On $(1,2)^*$ - π wg-Homeomorphisms

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ABSTRACT

This paper introduces the concepts of $(1,2)^*$ - π wg-closed maps and study some of their properties. Further, we introduce $(1,2)^*$ - π wg-homeomorphisms, $(1,2)^*$ - π wgC-homeomorphism and designs to study their properties. Also, we discuss its relationship with other weaker and stronger forms of $(1,2)^*$ - π wg-homeomorphism.

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Key Words:

$(1,2)^*$ - π wg-closed maps, $(1,2)^*$ - M- π wg-closed map, $(1,2)^*$ - π wg-homeomorphism, $(1,2)^*$ - π wgC-homeomorphism.

1. INTRODUCTION

Malghan[7] introduced and studied generalized closed mappings. Regular open sets have been introduced and investigated by Stone[20]. Sundaram and Sheik John [21] initiated the study of weakly closed sets in topological spaces. Weak generalized closed maps (wg-closed maps), rwg-closed maps were found and research was done in this field by Nagaveni[8]. Further, Lellis Thivagar and Ravi.O[5,19] unleashed the study of $(1,2)^*$ -g-closed map, $(1,2)^*$ -sg-closed map and $(1,2)^*$ -gs-closed map in bitopological spaces. Research on a broader scale was conducted by Maki et al [6] who introduced generalized homeomorphism and gC-homeomorphism which are nothing but the generalizations of homeomorphism in topological spaces.

This paper is an attempt to introduce a new class of maps called $(1,2)^*$ - π wg-closed map and an innovative class of mappings called $(1,2)^*$ - π wg-homeomorphism, $(1,2)^*$ - π wgC-homeomorphism in bitopological spaces and delved deep to study their results.

2. PRELIMINARIES

Throughout this paper, $(X, \tau_1, \tau_2), (Y, \sigma_1, \sigma_2)$ and (Z, η_1, η_2) (briefly X, Y and Z) will denote bitopological spaces.

Definition 2.1

Let A be a subset of X . Then A is called $\tau_{1,2}$ -open if $A = A_1 \cup B_1$, where $A_1 \in \tau_1, B_1 \in \tau_2$. The complement of $\tau_{1,2}$ -open set [1,13] is $\tau_{1,2}$ -closed set. The family of all $\tau_{1,2}$ -open (resp. $\tau_{1,2}$ -closed) sets of X is denoted by $(1,2)^*$ - $O(X)$ (resp. $(1,2)^*$ - $C(X)$).

Definition 2.2

Let A be a subset of a bitopological space X . Then (i) $\tau_{1,2}$ -closure of A [1,13] denoted by $\tau_{1,2}$ -cl(A) is defined by the intersection of all $\tau_{1,2}$ -closed sets containing A .
(ii) $\tau_{1,2}$ -interior of A [1,13] denoted by $\tau_{1,2}$ -int(A) is defined by the union of all open sets contained in A .

Remark 2.3

Notice that $\tau_{1,2}$ -open subsets of X need not necessarily form a topology.

Now, we recall some definitions and results which are used in this paper.

Definition 2.4

A subset A of a bitopological space X is said to be

- (i) $(1,2)^*$ -semi open [15] if $A \subset \tau_{1,2}$ -cl($\tau_{1,2}$ -int(A)).
- (ii) regular $(1,2)^*$ -open [9] if $A = \tau_{1,2}$ -int($\tau_{1,2}$ -cl(A)).
- (iii) $(1,2)^*$ - α -open [15] if $A \subset \tau_{1,2}$ -int($\tau_{1,2}$ -cl($\tau_{1,2}$ -int(A)).
- (iv) $(1,2)^*$ - π -open [16] if A is the finite union of regular $(1,2)^*$ -open sets.

The complements of all the above mentioned open sets are called their respective closed sets. The family of all $(1,2)^*$ -open sets [$(1,2)^*$ -semi open, $(1,2)^*$ -regular open, $(1,2)^*$ - α -open, $(1,2)^*$ - π -open] of X will be denoted by $(1,2)^*$ - $O(X)$ (resp. $(1,2)^*$ - $SO(X)$, $(1,2)^*$ - $RO(X)$, $(1,2)^*$ - $\alpha O(X)$, $(1,2)^*$ - $\pi O(X)$).

Definition 2.5

A subset A of bitopological space X is said to be

- (i) a $\tau_{1,2}$ - ω -closed [3] if $\tau_{1,2}$ -cl(A) $\subset$ U whenever $A \subset U$ and $U \in (1,2)^*$ - $SO(X)$.
- (ii) a $(1,2)^*$ -generalized closed set [11] ($(1,2)^*$ -g closed set) if $\tau_{1,2}$ -cl(A) $\subset$ U whenever $A \subset U$ and $U \in (1,2)^*$ - $O(X)$.
- (iii) a regular $(1,2)^*$ -generalized closed [14] (briefly $(1,2)^*$ -rg closed set) if $\tau_{1,2}$ -cl(A) $\subset$ U whenever $A \subset U$ and $U \in (1,2)^*$ - $RO(X)$.
- (iv) a weakly $(1,2)^*$ -generalized closed [17] (briefly $(1,2)^*$ -wg-closed) if $\tau_{1,2}$ -cl($\tau_{1,2}$ -int(A)) $\subset$ U whenever $A \subset U$ and $U \in (1,2)^*$ - $O(X)$.
- (v) a $(1,2)^*$ - π -generalized closed [16] (briefly $(1,2)^*$ - π g-closed set) if $\tau_{1,2}$ -cl(A) $\subset$ U whenever $A \subset U$ and $U \in (1,2)^*$ - $\pi O(X)$.
- (vi) a $(1,2)^*$ - $\pi g\alpha$ -closed set [2] if $\tau_{1,2}$ - α cl(A) $\subset$ U whenever $A \subset U$ and $U \in (1,2)^*$ - $\pi O(X)$.

(vii) a $(1,2)^*$ - regular semi open set [10] if there is a $(1,2)^*$ - RO(X) , U such that $U \subset A \subset \tau_{1,2} - cl(U)$.

(viii) a $(1,2)^*$ - $\tau_{1,2}$ - closed set [9] if $\tau_{1,2} - cl(A) \subset U$, whenever $A \subset U$ and U is $(1,2)^*$ - regular semi open set in X.

(ix) a $(1,2)^*$ - regular α -open [9] in X if there is a $(1,2)^*$ - regular open set U such that $U \subset A \subset \tau_{1,2} - \alpha cl(U)$.

(x) a regular $(1,2)^*$ - generalized α - closed set [9] (briefly $(1,2)^*$ - $rg\alpha$ - closed set) if $\tau_{1,2} - \alpha cl(A) \subset U$ whenever $A \subset U$ and $\alpha \in (1,2)^*$ - $R\alpha O(X)$. [$R\alpha O(X)$ - Collection of all regular $(1,2)^*$ - α -open set in X.]

(xi) a regular $(1,2)^*$ -weakly generalized closed [9] (briefly $(1,2)^*$ - rwg closed) if $\tau_{1,2} - cl(\tau_{1,2} - int(A)) \subset U$ whenever $A \subset U$ and $U \in (1,2)^*$ -RO(X).

(xii) A subset A of X is called $(1,2)^*$ - π wg- closed set [4] in X if $\tau_{1,2} - cl(\tau_{1,2} - int(A)) \subset U$ whenever $A \subset U$ and $U \in (1,2)^*$ - $\pi O(X)$.

Definition 2.6

A map $f: X \rightarrow Y$ is said to be

- (i) $(1,2)^*$ - continuous [11] if $f^{-1}(V)$ is $\tau_{1,2}$ -closed in X for every $\sigma_{1,2}$ - closed set V in Y.
- (ii) $(1,2)^*$ - ω - continuous [12] if $f^{-1}(V)$ is $(1,2)^*$ - ω - closed in X for every $\sigma_{1,2}$ -closed set V in Y.
- (iii) $(1,2)^*$ - rg -continuous [15] if $f^{-1}(V)$ is $(1,2)^*$ - rg closed in X for every $\sigma_{1,2}$ - closed set V in Y.
- (iv) $(1,2)^*$ - π -continuous [16] if $f^{-1}(V)$ is $(1,2)^*$ - π closed in X for every $\sigma_{1,2}$ - closed set V in Y.
- (v) $(1,2)^*$ - g-continuous [11] if $f^{-1}(V)$ is $(1,2)^*$ - g closed in X for every $\sigma_{1,2}$ - closed set V in Y.
- (vi) $(1,2)^*$ - wg-continuous [17] if $f^{-1}(V)$ is $(1,2)^*$ - wg-closed in X for every $\sigma_{1,2}$ -closed set V in Y.
- (vii) a $(1,2)^*$ - $rg\alpha$ -continuous [9] if $f^{-1}(V)$ is $(1,2)^*$ - $rg\alpha$ -closed in X for every $\sigma_{1,2}$ -closed set V in Y.
- (viii) a $(1,2)^*$ -rwg-continuous [9] if $f^{-1}(V)$ is $(1,2)^*$ - rwg-closed in X for every $\sigma_{1,2}$ -closed set V in Y.
- (ix) $(1,2)^*$ - ω - irresolute [12] if $f^{-1}(V)$ is $(1,2)^*$ - ω - closed in X for every ω - closed set V in Y.
- (x) A function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $(1,2)^*$ - π wg- continuous [4] if every $f^{-1}(V)$ is $(1,2)^*$ - π wg-closed in (X, τ_1, τ_2) for every closed set V of (Y, σ_1, σ_2) .
- (xi) A function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $(1,2)^*$ - π wg- irresolute [4] if $f^{-1}(V)$ is $(1,2)^*$ - π wg- closed in (X, τ_1, τ_2) for every $(1,2)^*$ - π wg- closed set V of (Y, σ_1, σ_2) .

Definition 2.7

A bijection $f: X \rightarrow Y$ is said to be

(i) a $(1,2)^*$ -homeomorphism [14] if both f and f^{-1} are $(1,2)^*$ -continuous in X

(ii) a $(1,2)^*$ - ω -homeomorphism [12] if both f and f^{-1} are $(1,2)^*$ - ω -continuous in X.

(iii) a $(1,2)^*$ - g-homeomorphism [18] if both f and f^{-1} are $(1,2)^*$ - g -continuous in X.

(iv) a $(1,2)^*$ - rw-homeomorphism [9] if both f and f^{-1} are $(1,2)^*$ - rw- continuous in X.

(v) a $(1,2)^*$ - $rg\alpha$ -homeomorphism [9] if both f and f^{-1} are $(1,2)^*$ - $rg\alpha$ - continuous in X.

(vi) a $(1,2)^*$ -rwg-homeomorphism [9] if both f and f^{-1} are $(1,2)^*$ - rwg- continuous in X.

Definition 2.8

A bijection $f: X \rightarrow Y$ is said to be

(i) a $(1,2)^*$ - ω *-homeomorphism [9] if both f and f^{-1} are $(1,2)^*$ - ω -irresolute.

(ii) a $(1,2)^*$ -gC-homeomorphism [18] if both f and f^{-1} are $(1,2)^*$ - g- irresolute in X.

(iii) a $(1,2)^*$ -rwC-homeomorphism [9] if both f and f^{-1} are $(1,2)^*$ - rw- irresolute in X.

(iv) a $(1,2)^*$ - $rg\alpha$ C-homeomorphism [9] if both f and f^{-1} are $(1,2)^*$ - $rg\alpha$ - irresolute in X.

Definition 2.9

A map $f: X \rightarrow Y$ is called a

- (i) a $(1,2)^*$ -closed map [19] if $f(U)$ is $\sigma_{1,2}$ -closed set in Y for every $\tau_{1,2}$ -closed set U in X.
- (ii) a $(1,2)^*$ -g-closed map [9] if $f(U)$ is $(1,2)^*$ -g -closed set in Y for every $\tau_{1,2}$ -closed set U in X.
- (iii) a $(1,2)^*$ - $\pi g\alpha$ -closed map [2] if $f(U)$ is $(1,2)^*$ - $\pi g\alpha$ -closed in Y for every $\tau_{1,2}$ -closed set U in X.

3. $(1,2)^*$ - π WG - CLOSED MAPS.

Definition 3.1

A map $f: X \rightarrow Y$ is said to be $(1,2)^*$ - π wg-closed if for every $\tau_{1,2}$ -closed V of X, $f(V)$ is $(1,2)^*$ - π wg-closed in Y.

The complement of the $(1,2)^*$ - π wg-closed map is $(1,2)^*$ - π wg-open map.

Theorem 3.2

Every $\tau_{1,2}$ -closed map (resp. $(1,2)^*$ -g-closed map is $(1,2)^*$ - π wg-closed map.

Proof: Straight forward.

Remark 3.3

The converse of the above need not be true as seen in the following example.

Example 3.4

Let $X = \{a, b, c\} = Y$, $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\emptyset, X, \{b\}, \{a, b\}\}$, $\sigma_1 = \{\emptyset, Y, \{a\}, \{a, c\}\}$, $\sigma_2 = \{\emptyset, Y, \{a, b\}\}$. Define $f: X \rightarrow Y$ be defined by $f(a) = c$, $f(b) = a$, $f(c) = b$. Here the map f is $(1,2)^*$ - π wg-closed map but not $\tau_{1,2}$ -closed map (resp. $(1,2)^*$ -g-closed map).

Proposition 3.5

If $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $(1,2)^*$ - π wg-closed and A is $\tau_{1,2}$ -closed subset of X , then $f|_A: A \rightarrow Y$ is $(1,2)^*$ - π wg-closed.

Proof:

Let $B \subset A$ and A be a $\tau_{1,2}$ -closed set in X . Then B is $\tau_{1,2}$ -closed in X . Now, f is $(1,2)^*$ - π wg-closed map implies $f(B)$ is $(1,2)^*$ - π wg-closed in Y . But $f(B) = (f|_A)(B)$. So, $f|_A$ is $(1,2)^*$ - π wg-closed in Y .

Proposition 3.6

If $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(1,2)^*$ -continuous, $(1,2)^*$ - π wg-closed and A is $(1,2)^*$ - π wg-closed subset of X , then

$f|_A: A \rightarrow Y$ is $(1,2)^*$ -continuous and $(1,2)^*$ - π wg-closed.

Proof:

Let F be a closed subset of A , then F is $(1,2)^*$ - π wg-closed subset of X . Then by proposition 3.5, it follows that $f|_A(F) = f(F)$ is $(1,2)^*$ - π wg-closed set of Y . Then the map $f|_A: A \rightarrow Y$ is $(1,2)^*$ - π wg-closed and hence $(1,2)^*$ - π wg-continuous.

Theorem 3.7

Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be two maps such that $(g \circ f)$ is $(1,2)^*$ - π wg-closed map. Then (i) If f is $(1,2)^*$ -continuous and surjective, then g is $(1,2)^*$ - π wg-closed map.

(ii) If g is $(1,2)^*$ - π wg-irresolute and injective, then f is $(1,2)^*$ - π wg-closed map.

Proof:

(i) Let V be a $\sigma_{1,2}$ -closed subset of Y . Since f is $(1,2)^*$ -continuous, $f^{-1}(V)$ is $\tau_{1,2}$ -closed set in X and $(g \circ f)(f^{-1}(V))$ is $(1,2)^*$ - π wg-closed in Z . Thus g is $(1,2)^*$ - π wg-closed.

(ii) Let A be a $(1,2)^*$ - π wg-closed set in X . Since $(g \circ f)(A)$ is $(1,2)^*$ - π wg-closed in Z . Since g is $(1,2)^*$ - π wg-continuous, $g^{-1}((g \circ f)(A)) = f(A)$ is $(1,2)^*$ - π wg-closed in Y . Hence f is $(1,2)^*$ - π wg-closed map.

Theorem 3.8

Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a surjective, $(1,2)^*$ - π -irresolute and a $(1,2)^*$ -closed map. Then $f(A)$ is $(1,2)^*$ - π wg-closed set of Y for every $(1,2)^*$ - π wg-closed set A of X .

Proof:

Let A be a $(1,2)^*$ - π wg-closed set of (X, τ_1, τ_2) . Let U be a $(1,2)^*$ - π -open set of (Y, σ_1, σ_2) such that $f(A) \subset U$. Since f is surjective and $(1,2)^*$ - π -irresolute, $f^{-1}(U)$ is a $(1,2)^*$ - π -open set of X . Since $A \subset f^{-1}(U)$ and A is $(1,2)^*$ - π wg-closed set of X , $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) \subset f^{-1}(U)$. Then $f(\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A))) \subset f(f^{-1}(U)) = U$. Since f is $(1,2)^*$ -closed map, $f(\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A))) = \tau_{1,2}\text{-cl}(f(\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A))))$. The above implies $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) \subset \tau_{1,2}\text{-cl}(f(\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)))) = f(\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A))) \subset U$.

Hence $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) \subset U$, whenever $A \subset U$ and U is $(1,2)^*$ - π -open in X . Therefore, $f(A)$ is a $(1,2)^*$ - π wg-closed set in Y .

Theorem 3.9

Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(1,2)^*$ - π wg-closed in Y , then for every subset A of X , $(1,2)^*$ - π wg-cl($f(A)$) $\subset f(\sigma_{1,2}\text{-cl}(A))$

Proof:

Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a $(1,2)^*$ - π wg-closed map and let $A \subset X$. Then $f(\sigma_{1,2}\text{-cl}(A))$ is $(1,2)^*$ - π wg-closed in Y . We have, $f(A) \subset f(\sigma_{1,2}\text{-cl}(A))$. The above implies $(1,2)^*$ - π wg-cl($f(A)$) $\subset (1,2)^*$ - π wg-cl($f(\sigma_{1,2}\text{-cl}(A))$). Then $(1,2)^*$ - π wg-cl($f(A)$) $\subset f(\sigma_{1,2}\text{-cl}(A))$

Theorem 3.10

If $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(1,2)^*$ - π wg-closed and (Y, σ_1, σ_2) is $(1,2)^*$ - $T_{\pi\text{wg}}$ -space, then their composition $(g \circ f): (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is a $(1,2)^*$ - π wg-closed map.

Proof:

Let A be a $\tau_{1,2}$ -closed set of X . Then by assumption $f(A)$ is $(1,2)^*$ - π wg-closed in Y . Since Y is a $T_{\pi\text{wg}}$ -space, $f(A)$ is $\tau_{1,2}$ -closed in Y . Since g is $(1,2)^*$ - π wg-closed map, $g(f(A))$ is $(1,2)^*$ - π wg-closed in Z . (i.e), $(g \circ f)$ is $(1,2)^*$ - π wg-closed in Z .

Definition 3.11

Let x be a point of X and V be a subset of X . Then V is called a $(1,2)^*$ - π wg-neighborhood of x in X if there exists a π wg-open set U of X such that $x \in U \subset V$.

Theorem 3.12

Suppose $(1,2)^*$ - $\pi\text{wgO}(X, \tau_1, \tau_2)$ is closed under arbitrary unions. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a mapping. Then the following statements are equivalent.

(i) f is $(1,2)^*$ - π wg-open mapping.

(ii) For a subset A of X ,

$$f(\tau_{1,2}\text{-int}(A)) \subset (1,2)^*\text{-}\pi\text{wg-int}(f(A))$$

(iii) For each $x \in X$ and for each neighborhood U of x in X , there exists a $(1,2)^*$ - π wg-neighborhood W of $f(x)$ in Y such that $W \subset f(U)$

Proof:

(i) $\Rightarrow$ (ii) Suppose f is $(1,2)^*$ - π wg-open. Let $A \subset X$. Since $\tau_{1,2}\text{-int}(A)$ is open in X , $f(\tau_{1,2}\text{-int}(A))$ is $(1,2)^*$ - π wg-open in Y . Hence $f(\tau_{1,2}\text{-int}(A)) \subset f(A)$. Then $f(\tau_{1,2}\text{-int}(A)) \subset (1,2)^*\text{-}\pi\text{wg-int}(f(A))$

(ii) $\Rightarrow$ (iii) Suppose (ii) holds. Let U be any arbitrary neighborhood of x in X . Then there exists an open set G such that $x \in G \subset U$. By assumption $f(G) = f(\text{int}(G)) \subset \pi\text{wg-int}(f(G))$ and $f(G) = (1,2)^*\text{-}\pi\text{wg-int}(f(G))$. Hence $f(G)$ is $(1,2)^*\text{-}\pi\text{wg-open}$ in Y and $f(x) \in f(G) \subset f(U)$ and by taking $W = f(G)$, the result (iii) holds.

(iii) $\Rightarrow$ (i) Let U be an open set in X such that $x \in U$ and $f(x) = y$. Then $x \in U$ for each $y \in f(U)$, there exists a $(1,2)^*$ - π wg-neighborhood W_y of y in Y such that $W_y \subset f(U)$. Since W_y is a $(1,2)^*$ - π wg-neighborhood of y , there exists a $(1,2)^*$ - π wg-open set V_y in Y such that $y \in V_y \subset W_y$. Therefore $f(U) = \bigcup \{V_y : y \in f(U)\}$. Hence $f(U)$ is a $(1,2)^*$ - π wg-open set of Y and f is a $(1,2)^*$ - π wg-open mapping.

Definition 3.13

A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be strongly $(1,2)^*$ - π wg-irresolute if $f^{-1}(V)$ is open in X for every $(1,2)^*$ - π wg-open set V of Y .

Theorem 3.14

Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be two mappings and let $(gof): (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ be $(1,2)^*$ - π wg-closed. Then

- (i) If f is $(1,2)^*$ -continuous and surjective, then g is $(1,2)^*$ - π wg-closed.
- (ii) If g is $(1,2)^*$ - π wg-irresolute and injective, then f is $(1,2)^*$ - π wg-closed.
- (iii) If f is $(1,2)^*$ - π wg-continuous, surjective and X is $T_{\pi wg}$ -space, then g is $(1,2)^*$ - π wg-closed.
- (iv) If g is strongly $(1,2)^*$ - π wg-irresolute and injective, then f is closed.

Proof:

(i) If f is continuous, then for any closed set A of Y , $f^{-1}(A)$ is closed in X . Also, (gof) is $(1,2)^*$ - π wg-closed implies $(gof)(f^{-1}(A)) = g(f(f^{-1}(A))) = g(A)$ is $(1,2)^*$ - π wg-closed in Z and g is $(1,2)^*$ - π wg-closed map.

(ii) Let A be closed in X . Then $(gof)(A)$ is $(1,2)^*$ - π wg-closed in Z and $g^{-1}((gof)(A)) = f(A)$ is $(1,2)^*$ - π wg-closed in Y . Hence f is $(1,2)^*$ - π wg-closed.

(iii) Let A be a closed set of Y . Then $f^{-1}(A)$ is $(1,2)^*$ - π wg-closed in X and $g(A)$ is $(1,2)^*$ - π wg-closed in Z .

(iv) Let A be closed in X , then $(gof)(A)$ is $(1,2)^*$ - π wg-closed in Z . g is strongly $(1,2)^*$ - π wg-irresolute implies $g^{-1}((gof)(A)) = f(A)$ is closed in Y and f is a closed map.

Theorem 3.15

A surjection $f: X \rightarrow Y$ is $(1,2)^*$ - π wg-closed iff for each subset S of Y and each open set U containing $f^{-1}(S)$, there exists a $(1,2)^*$ - π wg-open set V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof:

Suppose that f is $(1,2)^*$ - π wg-closed map. Let S be a subset of Y and U be an open subset of X containing $f^{-1}(S)$. Then $V = Y - f(X - U)$ is a $(1,2)^*$ - π wg-open set of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Conversely, let F be any $\tau_{1,2}$ -closed set. Then $f^{-1}(Y - f(F)) \subseteq X - F$ and $X - F$ is $\tau_{1,2}$ -open in X . By hypothesis, there exists a $(1,2)^*$ - π wg-open set V of Y such that $Y - f(F) \subseteq V$ and $f^{-1}(V) \subseteq X - F$. Therefore, $F \subseteq X - f^{-1}(V)$ and $Y - V \subseteq f(F) \subseteq f(X - f^{-1}(V)) \subseteq Y - V$. Hence $f(F) = Y - V$ $(1,2)^*$ - π wg-closed in Y , and hence f is $(1,2)^*$ - π wg-closed map.

Remark 3.16

The composition of two $(1,2)^*$ - π wg-closed maps need not be $(1,2)^*$ - π wg-closed.

Example 3.17

Let $X = \{a, b, c\} = Y = Z$, $\tau_1 = \{\emptyset, X, \{b, c\}\}$, $\tau_2 = \{\emptyset, X, \{a\}\}$, $\sigma_1 = \{\emptyset, Y, \{a\}\}$, $\sigma_2 = \{\emptyset, Y, \{b\}\}$, $\eta_1 = \{\emptyset, Z, \{b\}\}$, $\eta_2 = \{\emptyset, Z, \{a\}, \{a, b\}\}$. Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be identity maps. Here f and g are $(1,2)^*$ - π wg-closed. But for the $\tau_{1,2}$ -closed set $\{a\}$, $(gof)(a) = \{a\}$ is not $(1,2)^*$ - π wg-closed in Z . Hence (gof) is not a $(1,2)^*$ - π wg-closed map.

Proposition 3.18

If $f: X \rightarrow Y$ is $\tau_{1,2}$ -closed map and $g: Y \rightarrow Z$ is a $(1,2)^*$ - π wg-closed map, then $gof: X \rightarrow Z$ is $(1,2)^*$ - π wg-closed map.

Proof:

Let A be a $\tau_{1,2}$ -closed set in X , then $f(A)$ is $\sigma_{1,2}$ -closed in Y . Since g is $(1,2)^*$ - π wg-closed map, $g(f(A))$ is $(1,2)^*$ - π wg-closed in Z . Hence (gof) is $(1,2)^*$ - π wg-closed map.

Proposition 3.19

If $f: X \rightarrow Y$ is a bijection mapping, then the following statements are equivalent.

- (i) f is $(1,2)^*$ - π wg-open map.
- (ii) f is a $(1,2)^*$ - π wg-closed map.
- (iii) $f^{-1}: Y \rightarrow X$ is $(1,2)^*$ - π wg-continuous.

Proof: Straight Forward

Definition 3.20

A function $f: X \rightarrow Y$ is said to be M -($1,2$)*- π wg-closed map if the image $f(A)$ is $(1,2)^*$ - π wg-closed in Y for every $(1,2)^*$ - π wg-closed set A in X .

The complement of M -($1,2$)*- π wg-closed map is said to be M -($1,2$)*- π wg-open map.

Theorem 3.21

Every M -($1,2$)*- π wg-closed map is $(1,2)^*$ - π wg-closed map.

Proof:

Straight forward.

Remark 3.22

The converse of the above need not be true as shown in the following example.

Example 3.23

Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$, $\tau_2 = \{\emptyset, X, \{a\}\}$, $\sigma_1 = \{\emptyset, Y, \{a\}\}$, $\sigma_2 = \{\emptyset, Y, \{b\}, \{a, b\}\}$. Let $f: X \rightarrow Y$ be an identity map. Then for every $(1,2)^*$ - π wg-closed set A in X , $f(A)$ is not $(1,2)^*$ - π wg-closed in Y . Hence f is not a M -($1,2$)*- π wg-closed map, but the $\tau_{1,2}$ -closed set A in X , $f(A)$ is $(1,2)^*$ - π wg-closed in Y . Thus f is a $(1,2)^*$ - π wg-closed map but not a M -($1,2$)*- π wg-closed map.

Proposition 3.24

For any bijection $f: X \rightarrow Y$, the following statements are equivalent.

- (i) $f^{-1}: Y \rightarrow X$ is a $(1,2)^*$ - π wg-irresolute map.
- (ii) f is M -($1,2$)*- π wg-open map.
- (iii) f is M -($1,2$)*- π wg-closed map.

Proof: Straight Forward.

4. $(1,2)^*$ - π WG - HOMEOMORPHISMS.

Definition 4.1

- (i) A bijection $f: X \rightarrow Y$ is called a $(1,2)^*$ - π wg-homeomorphism if both f and f^{-1} are $(1,2)^*$ - π wg-continuous in X .
- (ii) A bijection $f: X \rightarrow Y$ is called a $(1,2)^*$ - π wgC-homeomorphism if both f and f^{-1} are $(1,2)^*$ - π wg-irresolute in X .

For a topological space (X, τ_1, τ_2) , we define the following three collection of functions.

- 1) $(1,2)^*$ - π wgCH $(X) = \{f: X \rightarrow Y \text{ is a } (1,2)^*$ - π wgC-homeomorphism\}
- 2) $(1,2)^*$ - π wgH $(X) = \{f: X \rightarrow Y \text{ is a } (1,2)^*$ - π wg-homeomorphism\}
- 3) $(1,2)^*$ -H $(X) = \{f: X \rightarrow Y \text{ is a } (1,2)^*$ -homeomorphism\}

Theorem 4.2

If f is a homeomorphism, then f and f^{-1} are $(1,2)^*$ - π wg-irresolute.

Proof:

First, we prove that f^{-1} is $(1,2)^*$ - π wg-irresolute. Let A be a $(1,2)^*$ - π wg-closed set of X . To show $(f^{-1})^{-1}(A) = f(A)$ is $(1,2)^*$ - π wg-closed in Y . Let U be a $(1,2)^*$ - π -open set such that $f(A) \subset U$. Then $A = f^{-1}(f(A)) \subset f^{-1}(U)$. Since A is $(1,2)^*$ - π wg-closed, $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) \subset f^{-1}(U)$. We have, $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(f(A))) \subset f(\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A))) \subset f(f^{-1}(U)) = U$. Then $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) \subset U$ and $f(A)$ is $(1,2)^*$ - π wg-closed. The above implies f is $(1,2)^*$ - π wg-irresolute and hence $f(A)$ is $(1,2)^*$ - π wg-closed. Also, $f^{-1}(f(A)) = A$ is $(1,2)^*$ - π wg-closed. Hence f and f^{-1} are $(1,2)^*$ - π wg-irresolute.

Theorem 4.3

1. Every $(1,2)^*$ -homeomorphism is $(1,2)^*$ - π wg-homeomorphism.
2. Every $(1,2)^*$ -g-homeomorphism is $(1,2)^*$ - π wg-homeomorphism.
3. Every $(1,2)^*$ - ω -homeomorphism is $(1,2)^*$ - π wg-homeomorphism.
4. Every $(1,2)^*$ -wg-homeomorphism is $(1,2)^*$ - π wg-homeomorphism
5. Every $(1,2)^*$ - π wg-homeomorphism is $(1,2)^*$ -rwg-homeomorphism.

Proof : Straight forward.

Remark 4.4

The converse of the above results need not be true as seen in the following examples.

Example 4.5

Let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$, $\tau_2 = \{\emptyset, X, \{b\}, \{a, b\}, \{a, b, c\}\}$, $\sigma_1 = \{\emptyset, Y, \{a\}\}$, $\sigma_2 = \{\emptyset, Y, \{b\}, \{a, b\}\}$. Let $f: X \rightarrow Y$ be defined as $f(a)=c$, $f(b)=a$, $f(c)=b$ and $f(d)=d$. Here f is $(1,2)^*$ - π wg-homeomorphism, But not a $(1,2)^*$ -homeomorphism. Thus $(1,2)^*$ - π wg-homeomorphism need not be $(1,2)^*$ -homeomorphism.

Example 4.6

Let $X = \{a, b, c\} = Y$, $\tau_1 = \{\emptyset, X, \{a\}, \{a, c\}\}$, $\tau_2 = \{\emptyset, X, \{a\}, \{b, c\}\}$, $\sigma_1 = \{\emptyset, Y, \{c\}\}$, $\sigma_2 = \{\emptyset, Y, \{b, c\}\}$. Let $f: X \rightarrow Y$ be defined by $f(a)=c$, $f(b)=b$, $f(c)=a$. Here f is $(1,2)^*$ - π wg-homeomorphism but not a $(1,2)^*$ -g-homeomorphism.

Example 4.7

In example 4.6, the inverse image and image of $\sigma_{1,2}$ -closed sets are $(1,2)^*$ - π wg-closed, but not $(1,2)^*$ - ω -closed in X . Hence $(1,2)^*$ - π wg-homeomorphism need not be $(1,2)^*$ - ω -homeomorphism.

Example 4.8

Let $X = \{a, b, c\} = Y$, $\tau_1 = \{\emptyset, X, \{a\}, \{a, c\}\}$, $\tau_2 = \{\emptyset, X, \{a\}, \{b, c\}\}$, $\sigma_1 = \{\emptyset, Y, \{c\}\}$, $\sigma_2 = \{\emptyset, Y, \{b, c\}\}$. Define $f: X \rightarrow Y$ be defined by $f(a)=b$, $f(b)=c$, $f(c)=a$. Here f is $(1,2)^*$ - π wg-homeomorphism but not a $(1,2)^*$ -wg-homeomorphism.

Example 4.9

Let $X = \{a, b, c\} = Y$, $\tau_1 = \{\emptyset, X, \{b\}, \{b, c\}\}$, $\tau_2 = \{\emptyset, X, \{c\}\}$, $\sigma_1 = \{\emptyset, Y, \{a\}\}$, $\sigma_2 = \{\emptyset, Y\}$. Let $f: X \rightarrow Y$ be an identity map. Here f is $(1,2)^*$ -rwg-homeomorphism but not a $(1,2)^*$ - π wg-homeomorphism.

Remark 4.10

The concepts of $(1,2)^*$ - π wg-homeomorphism, $(1,2)^*$ -rg α -homeomorphism are independent.

Example 4.11

Let $X = \{a, b, c, d\} = Y$, $\tau_1 = \{\emptyset, X, \{b\}\}$, $\tau_2 = \{\emptyset, X, \{c\}, \{b, c\}, \{b, c, d\}\}$, $\sigma_1 = \{\emptyset, Y, \{a, b\}\}$, $\sigma_2 = \{\emptyset, Y\}$. Let $f: X \rightarrow Y$ be an identity map. Here the map f is $(1,2)^*$ - π wg-homeomorphism but not $(1,2)^*$ -rg α -homeomorphism.

Example 4.12

Let $X = \{a, b, c, d\} = Y$, $\tau_1 = \{\emptyset, X, \{b\}\}$, $\tau_2 = \{\emptyset, X, \{c\}, \{b, c\}, \{b, c, d\}\}$, $\sigma_1 = \{\emptyset, Y, \{a, d\}\}$, $\sigma_2 = \{\emptyset, Y\}$. Let $f: X \rightarrow Y$ be an identity map. Here f is $(1,2)^*$ -rg α -homeomorphism but not a $(1,2)^*$ - π wg-homeomorphism.

Remark 4.13

The concepts of $(1,2)^*$ - π wg-homeomorphism, $(1,2)^*$ -rw-homeomorphism are independent.

Example 4.14

Let $X = \{a, b, c, d\} = Y$, $\tau_1 = \{\emptyset, X, \{a\}, \{c, d\}, \{a, c, d\}\}$, $\tau_2 = \{\emptyset, X, \{c\}, \{a, c\}, \{b, c, d\}\}$, $\sigma_1 = \{\emptyset, Y, \{a\}, \{a, b, c\}\}$, $\sigma_2 = \{\emptyset, Y, \{b\}, \{a, b\}\}$. Let $f: X \rightarrow Y$ be an identity map. Here the map f is $(1,2)^*$ - π wg-homeomorphism but not a $(1,2)^*$ -rw-homeomorphism.

Example 4.15

Let $X = \{a, b, c, d\} = Y$, $\tau_1 = \{\emptyset, X, \{b\}\}$, $\tau_2 = \{\emptyset, X, \{c\}, \{b, c\}, \{b, c, d\}\}$, $\sigma_1 = \{\emptyset, Y, \{a, d\}\}$, $\sigma_2 = \{\emptyset, Y\}$. Let $f: X \rightarrow Y$ be an identity map. Here the function f is $(1,2)^*$ -rw-homeomorphism but not a $(1,2)^*$ - π wg-homeomorphism.

Theorem 4.16

- (i) Every $(1,2)^*$ - ω^* -homeomorphism is an $(1,2)^*$ - π wg-homeomorphism.
- (ii) Every $(1,2)^*$ -gC-homeomorphism is $(1,2)^*$ - π wg-homeomorphism.

Proof: Straight forward

Remark: 4.17

The converse of the above need not be true as shown in the following examples.

Example 4.18

Let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\emptyset, X, \{a\}, \{a, b, c\}\}$, $\sigma_1 = \{\emptyset, Y, \{a\}, \{c\}, \{a, c\}\}$, $\sigma_2 = \{\emptyset, Y, \{a, b, d\}\}$. Let $f: X \rightarrow Y$ be an identity map. Here the function f is $(1,2)^*$ - π wg-homeomorphism but not a $(1,2)^*$ - ω^* -homeomorphism.

Example 4.19

Let $X = \{a, b, c\} = Y$, $\tau_1 = \{\emptyset, X, \{a\}, \{a, c\}\}$, $\tau_2 = \{\emptyset, X, \{a\}, \{b, c\}\}$, $\sigma_1 = \{\emptyset, Y, \{c\}\}$, $\sigma_2 = \{\emptyset, Y, \{b, c\}\}$. Let $f: X \rightarrow Y$ be defined by $f(a) = c$, $f(b) = b$, $f(c) = a$. Here f is $(1,2)^*$ - π wg - homeomorphism but not $(1,2)^*$ - gC - homeomorphism.

Remark 4.20

The Composition of two $(1,2)^*$ - π wg-homeomorphism need not be $(1,2)^*$ - π wg-homeomorphism is shown in the following example.

Example 4.21

Let $X = \{a, b, c, d\} = Y = Z$, $\tau_1 = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$, $\tau_2 = \{\emptyset, X, \{a, b, c\}\}$, $\sigma_1 = \{\emptyset, Y, \{a\}\}$, $\sigma_2 = \{\emptyset, Y, \{c, d\}, \{a, c, d\}\}$. Let $f: X \rightarrow Y$ be an identity map. Let $\eta_1 = \{\emptyset, Z, \{b\}\}$, $\eta_2 = \{\emptyset, Z, \{b, d\}, \{a, b, d\}\}$. Define $g: Y \rightarrow Z$ by $g(a) = b$, $g(b) = a$, $g(c) = c$, $g(d) = d$. Here f and g are $(1,2)^*$ - π wg- homeomorphisms. But the image and inverse image of their composition ($g \circ f$) is not $(1,2)^*$ - π wg-closed in X and Y . Hence ($g \circ f$) is not a $(1,2)^*$ - π wg-homeomorphism.

Theorem 4.22

Every $(1,2)^*$ - π wgC-homeomorphism is a $(1,2)^*$ - π wg-homeomorphism

Proof:

Let $f: X \rightarrow Y$ be an $(1,2)^*$ - π wgC-homeomorphism. Then f and f^{-1} are $(1,2)^*$ - π wg-irresolute and f is a bijection. Then f and f^{-1} are $(1,2)^*$ - π wg-continuous and hence f is a $(1,2)^*$ - π wg-homeomorphism.

Remark 4.23

The converse of the above need not be true as shown in the following example.

Example 4.24

Let $X = \{a, b, c, d\} = Y$, $\tau_1 = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$, $\tau_2 = \{\emptyset, X, \{a, b, c\}\}$, $\sigma_1 = \{\emptyset, Y, \{a\}\}$, $\sigma_2 = \{\emptyset, Y, \{c, d\}, \{a, c, d\}\}$. Let $f: X \rightarrow Y$ be an identity map. Here the map f is $(1,2)^*$ - π wg-homeomorphism but not a $(1,2)^*$ - π wgC-homeomorphism.

Remark: 4.25

From the above discussions and known results we have the following implications.

- (1) $(1,2)^*$ -homeomorphism
- (2) $(1,2)^*$ - g -homeomorphism,
- (3) $(1,2)^*$ - ω -homeomorphism
- (4) $(1,2)^*$ - ω^* -homeomorphism
- (5) $(1,2)^*$ -wg-homeomorphism.

(6) $(1,2)^*$ - gC -homeomorphism.

(7) $(1,2)^*$ - π wgC-homeomorphism.

(8) $(1,2)^*$ -rw-homeomorphism

(9) $(1,2)^*$ -rwg-homeomorphism

(10) $(1,2)$ -rg α -homeomorphism.

Theorem 4.26

If $f: X \rightarrow Y$ is a $(1,2)^*$ - π wgCH(X), then $(1,2)^*$ - π wg-cl($f^{-1}(B)$) = $f^{-1}((1,2)^*$ - π wg-cl(B)) for all $B \subset Y$.

Proof:

Since f is $(1,2)^*$ - π wg-irresolute and $(1,2)^*$ - π wg-cl($f(B)$) is a $(1,2)^*$ - π wg-closed set in Y , $f^{-1}((1,2)^*$ - π wg-cl($f(B)$)) is a $(1,2)^*$ - π wg -closed set in X . Now $f^{-1}(B) \subset f^{-1}((1,2)^*$ - π wg-cl($f(B)$)) implies $(1,2)^*$ - π wg-cl($f^{-1}(B)$) $\subset$ $(1,2)^*$ - π wg-cl($f^{-1}((1,2)^*$ - π wg-cl($f(B)$))) $\subset f^{-1}((1,2)^*$ - π wg-cl($f(B)$)). Since f^{-1} is $(1,2)^*$ - π wg-irresolute, $(1,2)^*$ - π wg-cl($f^{-1}(B)$) is a $(1,2)^*$ - π wg-closed set in X . Hence $(f^{-1})^{-1}((1,2)^*$ - π wg-cl($f^{-1}(B)$)) = $f((1,2)^*$ - π wg-cl($f^{-1}(B)$)) is a $(1,2)^*$ - π wg-closed set in Y . Now, $B \subset (f^{-1})^{-1}((1,2)^*$ - π wg-cl($f^{-1}(B)$)) implies $(1,2)^*$ - π wg-cl(B) $\subset f((1,2)^*$ - π wg-cl($f^{-1}(B)$)). Therefore, $f^{-1}((1,2)^*$ - π wg-cl(B)) $\subset (1,2)^*$ - π wg-cl($f^{-1}(B)$)). Hence $(1,2)^*$ - π wg-cl($f^{-1}(B)$) = $f^{-1}((1,2)^*$ - π wg-cl(B)).

Theorem 4.27

If $f: X \rightarrow Y$ is a $(1,2)^*$ - π wgCH(X), then

(i) $(1,2)^*$ - π wg-cl($f(B)$) = $f((1,2)^*$ - π wg-cl(B)) for all $B \subset Y$.

(ii) $f((1,2)^*$ - π wg-int(B)) = $(1,2)^*$ - π wg- int ($f(B)$) for all

$B \subset X$.

(iii) $f^{-1}((1,2)^*$ - π wg-int(B)) = $(1,2)^*$ - π wg-int ($f^{-1}(B)$) for all

$B \subset Y$.

Proof:

(i) Since $f: X \rightarrow Y$ is a $(1,2)^*$ - π wgCH(X), $f^{-1}: Y \rightarrow X$ is also a $(1,2)^*$ - π wgCH (Y). Therefore by theorem 4.26 , $(1,2)^*$ - π wg-cl($(f^{-1})^{-1}(B)$) = $(f^{-1})^{-1}((1,2)^*$ - π wg-cl(B)) for all $B \subset X$ (i.e) $(1,2)^*$ - π wg-cl($f(B)$) = $f((1,2)^*$ - π wg-cl(B)) .

(ii) For any set $B \subset X$, $(1,2)^*$ - π wg-int(B) = $((1,2)^*$ - π wg-cl(B^c))^c. Thus by (i) , we obtain $f((1,2)^*$ - π wg-int(B)) = $f((1,2)^*$ - π wg-cl(B^c))^c = $(1,2)^*$ - π wg-int $f(B)$

(iii) Since $f^{-1}: Y \rightarrow X$ is also a $(1,2)^*$ - π wgCH(X), $f^{-1}((1,2)^*$ - π wg-int (B)) = $(1,2)^*$ - π wg- int ($f^{-1}(B)$).

Proposition 4.28

Let $f: X \rightarrow Y$ be a bijective $(1,2)^*$ - π wg-continuous map. Then the following statements are equivalent to one another.

(i) f is $(1,2)^*$ - π wg-open map.

(ii) f is $(1,2)^*$ - π wg-homeomorphism.

(iii) f is $(1,2)^*$ - π wg-closed map.

Proof: Straight forward

Remark 4.29

Composition of two $(1,2)^*$ - π wgC-homeomorphism is $(1,2)^*$ - π wgC-homeomorphism.

Proof:

Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be two $(1,2)^*$ - π wgC-homeomorphic mappings. Then f, f^{-1}, g and g^{-1} are all $(1,2)^*$ -irresolute maps. The composition of two $(1,2)^*$ -irresolute maps is also a $(1,2)^*$ - π wg-irresolute. Then ($g \circ f$) and ($g \circ f$)⁻¹ are also

$(1,2)^*$ - πwg -irresolute and hence $(1,2)^*$ - πwgC -homeomorphism.

Proposition 4.30

(i) The set $(1,2)^*$ - πwgC -homeomorphism of X forms a group under composition of functions.

(ii) The set $H(X, \tau_1, \tau_2)$ is a subgroup of $(1,2)^*$ - $\pi\text{wgCH}(X, \tau_1, \tau_2)$

Proof:(i)

(a) Define the map $\phi: (1,2)^*\text{-}\pi\text{wgCH}(X) \times (1,2)^*\text{-}\pi\text{wgCH}(X) \rightarrow (1,2)^*\text{-}\pi\text{wgCH}(X)$ by $\phi(f,g) = \text{gof}$ for every $f, g \in (1,2)^*\text{-}\pi\text{wgCH}(X)$. Since the composition of two $(1,2)^*\text{-}\pi\text{wgCH}$ is a $(1,2)^*\text{-}\pi\text{wgCH}(X)$. Then $\text{gof} \in (1,2)^*\text{-}\pi\text{wgCH}(X)$. Hence $(1,2)^* \in (1,2)^*\text{-}\pi\text{wgCH}(X)$ is closed.

(b) The composition of functions is associative.

(c) Clearly, the identity map $i: X \rightarrow X$ is a $(1,2)^*\text{-}\pi\text{wgC}$ -homeomorphism and $i \in (1,2)^*\text{-}\pi\text{wgCH}(X)$. Also, $\text{iof} = \text{foi} = f$ for every $f \in (1,2)^*\text{-}\pi\text{wgCH}(X)$.

(d) For any $f \in (1,2)^*\text{-}\pi\text{wgCH}(X)$, $\text{fof}^{-1} = f^{-1} \circ f = i$. Hence inverse exists for each element of $(1,2)^*\text{-}\pi\text{wgCH}(X)$.

Hence $(1,2)^*\text{-}\pi\text{wgCH}(X)$ forms a group under the composition of maps.

(ii) Since $(1,2)^*\text{-}H(X) \subset (1,2)^*\text{-}\pi\text{wgCH}(X) \subset (1,2)^*\text{-}\pi\text{wgH}(X)$. Hence the set $H(X, \tau_1, \tau_2)$ is a subgroup of $(1,2)^*\text{-}\pi\text{wgCH}(X, \tau_1, \tau_2)$

Theorem 4.31

If $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a $(1,2)^*\text{-}\pi\text{wgCH}$, then it induces an isomorphism $F_f: (1,2)^*\text{-}\pi\text{wgCH}(X, \tau_1, \tau_2) \rightarrow (1,2)^*\text{-}\pi\text{wgCH}(Y, \sigma_1, \sigma_2)$

Proof:

Define $F_f: (1,2)^*\text{-}\pi\text{wgCH}(X, \tau_1, \tau_2) \rightarrow (1,2)^*\text{-}\pi\text{wgCH}(Y, \sigma_1, \sigma_2)$ by $F_f(h) = \text{fohof}^{-1}$ for every $h \in (1,2)^*\text{-}\pi\text{wgCH}(X)$. Then F_f is a bijection. Further, for all $h_1, h_2 \in (1,2)^*\text{-}\pi\text{wgCH}(X)$.

$$\begin{aligned} \text{Then } F_f(h_1 \circ h_2) &= \text{fo}((h_1 \circ h_2) \circ f^{-1}) \\ &= [\text{fo}((h_1 \circ h_2))] f^{-1} \\ &= [(\text{fo}h_1) \circ (\text{fo}h_2)] f^{-1} \\ &= (\text{fo}h_1 \circ f^{-1}) \circ (\text{fo}h_2 \circ f^{-1}) \\ &= F_f(h_1) \circ F_f(h_2) \end{aligned}$$

Therefore, F_f is a homomorphism and hence F_f is an isomorphism induced by f .

Remark 4.32

The converse of the above need not be true as shown in the following example.

Example 4.33

Let $X = \{a, b, c, d\} = Y, \tau_1 = \{\emptyset, X, \{a\}, \{b, d\}, \{a, b, d\}\}, \tau_2 = \{\emptyset, X\}, \sigma_1 = \{\emptyset, Y, \{a\}\}, \sigma_2 = \{\emptyset, Y, \{c\}, \{c, d\}, \{a, c\}, \{a, c, d\}\}$. Let $f: X \rightarrow Y$ be defined by $f(a) = b, f(b) = c, f(c) = d$ and $f(d) = a$. Then $(1,2)^*\text{-}\pi\text{wgC}(X) = \{\emptyset, X, \{b\}, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$ and $(1,2)^*\text{-}\pi\text{wgC}(Y) = \{\emptyset, X, \{b\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$.

Now, define the functions $h_a: (X, \tau_1, \tau_2) \rightarrow (X, \tau_1, \tau_2)$ by $h_a(a) = a, h_a(b) = d, h_a(c) = c, h_a(d) = b$ and $h_b: (Y, \sigma_1, \sigma_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by $h_b(a) = c, h_b(b) = b, h_b(c) = a, h_b(d) = d$. Then $(1,2)^*\text{-}\pi\text{wgCH}(X, \tau_1, \tau_2) = \{I_X, h_a\}$ and $(1,2)^*\text{-}\pi\text{wgCH}(Y, \sigma_1, \sigma_2) = \{I_Y, h_b\}$. Hence the map $F_f: (1,2)^*\text{-}\pi\text{wgCH}(X, \tau_1, \tau_2) \rightarrow$

$(1,2)^*\text{-}\pi\text{wgCH}(Y, \sigma_1, \sigma_2)$ is defined by $F_f(h_a) = h_b$ is an isomorphism, but f is not a $(1,2)^*\text{-}\pi\text{wgC}$ -homeomorphism.

Proposition 4.34

$(1,2)^*\text{-}\pi\text{wgC}$ -homeomorphism is an equivalence relation.

Proof:

It is immediate that $(1,2)^*\text{-}\pi\text{wgC}$ -homeomorphism is reflexive and symmetric. Since the composition of two $(1,2)^*\text{-}\pi\text{wgC}$ -homeomorphism is $(1,2)^*\text{-}\pi\text{wgC}$ -homeomorphism, it follows that $(1,2)^*\text{-}\pi\text{wgC}$ -homeomorphism is transitive. Hence $(1,2)^*\text{-}\pi\text{wgC}$ -homeomorphism is an equivalence relation.

CONCLUSION

The study of this concept has led to certain findings and conclusions. A comparison has been struck out between $(1,2)^*\text{-}\pi\text{wg}$ -homeomorphisms with other existing sets in bitopological spaces. An attempt has also been made to unearth the characteristics of $(1,2)^*\text{-}\pi\text{wg}$ -closed maps. This study focuses on the extension of $(1,2)^*\text{-}\pi\text{wg}$ -closed maps and $(1,2)^*\text{-}\pi\text{wg}$ continuous maps in bitopological settings.

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