Image Denoising based on Spatial/Wavelet Filter using Hybrid Thresholding Function

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ABSTRACT

In this paper a hybrid denoising algorithm which combines spatial domain bilateral filter and hybrid thresholding function in the wavelet domain is proposed. The wavelet transform is used to decompose the noisy image into its different subbands namely LL, LH, HL, and HH. A two stage spatial bilateral filter is applied. The first stage is applied on the noisy image before wavelet decomposition. This stage will be called a pre-processing stage. The second stage spatial bilateral filtering is applied on the low frequency subband of the decomposed noisy image namely subband LL. This stage will tend to cancel or at least attenuate any residual low frequency noise components. The intermediate stage deal with high frequency noise components by thresholding detail subbands LH, HL, and HH using hybrid thresholding function. The experimental results show that the performance of the proposed denoising algorithm is superior to that of the conventional denoising approach.

General Terms

Image Denoising, Wavelet Transform.

Keywords

Image Denoising, Spatial Bilateral Filter, Thresholding Function.

1. INTRODUCTION

In the image denoising process, information about the type of noise present in the original image plays a significant role. Denoising of electronically distorted images is an old, there are many different cases of distortions. One of the most prevalent cases is distortion due to noise. Typical images are corrupted with noise modeled with either a Gaussian, uniform, Rician, or salt and pepper distribution. Another typical noise is a speckle noise, which is multiplicative in nature. Speckle noise [1] is observed in ultrasound images, whereas Rician noise [2] affects MRI images. Mostly, noise in digital images is found to be additive in nature with uniform power in the whole bandwidth and with Gaussian probability distribution. Such a noise is referred to as Additive White Gaussian Noise(AWGN). White Gaussian noise can be caused by poor image acquisition or by transferring the image data in noisy communication channel. Most denoising algorithms use images artificially distorted with well defined white Gaussian noise to achieve objective test results[3-7].

Image denoising is often a necessary and primary step in any further image processing tasks like segmentation, object recognition, computer vision, ...etc. Among several denoising algorithms, denoising that based on spatial linear filtering techniques, such as Wiener filter or match filter, finds wide range of applications for many years. Generally, the main

weaknesses of linear filter are its inability to preserve image fine details and its poor performance in dealing with heavy tailed noise. Due to these facts, an alternative spatial nonlinear filtering technique are widely used. Many successful works [8-14] have been reported on image denoising using spatial nonlinear filters. Among several spatial non linear filters, the bilateral filter finds wide range of applications [9] due to its robustness in smoothing out noise while preserving image fine details. Besides spatial filters, denoising that based on wavelet transform for cancelling white Gaussian noise finds wide range of applications since the pioneer work by Donoho and Johnstone[15-17]. In wavelet based denoising algorithms, the noise is estimated and wavelet coefficients are thresholded to separate signal and noise using appropriate threshold value. Since the threshold plays a key role in this appealing technique, variant methods appeared later to set an appropriate threshold value[3-7]. Among various approaches to nonlinear wavelet-based denoising, BayesShrink wavelet denoising based on Bayesian framework has been widely used for image denoising [3]. Unlike the universal threshold[15], which depends only on the number of pixels and the variance of the noise, BayesShrink threshold is a Data-Driven adaptive to the features of the image and provide better results.

Recently, a number of different algorithms[3-14] have been proposed for digital image denoising, some of these algorithms are applied in frequency domain others in spatial domain. Most of these algorithms assume that the true image is smooth or piecewise smooth which means that the true image or patches of it contains only low frequency components and also assume that the noise is oscillatory or non smooth and hence contains only high frequency components. However, this assumption is not always true. Images can contain fine details and structures which have high frequency components. On the other hand, Noise in an image has low as well as high frequency components. Though the high frequency components can easily be removed through linear and non linear filtering, it is challenging to eliminate low frequency noise components as it is difficult to distinguish between real signal and low frequency noise components. Generally, these algorithms fully succeeded in removing high-frequency noise components but at the expense of removing the details of the image too which cause blurring effect. While, these algorithms keep the low frequency noise components untouched due to the assumption that the noise contains mainly high frequency components. To improve these denoising algorithms performance, a hybrid denoising algorithm that uses both spatial and frequency domain is proposed. The spatial domain filtering is designed in such a way that enables dealing with low frequency noise components, while the wavelet thresholding is designed to deal with high frequency noise components. For the spatial

part of the proposed denoising algorithm, although any spatial filter can be used, we suggest to use bilateral filter due to its robustness[9]. The rest of the paper is organized as follows. Section 2 and 3 briefly reviews the wavelet thresholding and the Bayesian threshold calculation. Section 4 presents hybrid thresholding function. In section 5, we introduce the spatial bilateral filter main concept. In section 6, we explain the proposed image denoising algorithm. Section 7, provides an empirical study for setting proposed denoising algorithm parameters. The results of our proposed denoising algorithm will be compared with BayesShrink[3], bilateral filter[9], and SURENeighShrink[7] in section 8. Finally, the concluding remarks are given in section 9.

2. WAVELET THRESHOLDING

Thresholding is a simple non-linear technique in which wavelet coefficient is thresholded by comparing against a threshold. Any coefficient that is smaller than the selected threshold is set to zero while keeping or modifying others. Estimation of suitable threshold value is a major problem in this field. It has been shown that[3], BayesShrink is simple and effective threshold estimation algorithm.

3. BAYES THRESHOLD CALCULATION

Bayesian based threshold calculation was proposed by Chang, et al [3]. The goal of this method is to estimate a threshold value that minimizes the Bayesian risk assuming Generalized Gaussian Distribution (GGD) prior. It has been shown that BayesShrink[3] outperforms SUREShrink[17] most of the times in terms of PSNR values over a wide range of noisy images. It uses soft thresholding and is subband-dependent, which means that thresholding is done at each band of resolution in the wavelet decomposition. The Bayes threshold, T_{Bayes} , is defined as:-

$$T_{\text{Bayes}} = \frac{\hat{\sigma}_n^2}{\hat{\sigma}_f^2} \quad (1)$$

Where, $\hat{\sigma}_n^2$ is an estimate of noise variance, and $\hat{\sigma}_f^2$ is an estimate of the original noise free signal variance. The noise standard deviation σ_n is estimated from the subband HH1, using the formula:-

$$\hat{\sigma}_n = \frac{\text{median}(|Y_{ij}|)}{0.6475}, Y_{ij} \in HH_1 \quad (2)$$

Where Y_{ij} are the detail coefficients in the diagonal subband HH_1 . From the definition of additive noise we have:-

$$r(x, y) = f(x, y) + n(x, y) \quad (3)$$

Where $r(x, y)$, $f(x, y)$, and $n(x, y)$ are the observed, original, and noise signals respectively.

Since the noise and the signal are independent of each other, it can be stated that:-

$$\sigma_r^2 = \sigma_f^2 + \sigma_n^2 \quad (4)$$

The observed signal variance σ_r^2 can be estimated using:-

$$\hat{\sigma}_r^2 = \frac{1}{M^2} \sum_{x,y=1}^M r^2(x, y) \quad (5)$$

The variance of the signal, σ_f^2 is estimated according to:-

$$\hat{\sigma}_f^2 = \max(\hat{\sigma}_r^2 - \hat{\sigma}_n^2, 0) \quad (6)$$

Knowing $\hat{\sigma}_n^2$ and $\hat{\sigma}_f^2$, the Bayes threshold is computed from Equation (1).

4. HYBRID THRESHOLDING FUNCTION

For a given threshold, soft thresholding has smaller variance, however, higher bias than hard thresholding, especially for very large wavelet coefficients. If the coefficients distribute

densely close to the threshold, hard thresholding will show large variance and bias. On the other hand, soft thresholding exhibits smaller error when the coefficients are close to zero. Generally, soft thresholding is chosen for smoothness while hard thresholding is chosen for lower error. To get the benefit of both soft and hard thresholding functions, a hybrid thresholding function is newly proposed that scaled the wavelet coefficients according to:-

$$\theta_{\text{hybrid}}^T(f) = \begin{cases} \text{sign}(f)(|f| - |f|^{1-\beta} T^\beta) & \text{if } |f| \geq T \\ 0 & \text{if } |f| < T \end{cases} \quad (7)$$

Where f is the wavelet coefficient, T is the threshold value, and β is the parameter that controls the thresholding characteristics. When $\beta \rightarrow 1$, the thresholding rule approaches the soft thresholding function. On the other hand, when $\beta \rightarrow \infty$, the thresholding rule follows hard thresholding function. Thus, by selecting suitable value for β , a better thresholding can be achieved that gets the merits of both soft and hard thresholding functions.

5. SPATIAL BILATERAL FILTER

Bilateral filter is firstly presented by Tomasi and Manduchi in 1998[9]. It is a nonlinear, and non iterative technique which considers both intensity similarities and geometric closeness of the neighboring pixels. The concept of the bilateral filter was also presented in [8] as the SUSAN filter. It is mentionable that the Beltrami flow algorithm is considered as the theoretical origin of the bilateral filter which produces a spectrum of image enhancing algorithms ranging from the L_2 linear diffusion to the L_1 non-linear flows[10, 11]. The bilateral filter takes a weighted sum of the pixels in a local neighborhood, the weights depend on both the spatial distance and the intensity distance which can be described mathematically as:-

$$W(x, y) = W_s(x, y) \times W_i(x, y) \quad (8)$$

Where W_s and W_i are the spatial and intensity weights respectively which both are monotonically decreasing positive values.

Mathematically, at a pixel location p , the result of passing the image to be denoised to the bilateral filter can be expressed as follows:-

$$\text{img}(p) = \frac{\sum_{k \in N(p)} W(p) \times \text{img}(k)}{A} \quad (9)$$

Where $N(p)$ is the spatial neighborhood of the center pixel p and A is the weight normalization constant that preserve local mean which can be expressed as:-

$$A = \sum_{k \in N(p)} W(k) \quad (10)$$

Tomasi and Manduchi[9] suggest using Gaussian weight function for both W_s and W_i , accordingly, Eq.(8) can be rewritten as:-

$$W = e^{-\frac{\|p-k\|^2}{2\sigma_s^2}} \times e^{-\frac{|\text{img}(p) - \text{img}(k)|^2}{2\sigma_i^2}} \quad (11)$$

where σ_s and σ_i are the parameters that control the fall-off of weights in spatial and intensity domains respectively. Substituting (11) into (9) yields,

$$\text{img}(p) = \frac{\sum_{k \in N(p)} W_s(k) \times W_i(k) \times \text{img}(k)}{A} \quad (12)$$

Where,

$$W_s = e^{-\frac{\|p-k\|^2}{2\sigma_s^2}}$$

and

$$W_i = e^{-\frac{|\text{img}(p) - \text{img}(k)|}{2\sigma_i^2}}$$

Equation(12) state that every pixel is replaced by a weighted average of its neighbors. These non linear weightings are selected such that larger weights ($W \rightarrow 1$) for neighbors close spatially and radio metrically to the center pixel. On the other hand, smaller weights ($W \rightarrow 0$) for neighbors apart spatially and radiometrically from the center pixel.

6. PROPOSED ALGORITHM

To deal with both low and high frequency noise components, the noisy image is decomposed into its different frequency subbands and then filtering each subband separately to get access of both low and high frequency noise components. For image decomposition, wavelet transform will be used due to its robustness and low computational cost. The noisy image is filtered using two-stage spatial bilateral filter. The first stage is applied on the noisy image before wavelet decomposition. This stage will be called a pre-processing stage. The pre-processing stage paved the way for the wavelet thresholding based filtering part to operate effectively. Thereafter, a second stage spatial bilateral filtering is applied on the low frequency subband of the decomposed noisy image namely subband *LL*. This stage will tend to cancel or at least attenuate any residual low frequency noise components. The intermediate stage deal with high frequency noise components by thresholding all high frequency subbands of the decomposed image namely subbands *LH*, *HL*, and *HH*. Among several wavelet thresholding algorithms, Bayesian based threshold calculation that uses hybrid thresholding function will be adopted. Finally, the filtered decomposed image is reconstructed by applying inverse wavelet transform to get the denoised image. Figure(1) shows the flow chart that describes the internal processing of the proposed denoising algorithm.

7. PROPOSED ALGORITHM PARAMETERS SELECTION

Extensive simulation test was conducted to select the parameters that control the behavior of the proposed denoising algorithm namely σ_s , σ_i , N , and β . For hybrid thresholding function, the effect of the parameter β was examined over a wide range of image degradations and the optimum value for β was searched that maximizes the Peak Signal to Noise Ratio (PSNR) between the original and denoised image. The results are reported in figure(2). From this figure, it's clear that the optimum value for β is a function of noise level and it lies within the range $1 \rightarrow 1.5$. The spatial bilateral filter parameters namely σ_s , σ_i and N were examined extensively over a wide range of image degradations. Results show that, for the proposed denoising algorithm, these parameters can be set easily and accurately for denoising a wide range of images over a wide range of noise levels under test. Results also show that the parameter σ_i has higher effect on denoising performance as compared with the σ_s , and N and it has a linear relationship with the noise standard deviation. Figure(3)

Fig 1: Flowchart of the Proposed Denoising Algorithm

shows the result of simulation for 30 standard and nonstandard test images of different sizes averaged over ten runs where both σ_s and N are kept fixed at 1.7 and 11 respectively and optimum value for σ_i were searched that minimizes the mean square error (MSE) between the original and denoised image. Referring to Figure(3), it can be clearly seen that, there is a highly dependency between optimal σ_i values and the noise standard deviation changes σ_n . This is due to the fact that σ_i affects on fall-off of weights in the intensity domain and hence if σ_i is smaller than σ_n then the noisy pixels will be kept untouched which in turn degrades denoising operation. Extensive optimization has been carried out for the selection of optimum value for σ_i related to σ_n .

Results show that, setting $\sigma_i=1.1\sigma_n$ is a suitable choice over a wide range of images under test. The same procedure is followed to search for optimum values for σ_s and N . Results show that the optimal σ_s and N values are relatively

insensitive to the variation of the noise standard deviation σ_n . Setting $\sigma_s=1.7 \rightarrow 2$ and $N=9 \rightarrow 11$ shown to be suitable choice over the whole scope of noise levels.

Fig 2: Optimal Selection of β , Top Left($\sigma_n=10$ $\beta_{opt}=1$), Top Right($\sigma_n=30$ $\beta_{opt}=1.05$), Bottom Left($\sigma_n=75$ $\beta_{opt}=1.12$), Bottom Right($\sigma_n=100$ $\beta_{opt}=1.5$)

Fig 3: Linear Relation Ship between σ_i and σ_n ($\sigma_s=1.7$, $N=11$)

8. RESULTS AND DISCUSSIONS

For evaluation purposes, an experiment was conducted to assess the performance of the proposed denoising algorithm for denoising images corrupted with white Gaussian noise with zero mean and standard deviations 10, 20, 30, 50, 75, and 100. The wavelet transform employs Daubechie's least asymmetric compactly supported wavelet with eight vanishing moments. The noise standard deviation is estimated using robust Median Absolute Deviation (MAD) defined in Eq.(2). We shall use the Peak Signal to Noise Ratio(PSNR) as our quantitative measure of the relative denoising algorithms performance. In this experiment, we have compared the proposed denoising algorithm with the conventional BayesShrink[3], conventional bilateral filter[9], and SURENeighShrink[7]. BayesShrink and SURENeighShrink are frequency domain based denoising algorithms using 4-Level wavelet transform decomposition. The bilateral filter is a spatial domain based denoising algorithm. While, the proposed denoising algorithm uses both spatial and frequency domain as shown in figure(1) with single-level wavelet transform decomposition. The PSNR for various denoising

algorithms are recorded in Table(1) for a set of images. The data are collected from an average of ten runs. The best denoising algorithm among others in terms of PSNR value is highlighted in bold font for each test image. Referring to the results in Table(1), we can clearly see that the proposed denoising algorithm outperforms other denoising algorithms most of the time in terms of individual PSNR value. It outperforms other denoising algorithms all the time in terms of average PSNR value over the whole scope of noise levels and images under test. Also, we can see that SURENeighShrink achieves competitive image denoising performance. However, SURENeighShrink requires much processing time compared with the proposed denoising algorithm. This is due to the fact that SURENeighShrink search for optimal window size and threshold value for every wavelet subband by minimizing Stein's unbiased risk estimate which is a time consuming process especially for large size images. As an example, the average execution time of ten runs, shows that SURENeighShrink requires about 27.352 seconds for denoising image of size 512x512 while the proposed denoising algorithm did better results with about just

9.508 seconds. Thus we can deduce that the proposed denoising algorithm provides both good performance and low

computation cost.

Table 1. PSNR Results for Denoising Lena, Aircraft, Cameraman and House images

σ_n Algorithm	10	20	30	50	75	100	Average PSNR
Lena Image							
BayesShrink	33.468	30.352	28.644	26.419	24.348	22.550	27.630
Bilateral	33.791	30.356	28.259	25.383	23.123	21.524	27.073
SURENeighShrink	34.624	31.444	29.651	27.037	24.592	22.753	28.350
Proposed	34.401	31.261	29.484	27.068	25.037	23.409	28.443
Aircraft Image							
BayesShrink	34.803	32.149	30.812	29.059	26.532	23.858	29.536
Bilateral	36.590	32.357	29.623	26.399	24.003	22.038	28.502
SURENeighShrink	36.312	33.454	31.944	29.640	27.052	24.173	30.429
Proposed	37.010	33.926	32.082	29.845	27.398	25.199	30.910
Cameraman Image							
BayesShrink	31.206	27.170	25.040	22.487	20.455	18.928	24.214
Bilateral	32.458	28.564	25.907	22.774	20.610	19.097	24.902
SURENeighShrink	32.456	28.420	26.065	22.911	20.671	19.196	24.953
Proposed	32.719	28.812	26.586	23.690	21.378	19.785	25.495
House Image							
BayesShrink	33.032	29.697	28.002	25.654	23.637	21.855	26.980
Bilateral	33.847	30.149	27.902	25.088	22.904	21.381	26.879
SURENeighShrink	34.339	30.912	28.938	26.421	24.021	22.297	27.821
Proposed	34.497	31.168	29.275	26.770	24.557	22.937	28.201

Finally, it is important to compare the performance of the denoised images visually. Figure(4), shows that for very low noise level degradation ($\sigma_n \leq 10$), almost all denoising algorithms achieve nearly equivalent visual quality although SURENeighShrink exhibits higher PSNR value. Figure(5) through figure(8), show the effect of denoising for moderate to high noise levels. Noticeably, the proposed denoising algorithm exhibits both higher PSNR value and higher denoised image visual quality as compared with all other denoising algorithms. Also we can notice that the

BayesShrink, and bilateral filter leave considerable amount of residual low frequency noise unaltered (especially for $\sigma_n \geq 20$) which is more prominent in the uniform areas (as an example see the sky in figure(6c-d) and figure(7c-d)). While, SURENeighShrink, corrupts useful low frequency image information when it attempts to remove low frequency noise components(as an example see figure(6e), figure(7e), and figure(8e) respectively). On the other hand, we can see that the proposed denoising algorithm succeeded in distinguishing between low frequency noise components and useful low

Fig 4:Denoising results for Lena image: (a) Original image; (b) Noisy image ($\sigma_n=10$) PSNR=28.131 dB;(c) BayesShrink, PSNR=33.478 dB;(d) Bilateral filter, PSNR=33.807 dB;(e) SURENeighShrink, PSNR=34.609 dB; (f) Proposed algorithm, PSNR=34.415 dB.

Fig 5:Denoising results for Child image: (a) Original image; (b) Noisy image ($\sigma_n=15$) PSNR= 24.713 dB;(c) BayesShrink, PSNR=33.193 dB;(d) Bilateral filter, PSNR=33.621 dB;(e) SURENeighShrink, PSNR=34.407 dB; (f) Proposed algorithm, PSNR=34.580 dB.

Fig 6: Denoising results for House image: (a) Original image; (b) Noisy image($\sigma_n=20$) PSNR=22.172 dB; (c) BayesShrink, PSNR=29.701 dB; (d) Bilateral filter, PSNR=30.173 dB; (e) SURENeighShrink, PSNR=30.934 dB; (f) Proposed algorithm, PSNR=31.103 dB

Fig 7: Denoising results for Cameraman image: (a) Original image; (b) Noisy image($\sigma_n=30$) PSNR=19.067 dB; (c) BayesShrink, PSNR=25.026 dB; (d) Bilateral filter, PSNR=25.596 dB; (e) SURENeighShrink, PSNR=26.205 dB; (f) Proposed algorithm, PSNR=26.679 dB.

Fig 8: Denoising results for Aircraft image: (a) Original image; (b) Noisy image($\sigma_n=50$) PSNR=14.672 dB; (c) BayesShrink, PSNR=29.085 dB; (d) Bilateral filter, PSNR=27.121 dB; (e) SURENeighShrink, PSNR=29.839 dB; (f) Proposed algorithm, PSNR=29.859 dB.

frequency image information. This distinguishing property enable the proposed denoising algorithm to (cancel) or at least attenuate both low and high frequency noise component effectively. To summarize, Figure(9) shows graphically the relative average PSNR of the different denoising algorithms under test. Clearly, this figure states that the proposed

denoising algorithm outperforms all other denoising algorithms in terms of average PSNR values. As an example, for aircraft image, the proposed denoising algorithm achieves an average PSNR gain of 1.374, 2.408, and 0.481 dB as compared with BayesShrink, Bilateral filter, and SURENeighShrink respectively.

Fig 9: Average PSNR of Various Algorithms

9. CONCLUSIONS

In this paper, a new hybrid denoising algorithm was proposed. The performance of the proposed algorithm was compared with conventional bilateral filter[9], BayesShrink[3], and SURENeighShrink[7]. The subjective and objective quality of the proposed denoising algorithm reveals that it outperforms all other denoising algorithm under test and can deal with both low and high frequency noise components effectively. The performance of proposed denoising algorithm can further be improved by adaptively tuning the bilateral filter parameters(σ_s and σ_i) over the image based on the spatial noise levels. Moreover, we believe that, it is possible to improve the proposed denoising algorithm further by using better detailed-subband denoising through adopting neighborhood wavelet based thresholding instead of individual wavelet based thresholding. These issues are left as future work.

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