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PEDAGOGIA DO CÁLCULO NA ÍNDIA: UMA INVESTIGAÇÃO EMPÍRICA

PEDAGOGY OF CALCULUS IN INDIA: AN EMPIRICAL INVESTIGATION

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RESUMO

Quando os alunos aprendem uma construção de cálculo, tanto uma imagem conceitual quanto uma definição de conceito são impressas em sua mente e, por causa disso, exemplos concretos e da vida real se tornam um pré-requisito para um ambiente de aprendizado contextualmente rico para as abstrações inerentemente presentes no cálculo. À luz das proposições mencionadas, o presente estudo se concentra em investigar várias questões, algumas das mais importantes incluem a natureza epistemológica do currículo de cálculo nas escolas de ensino médio da Índia, papel dos professores de cálculo indianos na cognição dos alunos, possibilidade de enumeração de características de um professor de cálculo bem-sucedido em relação ao meio sociocultural da Índia, desafios relacionados à imersão completa do cálculo na manipulação de símbolos que eventualmente dão origem a obstáculos cognitivos, inter-relação entre o conhecimento do conteúdo de cálculo dos professores e suas práticas pedagógicas, efeito do cálculo da escola secundária sobre o desempenho do cálculo da faculdade de estudantes indianos e a natureza do efeito em alunos indianos que fazem cálculo na escola sobre seu desempenho processual e conceitual. Para este extenso estudo, foram coletados dados de PGTs e Professores Auxiliares / Associados com mais de 8 anos de experiência em ensino de cálculo em 76 escolas, faculdades e universidades diferentes, pertencentes a 23 estados e territórios sindicais da Índia. Participaram deste estudo 323 professores. Múltiplos métodos de coleta de dados foram utilizados, incluindo observação naturalista, entrevistas estruturadas, observações em sala de aula, entrevistas em grupo focadas e discussões informais, e estas foram realizadas antes e depois do ensino em sala de aula. O pesquisador transcreveu as entrevistas, identificou temas emergentes e repetidos e utilizou o software NVivo e Concordance para conduzir a análise de conteúdo e discurso em sala de aula, com métodos simples de contagem e abordagem aplicada à teoria fundamentada, na qual os dados empíricos foram categorizados tematicamente e, no processo, empregados. a abordagem de indução. O pesquisador analisou as transcrições usando N5 (NUD * IST 5.0; QSR International, Melbourne) com a abordagem da teoria fundamentada. Este estudo de pesquisa é de natureza puramente qualitativa e sua estrutura está dentro do paradigma interpretativo. O estudo atual foi realizado entre junho de 2016 e março de 2019. Os resultados indicam que existem muitos obstáculos cognitivos na compreensão dos conceitos incorporados no cálculo: dois dos mais destacados que saíram do estudo incluem o relacionado às intuições e o outro relacionados a aspectos lingüísticos / representacionais.

Palavras-chave: *Currículo; Diferenciação; Avaliação; Integração; Educação Matemática*.

ABSTRACT

When students learn a calculus construct, both a concept image as well as a concept definition is imprinted in their mind, and because of it, concrete and real-life examples become a prerequisite for a contextually rich learning environment for the abstractions inherently present in calculus. In the light of aforementioned propositions, the current study focusses on delving into several issues, few of the prominent ones include the epistemological nature of calculus curriculum in India's senior-secondary schools, role of Indian calculus teachers in students' cognition, possibility of enumeration of characteristics of a successful calculus teacher with regards to India's socio-cultural milieu, challenges regarding complete immersion of calculus in manipulation of symbols that eventually give rise to cognitive obstacles, interrelationship between teachers' calculus content knowledge and their pedagogical practices, effect of secondary school calculus on performance of Indian students' college calculus, and the nature of effect on Indian learners having calculus in school on their procedural and conceptual performance. For this extensive study, data were collected from

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PGTs and Assistant/Associate Professors having more than 8 years of calculus teaching experience drawn from 76 different schools, colleges and universities belonging to 23 different states and union territories of India. A total of 323 teachers took part in this study. Multiple methods of data collection were used including naturalistic observation, structured interviews, classroom observations, focussed group interviews, and informal discussions, and these were done both before and after the classroom teaching. The researcher transcribed the interviews, identified emerging and repeated themes, and used NVivo and Concordance software to conduct content and classroom discourse analysis, with simple counting methods and applied grounded theory approach using which empirical data were thematically categorized and in the process of it, employed the induction approach. The researcher analyzed the transcripts using N5 (NUD*IST 5.0; QSR International, Melbourne) with the grounded theory approach. This research study is purely qualitative in nature and its framework lies within the interpretative paradigm. The current study was carried out between June 2016 and March 2019. Findings indicate that there are lots of cognitive obstacles in understanding the concepts inbuilt in calculus: two of the prominent ones that came out from the study include the one related to intuitions and the other related to linguistic/representational aspects.

Keywords: *Curriculum; Differentiation; Evaluation; Integration; Mathematics Education*

1. INTRODUCTION

It's been several years that researchers have been discussing and debating about the very nature and purpose of making school and college students learn mathematics (Dossey, 1992; Orton & Wain, 1994). In most of the countries be it developed or developing, it is seen that mathematics in school has a position that is privileged over other subjects and that the status which it enjoys is because of its usefulness and application which is in stark contrast to others' beliefs who view mathematics as the highest form of culture and that which emphasizes abstractness having formal proof and that it focusses inside of itself (Gardiner, 1995; Neumark, 1995).

The recurrent problem of introducing integration and differentiation to newbies is the frequent reinforcement of certain typical questions that involves asking them to solve, graph, calculate, plot, compute, differentiate, sketch, determine, etc. (Ferrini-Mundy & Graham, 1991). Students' learning a concept or a construct without knowledge and comprehension of its meaning has been the issue of research for several decades (Hiebert & Carpenter, 1992). In the 1980s, because of the visible crisis pertaining to learning and teaching of calculus, the US witnessed a movement that inspired changes in the manner in which calculus was taught to students (A. Tucker & Leitzel, 1995).

There has been an attempt by several authors to expand the "Rule of Three" to incorporate enactive and formal representation (David Tall, 1996); representations using animations (Bowers, 1999; Leinbach, 1997); representation of real data (Kaput, 1998) wherein learners experiencing states of affairs that are close to reality and natural phenomenon and implanting the usage of functions in data that are real and representations that are verbal (Kennedy, 2000). (Asiala, Cottrill, Dubinsky, & Schwingendorf, 1997) have talked about what existing literature says with regards to learner's understanding of functions and reports that students have a conception or mental representation of functions that are pretty infirm and that they show a tendency of relying on algebraic formulas while evolving and formulating their conception of functions. (Koirala, 1997) sets the students' conceptual understanding while teaching calculus in the theoretical account that (Skemp, 1976) gave about the relational and instrumental cognition involving giving rules and formulas to students in solving problems on calculus and the application of those formulas by students in solving calculus problems that are of routine-nature which does not require students to put any brain to delve into its basics.

(Schwalbach & Dosemagen, 2000) emphasizes the use of concrete examples giving students a contextually rich environment to inquire into the abstractions inherently present in calculus. When a student learns a certain construct, a concept image, as well as a concept definition, is built in her mind (David Tall & Vinner, 1981). It is mostly by accident that process of concept construction occurs among learners because they learn using identical textbook problems which lead to naïve or intuitive structures that are immune to transformation and the belief of abstractions that are reflective (Piaget, 1985), which (Dubinsky, 2002) had conceptualized for summarization of traits that is present in constructively reflective abstractions emanating from the viewpoint of mathematical thinking of higher-order, constitutes co-ordination, encapsulation, internalizing, reversibility and generalization.

Using Dubinsky's model, (Repo, 1994) has come up with an explanation of reflective abstraction in the construction of cognitive structures on constructs pertaining to derivatives. It is very much possible that if learners could internalize concepts that are specific to derivatives, and work towards development of capabilities in conceiving a function of derivatives as the nodal unit for processing, they can in subsequence easily fabricate a novel inverse process for delving in operations of differentials, thus making it pretty alike to determination of integration of the originally chosen function. (David Tall, 1996) delineates the proposition given by (Sfard, 1992) that viewing of mathematics operationally is preceded by structurally viewing it, considering objects and the formal definitions and this could have prominent implications with regards to theories of teaching. The difference which is there among "*concept definition"* and the holistic impression of "*concept image"* is reprised by (David Tall, 1996) and this distinguishing difference made some mathematicians (David Tall & Vinner, 1981; Vinner & Dreyfus, 1989) explicate certain lack of successes in students' understanding of it. (Gray & Tall, 1994) propounded and used the notional belief of "*procept*", describing it as an amalgamation of process and concept, thereby laying claim that it is, in particular, conformed to the contemplation of calculus learning and its initial analysis. They opined that functions, integrals, derivatives and the notions of fundamental limits are all examples of *procepts*. When concepts are viewed in more than one setting and from diverse viewpoints, it becomes an essential noetic state of cognition that is visualized as the facet of "general idea of flexibility" talked about by (Dreyfus & Eisenberg, 2012). In their study, (De Guzmán, Hodgson, Robert, & Villani, 1998) show that at different stages of an individual's learning and education, students show varying levels of maturity while proving a theorem. It is specifically expected from tertiary level learners to showcase correct formalism while engaging with non-trivial proofs. An attempt was made, applicable to diverse mathematical areas to teach analysis. And was showcased in the study carried out by Legrand and the approach under consideration, often referred to as "scientific debate" (Artigue, 2001; Legrand, 1993), has its roots in a particular type of discourse among learners with regards to theorems' validity. If students encounter arguments that are structured with regards to mathematical content, deeper development of cognition of fundamental concepts is seen.

(Praslon, 1999) in his study has examined the excogitation of derivatives as a case study of non-continuities in transitioning from the school to university and the findings attest to the attempts that students make in adjusting their mathematically sound learning aids to situations that are baffling and complicated leading these attempts towards oversimplifications that are consequences of their limited field of experience. Most of the researches carried out till date are towards reforming calculus, emphasizing majorly on opinions made or descriptions given either on software programs that ease calculus learning, or of the contemporary curricula which eventually lead to mere informing the readers of *what* and *how* of calculus learning and teaching, relevant examples to which can be found in (A. H. Schoenfeld, 1995), (Douglas, 1995), and (Solow, 1994). In many instances of research in calculus education, the evaluations have made use of experimental and quasi-experimental designs that are straightaway borrowed from physical or natural sciences. They have used two groups, one where exposure is given and other where no exposure is given and comparison is drawn between the pre-test and post-test scores where randomization techniques were used for selection of samples in a relatively controlled environment making use of the reform approach. Tests sometimes were specifically designed for assessment of certain specific kinds of students' performance, like their proficiency in handling traditional algorithms or in the determination of their ability to solve problems that are conceptually driven. Relevant examples to it can be found in (Armstrong, Garner, & Wynn, 1994) and (Bookman & Friedman, 1994). Comparative studies of this kind have certain long-familiar limitations, [see e.g., (A. Schoenfeld, 1994)] prominent one is its poor suitability in studying a phenomenon that is as complicated as students' learning and teaching. Prima facie it is found that, in many cases of researches done towards improvement of pedagogy of calculus, what is identified turns out to be an interesting phenomenon but the findings, results and conclusions has less of bearing when it comes to its contribution on cognition of effective techniques to be implanted in teaching and learning of calculus. Identification of interesting occurrences, say for instance, determination of differential performance of students from different courses on exam questions wherein the study is using diverse designs of experimental research or observations of learners' problem-solving behavior while they are in calculus class, have the potential for generation of results possessing explanatory power for uncovering differences in performances of learners. In few of the researches, such methods are used in examining research questions that are non-comparative in nature, to read more on non-comparative studies and examples of it see (Selden, Selden, & Mason, 1994); (Palmiter, 1991); (Park & Travers, 1996); and (Bonsangue & Drew, 1995). It is unfortunate though that studies yielding results possessing impregnable explanatory power are scarcely available and studies that are comparative in nature undertaken on the teaching of calculus and reform in its curriculum have not been able to effectively advance our cognition about students' calculus learning and on how diverse pedagogical circumstances can positively impact their understanding.

It has been documented in several types of research that what appears to be pupils' impuissance in the cognition of concepts of calculus might not actually be the case; in fact, it may just be the manifestations of their pre-existent comprehension of associated concepts. Learners may, for example, comprehend the conceptions of functions in a certain way that serves them considerably in few particular situations but are not in consonance with, or are not supportive of the developments of a sturdy inference of derivatives, illustrations related to it are witnessed in (David Tall, 1992), (S. Monk & Nemirovsky, 1994), (Ferrini-Mundy & Graham, 1994), (Williams, 1991), and (White & Mitchelmore, 1996). A study conducted by (Kuh, Kinzie, Schuh, & Whitt, 2011) found that to accelerate pupil learning and collegiate excellence; thought-provoking and intellectually stimulating creative work is fundamental.

There are certain beliefs that are commonly held by secondary school students of mathematics that were outlined by (Garofalo, 1989) based on observations and conversations he engaged with his students during his long career as a mathematics teacher. These beliefs include: teachers and textbooks are the sole authority of knowledge and that most of the mathematical problems could be solved by directly applying the facts, theorems, rules, formulas and procedures emanating directly from teachers and textbooks, (p. 502); formulas rather than their derivations are important (p. 503); and that the teachers/textbooks are the sole dispensers of knowledge (p.503), concomitantly the way students engage with mathematics, the approach with which they solve math problems and their expectations with the nature of a mathematics classroom are directly affected by the belief systems students hold (Bookman, 1993; A. H. Schoenfeld, 1995). It has been recommended by (Smith & Moore, 1990, 1991) that school/college teachers shall involve less in delivering lectures and more on encouraging them engaging in group tasks/activities [see also (Bookman & Blake, 1996)].

Most of the prominent contemporary researches in mathematics education that focusses on reformation leading to enhancement of teaching efficiency are works that are either on mathematics pedagogy at elementary or secondary level or on calculus learning at school and university level (Douglas, 1986; N. C. o. T. o. M. C. o. S. f. S. Mathematics, 1989; N. C. o. T. o. M. C. o. T. S. f. S. Mathematics, 1991; M. Tucker, 1990), mathematical fraternity thus are now recognizing the immense importance of calculus and are thus getting all the more connected in the process (Young, 1987). A constructivist mathematician would state that learning math is a process in which students engage in reorganization of their activities for resolving problem areas that are found excessively difficult to them (Cobb et al., 1991) and in alignment with this take, constructivists agree on construction of mathematical knowledge in the classrooms via reflective abstraction, and that there is continuous development of cognitive structures (Noddings, Maher, & Davis, 1990).

In the field of mathematics education, it has been found that several researchers have showcased their interest in facets of learners' cognition of functions at the senior-secondary schools and colleges (Buck, 1970; Dreyfus & Eisenberg, 1983, 1984; D. H. Monk, 1987). Findings of these researches have shown that most of the learners at this level are stuck to a single definition of functions that is being staged by the correspondence rule whose domain is unvarying in its entirety (Ferrini-Mundy & Graham, 1991; Markovits, Eylon, & Bruckheimer, 1986; Vinner & Dreyfus, 1989). When there is movement in thought process from graphical mode to algebraic mode, piecewise definition of functions bring about massive difficulty for the learners (Ferrini-Mundy & Graham, 1991; Markovits et al., 1986; Vinner & Dreyfus, 1989), and additionally, learners for most of the time, ascertain whether or not graphs represent functions by its measure of acquaintance (Ferrini-Mundy & Graham, 1991). In the settings where students have both graphical and algebraic data, they often view them independently and oftentimes find comfortability using methods whose reasoning is contradictory (Ferrini-Mundy & Graham, 1991). Tendency of students have been noted by researchers, and while examining their behavior about a graph both locally and at any one point and also algebraically, students have shown the tendency of evaluating formulas for one domain value, while on the contrary, a holistic rendition is usually of utmost importance in cognizing concepts in calculus (Bell & Janvier, 1981; D. H. Monk, 1987).

It has been observed that researches on pupils' cognition of limits are not very extensive and learners often encounter conflicts among precise/formal and informal definitions that use interpretations in simple language and in natural discourses (Confrey, 1981; Graham & Ferrini-Mundy, 1989; D Tall & Schwarzenberger, 1978; Williams, 1991) and pupils often have this feeling that a limit can never be reached and are mostly anxious for the fear of encountering a mismatch between their instinctual knowledge and the solutions they come up with via mathematical processing (Ferrini-Mundy & Graham, 1991) and in connection to it (Davis & Vinner, 1986) figured out that learners keep holding a very similar visceral perception regarding limits of sequences. Easy accessibility and inexpensiveness of graphics calculators make students of pre-calculus and calculus, studying in formal educational institutions; use this technology extensively, thereby showcasing a substantial impact on classroom instruction.

It is widely claimed that usage of graphics calculators makes room for enhanced conceptual approaches to a problem-solving, refined understanding of the bond between graphic representation and symbolic algebra, and sharpened ability among students in solving mathematical problems. Students now take all the benefits emanating from it because earlier they were asked to solve problems the way traditional and formal mathematics required (Demana, Waits, & Clemens, 1993; N. C. o. T. o. M. C. o. S. f. S. Mathematics, 1989; DavId Tall & Blackett, 1986).

Considering the aforementioned issues egressing from the literature, the researcher aims to investigate the epistemological nature of calculus curriculum in India's senior-secondary schools, role of Indian calculus teachers in students' cognition, possibility of enumeration of characteristics of a successful calculus teacher with regards to India's socio-cultural milieu, challenges regarding complete immersion of calculus in manipulation of symbols that eventually give rise to cognitive obstacles, interrelationship between teachers' calculus content knowledge and their pedagogical practices, effect of secondary school calculus on performance of Indian students' college calculus, and the nature of effect on Indian learners having calculus in school on their procedural and conceptual performance.

Research assumptions

Frid, in his research, focuses on teaching the principles of calculus with different approaches: a study of comparisons of conceptual and infinitesimal approaches vs. technique-oriented approach of understanding calculus (Frid, 1994). This *frame of reference* reflects the diverse impact it has on using different approaches to students' sources of conviction and language use. There are a lot of cognitive obstacles in understanding the concepts of calculus, two of the prominent being: one related to intuitions and the other related to linguistic/representational aspects. Now that calculus is completely immersed in the manipulation of symbols, that too at the expense of students' mathematical understanding, consequently giving rise to cognitive obstacles through linguistic and/or representational aspects. Students are often pretty curious to know what they are being asked to learn (here it is calculus), and the possession of their intuition has an emphatic role to play in their construction of calculus concepts. (Norman & Prichard, 1994) in their research propose that a framework that is useful to fix securely the considerations of cognitively demanding obstacles that confines in the realm of the framework proposed by Krutetskian cognitive processes of flexibility, generalization and reversibility and to develop the understanding of calculus among learners, the importance underlying the connections between different representations be it visual-spatial, graphical, concrete, algebraic or numeric is bobbing up into existence.

Harvard Consortium Calculus text gave a guiding principle by the name "Rule of Three", which states that topics of the syllabus, as far as possible, shall be transacted using all three techniques, namely graphically, numerically and analytically whose rationale is to come up with a course where all the three viewpoints are balanced, and students get to view all the major concepts from all possible angles (Hallett, 2006). The most significant takeaway from the analyses of the researches done so far in learning and teaching of calculus is that the students before they immerse into symbolic manipulations, more emphasis they should place on learning conceptually and making use of multiple representations and connections.

2. MATERIALS AND METHODS

The author has traced the development of calculus-teaching at schools and colleges and identified commonalities and differences. For this extensive study, data were collected from post-graduate teachers (PGTs) and Assistant/Associate Professors having more than 8 years of calculus teaching experience, drawn from 76 different schools, colleges, and universities, belonging to 23 different states and union territories of India. Data were mostly collected using a schedule consisting of 24 items. A total of 323 teachers took part in this study. Data were also amassed making use of structured interviews, classroom observations, focussed group interviews, and informal discussions that were done both before and after classroom teaching. Semi-structured interviews were audio-taped and were carried out with each of 323 teachers (its duration was close to one hour per teacher).

Along with that, the researcher took field notes of their observations from teaching sessions that were conducted by 62 calculus teachers. Towards the end, the researcher was in possession of a large amount of data. This research study is purely qualitative in nature and lies within an interpretative paradigm/framework. This study was carried out between March 2016 and May 2019.

It was specifically taken care of that the 76 educational institutions chosen for the study were geographically separated from one another - this was done to ensure that the sample was a true representation of India. Interviews were transcribed, and those themes that kept egressing and echoing were keyed out. NVivo and Concordance software programs were used for the analysis of contents and classroom discourses, making use of plain enumeration methods.

The grounded theory approach was employed, using which, thematic categorizations of empirically derived data were done using the inductive approach. Validity checks allowed apparent cogency of the authors' experiential accounts, assaying éclaircissement and illustration of central ideas throughout the interview process, and devotion of concentration to aberrant illustrations, cases, and examples was done in extreme minute detail.

The teachers were selected after being granted due permission from the principals, directors, HoDs of schools, colleges, and universities. The time period of the study was 2 years and 9 months, i.e., from June 2016 till March 2019. Focused group interviews were conducted through schedules that were pre-designed as per the objectives of the research. Semi-structured designs were used in focussed group interviews. The length of each interview was between 90 and 120 minutes. Teachers were asked to consecrate to concealment by not quoting what other participants in their focus group discussed. Teachers were promised by the investigator of the prudence of what they revealed and were inspired with confidence to be as frank, forthright, blunt, and honest as existing in possibility. Tape-recording of interviews were done, they were then amply rewritten in a different script, and were examined for initial analysis by the researcher.

The final analytical investigation disclosed that the themes from the first two analyses, even though they were contrastingly clubbed, excerpted exactly identical situations and events from the empirically derived data. This the researcher took as ratification of the basing of the investigation of the data. To include it as a theme, affirming data were contained in focus groups from all 76 educational institutions and in all of the focus groups. Indexing of the extracts of the interviews was done, e.g., EdInt1, EdInt2, etc. Participants represented a mix of both the genders, and their ages varied from 29 to 57 years. In order to prevent the identification of the participating institutions, teachers were advised to make anonymous the details of the educational institutes and hide any such materials that could lead to the recognition of their schools/universities.

Researcher was an integral part of all the focus group interviews and ahead of each meeting, the researcher explicated the agenda of the research. It was clearly and categorically stated beforehand that each of the participants will be anonymized. After the interview got over, no contact was established with the participants. Copies of the transcripts were returned to the groups, and each participant understood that if the content in it is not what they intended to convey, the transcripts, in that case, will be discarded and won't be used in the study.

Separate analyses were done of transcribed tapes of each of the interviews and meetings. The grounded theory approach was used by the researcher in the development of theoretical and explanatory principles.

The coding of themes was done consistently and robustly following grounded theory rules, and all the emerging themes directly supported the verbatim data coming from the interviews. The overarching aim of generating theory from the findings was never an objective of this study. Probing questions were asked in the middle of the focus group interviews so that they easily open up and that no elements remain untouched. To establish rapport and to make them feel at ease, informal chit chat was done to attract their interest and it was ensured to them that whatever they say will be out in the open anonymously. These strategies were used to stimulate healthy discussion during focussed group interviews. A total of seven participants refused to participate after reading through the transcribed data, and 16 respondents did not differ in key characteristics.

After all the teachers were communicated about the goals and design of the research, a consent letter was taken into possession from the participants for interviewing them, which states that they were free to leave the focus group or interview sessions, if they wished to, at any stage of the research. Approvals were obtained from 11 local ethics committees, and in order to shield the identity of the teachers as well as that of the educational institutions, pseudonyms were rendered to them. Constant comparative method (Bogdan & Biklen, 2003) was used to analyze the transcribed verbatim interviews. Data were analyzed inductively, and the identification of common themes and concepts were made across experiences (Lincoln & Guba, 1985).

Once the researcher was done with the coding, and drawing up of the themes were accomplished, the comparison of all the themes was made. It eventually resulted in the final six themes for the expatiated study. Although identification of several codes was made, only the ones with the strongest bearing were expended (assorted with more than 50% of the interview sample). The array of codes that were with me in the beginning, the excerptions were aggrouped and gestated utilizing them and were then made part of the adoption factors. Complemental codes and constructs were, however, admitted if they were to egress in the analysis.

After the initial coding was discharged, the database of excerptions underneath every factor was reread again and again to ascertain coherent applications of excerpts, and the factors were systematically framed. In the course of these scrutinies, certain transfigurations to the coding were brought in. As an example, certain excerpts were situated underneath a dissimilar code. Multiple techniques of data collection were utilized by blending focussed group interviews, in-depth interviews, and naturalistic observation. Investigator analyzed transcripts making use of N5 (NUD*IST 5.0; QSR International, Melbourne) accompanying the approaches employed in grounded theory methodologies. For the purpose of revision, transcribed transcripts were returned to each of the respondents.

The researcher formulated themes from the transcripts. Marking and linking of the segments of texts were executed from different interviews that addressed similar concerns or mattes or experiences making use of NUD*IST. Considering the contexts of all the interviews, themes were conceived. It was impossible to develop inter-rater reliability scores because the interviews had very little similarity with respect to complex composition—scores of this nature are inappropriate for the data that have minuscule or no predefined coding. Triangulation method employing diaries, questionnaires, and interviews were consecrated to get over powerful criticisms of common method bias in the methods that were used predominantly. Diaries were effectively used in recording the data. This tool has been opted in spite of the complemental exertion demanded in the collection of data. In this case, diaries also playacted as a think-aloud mechanism, which eventually helped the researcher in effectively capturing teachers' cognitive processes.

3. RESULTS AND DISCUSSION

Integration and differentiation has two distinct conceptual settings, one geometric and the other dynamic. Calculus teachers are sufficiently comfortable moving between them that they often forget how difficult it can be for students to grasp their equivalence. It has been found that intervention programs like calculus workshops promoting excellence in academia and fruitful classroom interaction have a direct bearing on pupil's academic achievement on both basic calculus courses as well as advanced-level calculus courses and thus supports the suggestions detailed in (N. C. o. T. o. M. C. o. S.

f. S. Mathematics, 1989; N. C. o. T. o. M. C. o. T. S. f. S. Mathematics, 1991) and (Weissglass, 1992). Looking at the persistence, performance, and cost findings, the current study emphasizes on needs in schools and colleges to possess the facility to accurately pursue the scholastic achievement of the pupils across time. The achievement among students in calculus learning appears to be linked less to pre-college aptitude than to their in-college scholastic occurrences, activities, performances, and anticipations; and it has prominently surfaced that when students are taught calculus by "discovery" approach, they do as well on problems and manipulative skills as those with traditional instructions, but in addition, they have increased understanding thereby making discovery approach to hold possession of a superior knowledge of the fundamental theory and logical relations among parts of the calculus, consequently, leading to students, experiencing the thrill of discovery and the satisfaction of producing results through creative effort - all of which lead to greater enjoyment of mathematics and a deeper understanding of its nature and use thereby making students express ideas of calculus in their own language and undergo the stimulating and disciplinary experience of having their expressions and ideas sharpened through examination by other students as well as by the teacher.

The integration of digital tools in calculus learning and teaching has been initiated by a few schools in India. However, in most of the Indian schools, classroom teaching of calculus is traditional, and thus emphasize those aspects of knowledge that are truly procedural. In those instances where the emphasis is more on the application of concepts in calculus teaching, often contradictions exist with other contextual matters like for example, our evaluation system presses calculus learners to perform relatively fair in regular problems, thus indicating that it is being operated at the action level. It came along from the interviews from calculus teachers that, even though, pupils have the right answers, yet they lack understanding of the concepts.

Learners based on their previous mental assimilations frequently constructed their knowledge, relying by and large on thinking procedurally and not on thinking conceptually while solving tasks on calculus. (Hobden, 2006) in his research stressed on plausible reasons of learners successfully engaging with mathematical constructs, it is emphasized that learners require to be efficient and competent in both conceptually understanding the constructs as well as in developing procedural fluency. The current study found out that some groups successfully construct their knowledge of derivative with the notations' (e.g., dy/dx) instrumental understanding and for them it is the representation in every context with regards to derivatives and it appeared that construction of their mathematical knowledge took place as facts that were isolated and they found difficulty in seeing interrelationships between different constructs. Learners' responses to teaches' queries made the researcher believe that they struggled to build a connection between maxima/minima and different functions.

In few other cases, it appeared that calculus students were struggling to accommodate those concepts that they newly learned with the ones they had previously learned; e.g., the concept of minima/maxima is taught to students of 11th grade, but they find it difficult in applying that know-how to 12th-grade calculus, and if learners have quadratic equations at hand and to them as they have learned it, it helps generate parabolas and it is known how can one ascertain the turning point in similar cases but now when they engage in learning calculus, minima/maxima has altogether different meaning, and are in search of other mechanisms to deal with it which led the researcher to come up with few issues pertaining to calculus cognition that emerged in the research findings. Few of these include: 1) there was a lack in understanding about notations (e.g., $\frac{dy}{dx}$ by the learners, 2) construction of schema for derivative and maxima/minima is missing among the learners 3) modeling the problems is a weak spot for the calculus learners 4) preference learners give more to rules and formulas, and 5) incorrect application of algebraic notations are seen in calculus students. In this research, responses of teachers brought out that the functioning of the majority of the students is at the phase of action and wherever there is a requirement of rules and formulas, they effectively solve all the questions. Repeated substitution of area and volume formulas into the problems and application of differentiation rules are seen to be done by students to find the derivatives. Whereas on the other hand, some learners often interiorize the formulas on volumes in a process where volume is visualized as a cubic function and encapsulating derivatives as an object in finding the minima. It is evident that this schema has partial assimilation in learners' memory structure, yet mostly they fail in coordinating with schemas that are there already present, for instance, that of functions and gradients that are essential in solving questions on optimization and in viewing its schema.

Calculus teachers in their interviews revealed that learners have instrumental knowledge of optimization that occurs with both the action stage and with APOS theory's process stage. Most of the time, the learners' knowledge creation is confined only by action conception. This is mainly for the fact that they could solve only such problems that are in the requirement of extraneous stimulation. Causes of it may be attributed to the creation of stimulus within the organized nature of the problems: coordination of others with complete totality in respective objects making other actions and processes act on them. The findings revealed that the entire basis of learners' knowledge creation is rooted in those conceptions and procedural functions that are totally detached from one another.

This possibly is the consequence of the mannerisms in which learning and teaching occur, which bestow greater vehemence on aspects that are procedural in nature. Disregarding conceptual inferences of the constructs of calculus and the questions that pupils are solving in their calculus classrooms encourage their instrumental understanding. Learners eventually broaden their inferences into different aspects of their areas of knowledge. This occurs: 1) when learners are demanded to find out a curve's gradient which is delineated by $y = ax^2 + bx + c$; here learners cite 'a' as the first term's coefficient and contradicting by iterating that the coefficient in the first term of $y = mx + c$ is employed for straight line's gradient, and 2) to find the minima/maxima at the turning point for $y = ax^3 + bx^2 + cx + d$, learners use the formula $x = -b/2a$ to find the x value at the local maxima/minima. Evidently, the learners made use of the knowledge they acquired when they learned quadratic functions. Now, talking about teacher knowledge and its role in teachers' practices, the findings indicate that both have an effect on actual teaching because teachers have to deploy both mathematical as well as pedagogical knowledge in her teaching in order to take "effective" decisions.

3.1 Significance of Learning Calculus in Schools, Colleges, and Universities

To study the phenomenal changes in the physical world, calculus plays a pivotal role in effective engagement with advanced physics and students who major in mathematics start off with introductory calculus as a groundwork in engagement with advanced calculus which helps in examining the underlying theory which runs in the background of it and in comprehending more complex problems including partial and directional derivatives and the geometry of three-dimensional spaces. Certain advanced courses in statistics and computer programming require expertise in calculus, and in spite of mathematical subspecialty, all specialized disciplines have a profound background in problem-solving techniques and thought processes of calculus. Since the problems that are solved with the help of calculus are continuously evolving, studying calculus becomes all the more crucial for students majoring in computer science, and as a consequence, computer programmers specifically will be able to define and involve in problem-solving in a stepwise fashion using methods of calculus.

The subtleties of numerical analysis require the usage of calculus, and the basic logical and analytical processes that are fundamental to calculus are thus having incalculable worth for the many careers that are available for computer science engineers and programmers. Talking about America of early 1980s, there came a movement to replace first-year graduate calculus with discrete mathematics but this move faced massive resistance by defenders of calculus who powerfully and effectively defended the importance and requirement of including calculus to the core of university curriculum for mathematics and eventually this matter was resolved and calculus in the first year of college again found its place in the curriculum. (Bressoud, 1992) found it apt but was a little dissatisfied. He believed that if systemic changes are to be carried out to improve undergraduate mathematics education, then mathematics teachers must be clear about the "Why?" of teaching calculus. It was found that the recommendation of CUPM was completely wrong in not changing the syllabus of the first semester of calculus and the current syllabus was inadequate as it stands. A feeling of alarm or dread and expectation was there among the students in their approach to calculus.

Students believe that the road ahead in learning calculus is going to be tough, but at the same time, they expect that the course will draw together from the mathematics that they have learned in high school and that their learning of college calculus will transform their earlier gathered knowledge of mathematics in better comprehension of the world around them by acting as a potent instrument. This tool actually exists in calculus but is often missed by the students. Owing to this, the students are left disillusioned and disappointed. Two answers came up to the question - *Why Do We Teach Calculus?* The first is that, for now many disciplines are using calculus and that too in many different contexts, so if the mathematicians don't teach it to the pupils, then Biologists, Engineers and Physicists will have to take up the job of teaching calculus to them. So the mathematics teachers, shall not shy away from teaching calculus. The usefulness of calculus in varied disciplinary areas is an insufficient answer to the question - *Why Do We Teach Calculus?* If this is the sole criterion, then by this logic, linear programming, and more so statistical analysis will turn out to be even more useful to a majority of college mathematics students, so mathematics teachers should teach only the topics on discrete mathematics instead of teaching calculus! Thereby, there came the second answer to the question. This answer had consequences that were revolutionary to the way mathematics teachers make students learn calculus.

Calculus is situated at the core of mathematical paradigm and concepts of calculus have helped develop modern scientific thought and to see mathematics outside the context of calculus is somewhat meaningless, and thus calculus positions itself at the very foundation of scientific world view and by that means, development of calculus brought mathematics into being. The second answer emphasizes the reason for the teaching of calculus on the vehemence of calculus in learning of locating ourselves in the society that constitutes us. It will be tremendously gross conduct against the first-year pupils of calculus if the teachers of mathematics aren't able to convey the excitement of making them uncover the secrets of nature using calculus.

3.2 Main Issues in Learning and Teaching of Calculus

The following questions attempt to throw light on the importance of learning calculus while transitioning from senior-secondary school to college mathematics: What are the effective policies and practices to remove obstacles and to overcome difficulties that students face in such transitioning? What can be done to place students in the appropriate course when they join college? What can be done to ensure that the chosen course enables students to move up the ladder in the courses that are built upon them? The transition from senior-secondary school to college mathematics is often damaging to pupils' sense of self-efficacy and also of their mathematical identity, especially in the case of women. What could be the core issues here, and what could be done to address them? Is there a relationship between currently taught calculus at the senior secondary school and its true needs in effective learning of mathematically intensive university courses? How shall colleges and university departments respond to the increasing number of students taking up calculus in senior-secondary schools in designing/shaping what to teach and how they are to be taught? What measures shall be opted to ensure that both senior-secondary school and college/university teachers make use of the most effective/efficient methods for teaching calculus? How to make certain that there is ample opportunity for the students to develop their abilities pertaining to mathematical practices that are critical in nature when they transition from senior-secondary school to college mathematics?

It came out from the interview of calculus teachers that students shall learn calculus not because there is some significant residual associated with it; instead, they must have the drive to learn it. Where the very purpose of learning calculus is not to develop a deeper and abiding understanding or to playfully learn while engaging with the tools of calculus, and to mere pass the exams with good marks, reasons of it could either be that students undervalue calculus as an integral part that will help direct their career path or could be because they know that they will be studying calculus again when they join college, the learning in both the cases will be quite superficial. Consequently, many of the mathematics students who get themselves enrolled in calculus in senior-secondary schools are inadequately prepared for calculus when they join college. In the best case, the learning trajectory shall unfold among students in such a way that the formal mathematics emerges in every mathematical activity a student undertakes and this ideal case is associated with (Freudenthal, 1991), wherein this contention states that mathematics should start and shall stay within common sense and he wished that this adage be understood dynamically, further arguing that common sense is dynamic and not static and remarked that what is common sense for a mathematician may not be what it is for a layperson and that common sense is something that evolves in the course of learning.

3.3 Alternative Methodologies for Teaching Calculus

It is stressed by many calculus teachers that sophisticated knowledge of elementary functions, including trigonometric, algebraic, logarithmic, and exponential functions, should be taught thoroughly before taking up teaching and learning of calculus. Our school mathematics curriculum has never come up with a satisfactory program in geometry. To effectively teach calculus in schools, full treatment of analytic geometry should be made an essential prerequisite. Understanding of analytic geometry is extremely important, so much so that teaching of finite mathematics and matrix algebra shall be done only after rigorously presenting analytic geometry to the students. (Allendoerfer, 1963) therefore contends that 1) complete abandonment of analytic geometry by colleges is their greatest loss and if at all analytic geometry is to be taught, it must appear in the curriculum of high school mathematics. 2) being a straightforward subject, analytic geometry, vis-à-vis calculus and probability, makes all the mathematics teachers of all the schools to easily handle the teaching content without the need for any specialized training. The question then comes regarding the time of the academic life of a student when she/he shall be taught calculus and who should be bestowed with the responsibility of teaching calculus to the school students? With regard to the former question, it is maintained that until the school students are well versed with the concepts of analytic geometry and elementary functions, they shouldn't be introduced to calculus. If by marathon teaching, these concepts are taught before Grade-12, the teaching of calculus shall be considered. It has been long advocated for a 6-week short course in calculus towards the end of Grade-12, as a fresh transition to 1st year in college. It is feared of this course getting any longer than 6 weeks and shorter than 10-12 months, for then it will simply replace the college course and will waste student's time and consequently the appetite for college calculus will be lost. Now from the teacher's viewpoint, the teaching of calculus effectively is a tough task and thus, calculus should be handled by the best trained, most efficient and shall be chosen from "*the lot*" of most thoroughly experienced mathematics teachers. Findings to the research bewails the appointment of average mathematics school teachers for a course in calculus and laments teaching of inexperienced students who takes up calculus in their 1st year of college.

Learning calculus can be a wonderful and exciting experience when it is taught well by experienced teachers and contrarily a horrible one otherwise. It is therefore urged to school authorities to not offer calculus as a course if they don't have efficient and experienced teachers, and it is advised to teachers to ask their students to diligently choose colleges where the college employs their most able teaching staffs for the course in calculus. It is commonly believed that differentiation and integration is what is there in the study of calculus, but in the true sense, this viewpoint is superficial and rather completely flawed. The prominent idea of calculus emerges from the concept of *limits* and without the systematic understanding of *limits*, a course in calculus is a complete failure. A course in calculus often begins with little or no preparation or forethought to *limits* as something which is difficult to comprehend by the students and consequently, calculus teachers get marveled considering the reasons for student's inability to handle even the routine aspects of calculus.

Using the epsilon-delta technique for problems on limits is usually done at the beginning of the course in calculus. An intuitive preparation is required, for it being a difficult new idea, if the epsilon-delta technique is to be dealt in full detail. The very idea and notion among students about the concept of absolute value and inequality is naively understood and is utterly oppressive. There is a dire need for alternatives because both the aforementioned approaches lead to failure. Mathematics teachers shall start off calculus by solving a substantial number of problems on limits which students are familiar with, but care shall be taken that those problems shall not be trivial. The triviality of problems can be understood with an illustration like this:

$$\lim_{x\to 3}\frac{5x^2 - 8x - 13}{x^2 - 5}$$

Presenting such problems is a futile exercise in learning *limits*. Understanding of real number system in general and completeness property, in particular, is crucial that could be least said in the cognition of the concepts of *limits*. The least upper bound is how students at this stage will grasp about completeness property. With regards to the collection of examples on plane geometry dealt in the first set, often referred to in popular discourse as "*incommensurable cases*", shall be made an integral part of the calculus course, in their very first few weeks.

The theorem worth examining in greater depth and understanding is "*a line that is parallel to any one side of a triangle will divide the other two sides of the triangle into proportional segments*", and another problem to be extensively understood is of "*defining the area of a rectangle, one of whose side is irrational*". From here, calculus teachers can move on to define the length of an arc of a circle. Another interesting problem to be followed could be on irregular inscribed and circumscribed polygons: "*to show that the length of any inscribed polygon is less than that of any circumscribed polygon and that there is a pair consisting of an inscribed and a circumscribed polygon whose lengths are arbitrarily close together*". Here nothing is being done, but integral calculus. Till here, calculus teachers have not introduced the concepts of integrals and the formal machinery of integral calculus, i.e., comprehension and articulation of *ideas* without being hung up to the complex formulas. After these geometry examples, it is expected from calculus teachers to shed light on the infinite series. To come up with an answer or solution on summing the elements of infinite series is always pretty fascinating for students of senior secondary schools, consequently leading to a meaningful payoff in their cognition of *limits*.

There was a time when infinite series had a place in every algebra book. Before a student starts to learn basic calculus, it was earlier understood both by publishers of mathematics textbooks as well as by the mathematicians that *limits* had to be introduced in the above mentioned way. It is contended that their experiences have just been bluntly ignored. At this stage, only the series on constants shall be treated, but then calculus teachers also shall immediately jump on to power series as it couldn't be postponed for long. Now the time is ripe for introducing students to examples and illustrations on limits of functions, such as the ones shown below:

$$\lim_{x\to 0}\frac{\tan x - \sin x}{x^3} \qquad \lim_{x\to 5}\frac{\sqrt{x^2+11}-6}{x-5}$$

After learning the aforementioned, students must have gained the understanding so well that it will enable them to use freely the epsilon-delta notation. Also, they will be in a position to understand and comprehend the concept of continuous functions and their properties. This is the stage in mathematics student's life when the traditional concepts of differentiation, integration, differential equations shall be brought to them.

Another important point to make here is with regards to the popular college textbooks. The introduction of integration after differentiation is seen in them, which is pretty disturbing. It is the least effective way to teach calculus as it is suggested to start with integration and finding areas and volumes of infinite series by summing the series the same way as discussed above. This is said to be the best takeaway: "*an integral is the limit of the sequence of sums*". After the establishment of this idea, calculus teachers shall then move on to solve the differentiation of polynomials and establish its connection with integration and consequently prove the Fundamental Theorem. Transcendental functions shall be introduced only after discussing Taylor's theorem, power series, and the interval of convergence. Now the calculus teachers shall expand by their series, the exponential and trigonometric functions, and derive their calculus without investing any further time. There is no other better or more refined approach to teach calculus with understanding vis-à-vis the aforementioned proposed approach.

4. SUGGESTIONS AND RECOMMENDATIONS

Envisioning of learning in a socio-cultural context by breaking down complex calculus concepts into parts similar to the idea vigorously pressed by Vygotsky, and such similar innovative techniques shall be emphasized for student's cognition that is all together process-related that uses scaffolding and classroom dialogue. Calculus reform needs to concentrate not only on increasing the success rate of all students but also on improving their ability, to apply calculus creatively, and to accomplish this, might well require calculus teaching to be structured differently. The author recommend the incorporation of videos in calculus teaching wherever required, for it is common knowledge that teachers too enjoy when more students with better preparation turn up for their class.

For students looking out for quality resources online when not fetched forthwith by educators makes it essential for the educators to acquire familiarity with such online content and to learn to develop online tools of their own for easy navigation of course materials for their students. Calculus modules that call for a little bit of background understanding make the flipped classroom model effective and all the more important. For students to recall before class what was taught in previous calculus classes, effective use of videos and web resources can be made, eventually saving a lot of time to be dedicated to the topic actually thought of by the teacher to be taught in the class thereby providing learners, who are having fewer time and greater social pressure limitations, an invaluable learning opportunity.

Calculus teachers need to keep in mind the Guershon Harel's Necessity Principle, which states that students are most likely to learn when they see a *need* for what teachers intend to teach them, where, by '*need*', it is meant *intellectual need*. Students require experience with a variety of problems for which the easiest and most natural approach is in terms of accumulation that can be modeled with a sum of products, multiplying the rate of accumulation by the small increment in the independent variable, problems for which the evaluation of the definite integral is the last step. The heart of the intellectual activity should be on converting a problem into an accumulation that can be calculated via a limit of appropriate sums. The focus of calculus teachers shall be first on the dynamic understanding, and then to use this to build the geometric realization of the theorems in calculus.

The analysis of this study on course-repeating patterns among workshop and non-workshop students suggest that time-effective, cost-effective, and a highly academic intervention program, if carried out near the learners' college career, could improve this misfortunate picture. Findings to this study suggest that a good calculus teacher is one who provides explanations that are understandable, who listens carefully to pupils' questions and comments, who help students become a better calculus problem solver, who allows time to students for understanding of difficult calculus ideas, who makes the students feel comfortable in asking calculus questions during class, who presents more than one method for solving the same calculus problem, who makes the calculus class interesting, who asks questions to students to determine whether they understood what was being discussed, and discusses and emphasizes more on the applications of calculus.

To be an effective calculus teacher, ambitious teaching strategies require strong student-teacher rapport. The practices that a calculus teacher shall opt include frequently allowing students to work on calculus problems with one-another, assignments that are to be completed outside class time shall frequently be submitted as a group project, exam questions shall require students to mostly solve word problems on calculus, calculus teachers shall frequently ask students to pen-down the explanations of their solution in the notebook, both home assignments and exam questions shall require students to solve calculus problems that are unique and different unlike those done in class or are present in the book, calculus teachers shall hold whole-class discussions giving students the opportunity to explain and justify their solution to the entire class and also clear the doubts of their fellow mates who wishes for further clarification.

Calculus problems which have problem situations that are experientially real to the learners are referred to as context problems, and in concurrence with it is the overall goal of realistic mathematics education (RME) instructional design which is used to support the gradual emergence of a taken-as-shared mathematical reality and opting for RME will consequently make the calculus experience for its learners enriching, for now, they can reinvent their expanding common sense, thereby making these experiences non-dichotomous between their everyday life experiences and the calculus problems they solve while learning in school from textbooks written by foreign authors; making both part of the same reality and thereby arriving at a reflexive relationship between usage of context problems and development of experiential reality among calculus students making the context problems on calculus rooted in reality and consequently if more the context problems students solve on calculus, the larger they expand their reality. This connection that calculus offers to its students is incisively the cause that makes calculus students act in a meaningful manner from the very beginning. The researcher suggests that concern for understanding rather than manipulative skill alone is a matter of utmost importance in encouraging the attainment of the quality which calculus education should provide.

To continue the teaching of calculus by methods which are antithetical to those of the experience-discovery approach may deprive our students of the richness of mathematical education which they deserve and which is so imperative that they shall have it. To develop conceptual understanding and cognition of principles of advanced calculus and possibilities in teaching those to school students and to effectively teach “Visual Calculus” course which comprises mathematical topics few of which are multivariable calculus concepts on differentiation and integrated 3-D geometry, integration and optimization, etc., a reform in calculus teacher preparation is required so that efficient teachers could be tailor-made for teaching school calculus.

The perception of calculus teachers needs to be examined with regards to the early development of calculus concepts among learners. The correlation between teachers’ pedagogic knowledge and their content knowledge and examining their preparedness, and consequently, their confidence in teaching school students the concepts of calculus, requires profound attention. The onus lies on the teachers to be mindful of students’ internal conflicts in dealing with mathematical problems to help them strengthen the novel constructs students come across; and to help them counter it effectively, the teachers shall devise Itemised Genetic Decompositions (IGDs) for the tasks at hand, because it is these IGDs that make teachers learn about the mental configurations of their students and this critical assessment of their learners eventually help educators in gauging the effectiveness of their teaching.

5. CONCLUSIONS

It has been found that calculus under normal conditions in India is taught by inefficient teaches, at a fallacious time and in an inaccurate manner. In mathematics education, time is ripe to examine calculus teaching. Reasons for the same that came out from this research include: (1) calculus in today’s time is pretty common to be taught to senior secondary school students, and (2) the very nature and character of calculus being taught in senior secondary schools and colleges has a bearing on the nature and character of pre-calculus courses taught in middle and high schools. A marathon discussion needs to be initiated for the greater good. For now, the rsearcher sees how our schools and colleges are troubled by nature and approach to teaching calculus to the learners. Some of the pressing questions that the current study has found include: What is the nature of senior-secondary school calculus? Is there a rationale behind opting for one senior-secondary school calculus curriculum over another that helps better prepare students in the successful learning of university mathematics? What is the role of calculus teacher, and is it really the teacher who plays the most important role in making students learn calculus? If it is so, can the characteristics of a successful calculus teacher be enumerated? Under the present conditions of early introduction of calculus in the senior-secondary school mathematics curriculum, are the students *actually* encouraged to study mathematics and engineering sciences? Is it so that the students studying in schools wherein calculus is taught are significantly better mathematically, compared to those students, in whose school, calculus is not offered or to those students who do not take up calculus course in senior-secondary class e.g., those who choose to study humanities and social sciences after their 10th board exams? Concerning pedagogical approaches, what is the mathematical nature of teachers’ knowledge with regards to derivatives? What are teachers’ commonly used pedagogical practices? What are teachers’ views as for teaching and learning of calculus? Can different kinds of interrelationships be keyed between teachers’ mathematical knowledge and their pedagogical practices? Why teachers find it extremely difficult to come up with novel solutions to sort out the difficulties that a student face, and is it in anyway correlated to their mathematical and/or pedagogical knowledge? Does secondary school calculus affect performance in college calculus? What is the effect of the level of secondary school calculus background on the measures of their learning outcome? What is the nature of this effect on procedural competency and on conceptual performance? Is there a relationship between secondary school calculus background and continuation to first-semester college calculus?

These are some of the pressing issues vis-à-vis calculus learning and teaching, that require immediate attention and deliberation.

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