varphi_n)|^2 \cdot J \quad (\text{Eq. 10})$$

$$\alpha = H(\mathbf{Q}, \Theta)(\mathbf{p}). \quad (\text{Eq. 11})$$

$$X_i = F(\alpha_i). \quad (\text{Eq. 12})$$

$$\mathbf{y}_f = f\left(\sum_{i=1}^N (x_{f,i} w_i)\right) \quad (\text{Eq. 13})$$

$$\mathbf{y}_{b,i} = x_b g\left(\sum_{i=1}^N (x_{f,i} w_i)\right) \quad (\text{Eq. 14})$$

$$g(v) = df(v)/dv. \quad (\text{Eq. 15})$$

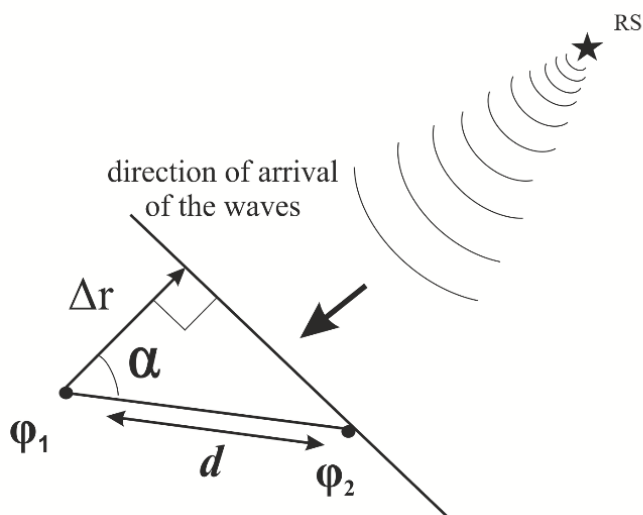


Figure 1. Received signals at two spatially separated points

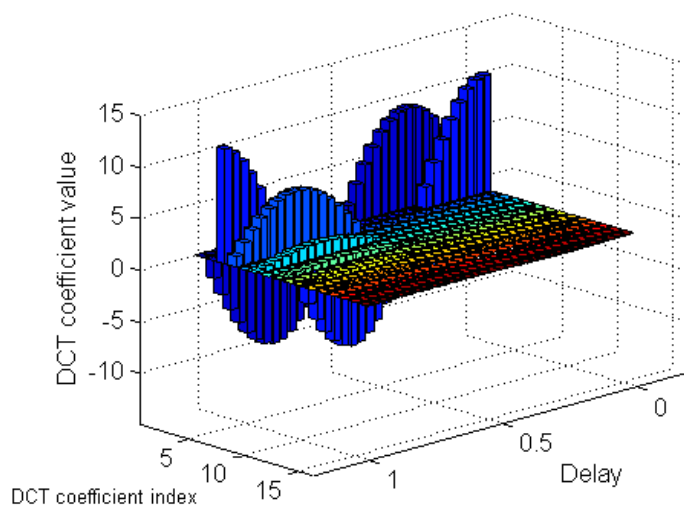


Figure 2. DCT energy compression

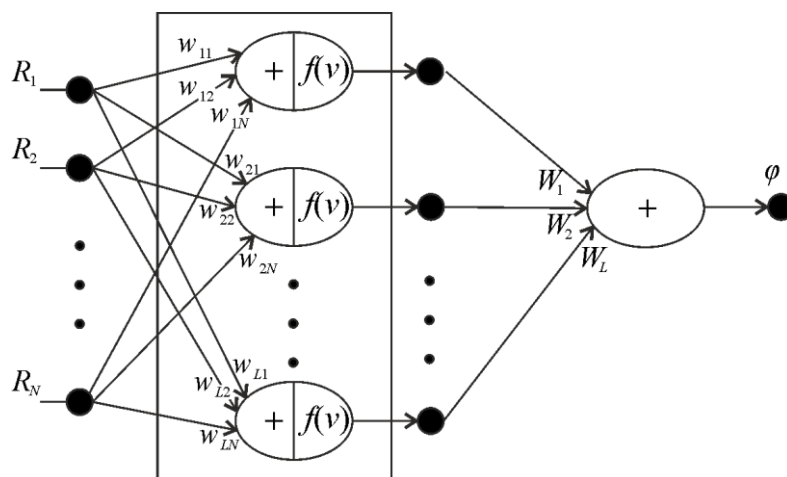


Figure 3. Multilayer perceptron as ANN angle evaluator

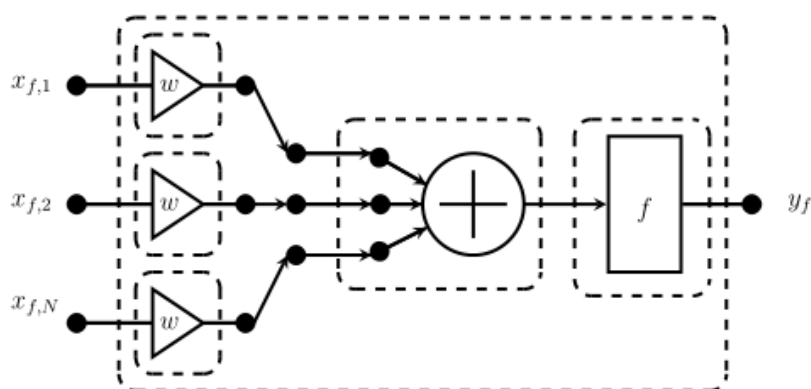


Figure 4. Regular neuron with three inputs during feed-forwarding

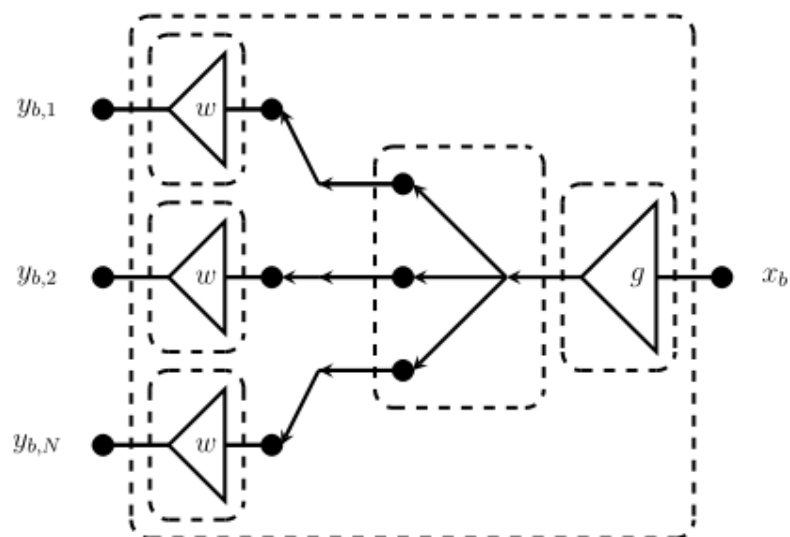


Figure 5. Three-input neuron during back-propagation

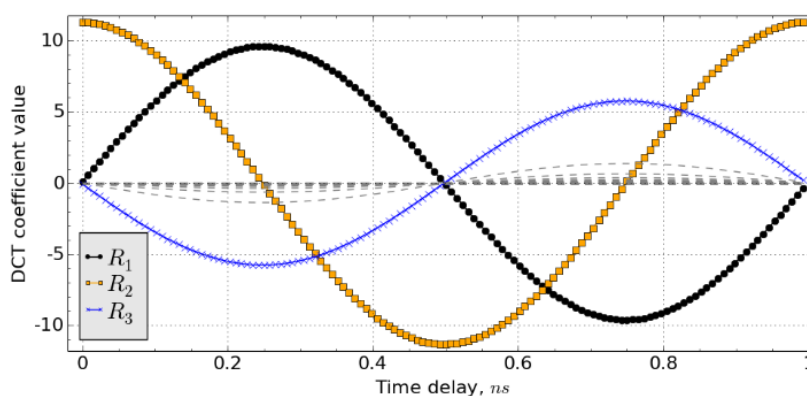


Figure 6. DCT values dependency on the phase shift

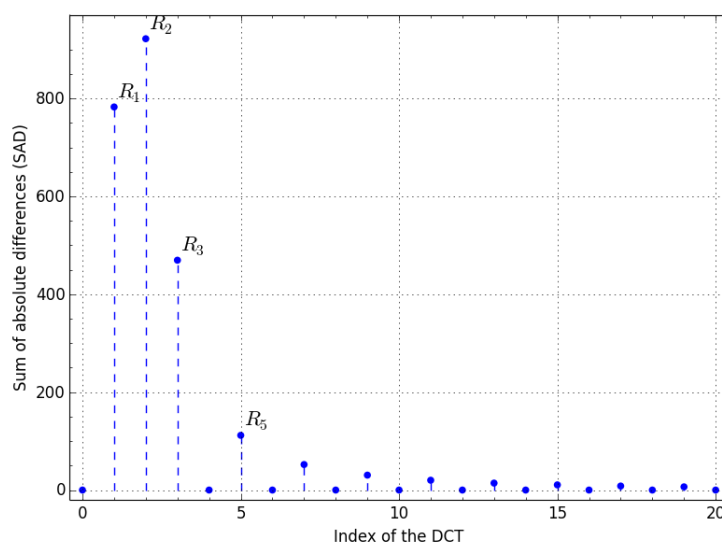


Figure 7. SAD evaluated for various DCT coefficients

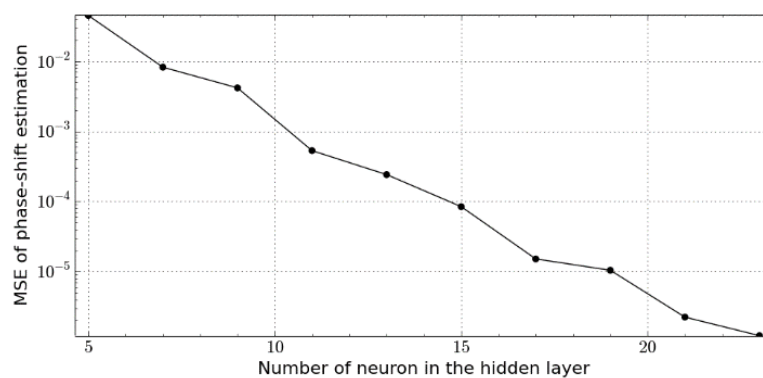


Figure 8. MSE against the number of neurons in the hidden layer

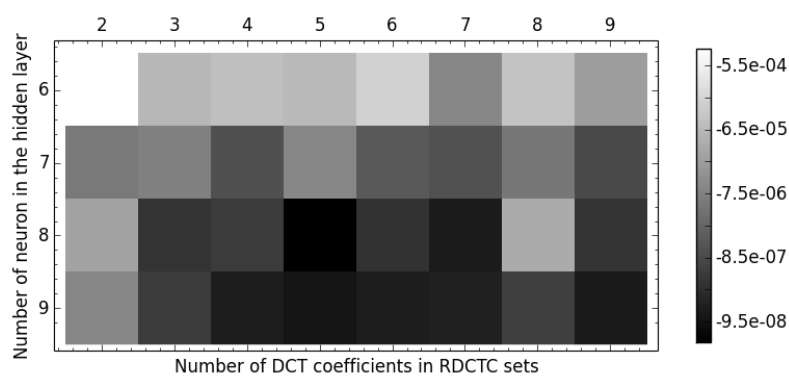


Figure 9. MSE against the number of neuron in the hidden layer and size of the RDCTS sets