Testing roundabout capacity models: a practical aid for choosing the right one based on total capacity calculation

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Abstract

The determination of efficiency measures, under a known traffic demand, is a central aspect of roundabouts analysis. Many capacity formulations have been available for some time in different countries relating to different roundabout layouts. This paper refers to fifteen roundabout entry capacity models used above all in the European countries, highlighting their input requirements and their algorithm complexity. The formulations reviewed are then compared referring to a test roundabout, with a simple and widespread layout. Monte Carlo simulation have been performed considering 5,000 traffic distribution matrices for each capacity model. The results obtained are critically assessed resorting to the calculation of roundabout total capacity. The probability distributions of total capacity allow finding a model that stands out among all as an advisable choice for a suitable capacity formulation, acting as a synthesis of all those examined. This can be of considerable use in view of practical applications, for the design of efficient and safe roundabout intersections in situations in which a country-specific capacity formulation is not available.

Keywords: roundabout capacity models; single lane roundabout; entry capacity, total capacity, probability distribution.

1. Introduction

As it is well known, the capacity of a highway facility describes the maximum throughput of traffic demand that can be accommodated under current operating conditions. For this reason, capacity plays a fundamental role in highway facilities research, and it is essential for their planning and design.

This situation also fully concerns roundabouts, which have now conquered a space of absolute prominence among at-grade road junctions, and which are the subject of constant research in highway engineering, also with innovative design solutions for greater safety performance (Tollazzi et al., 2016)

Many of these aspects concern the modelling of roundabouts performances towards road users, commonly expressed in terms of the delays to cross the junctions and the consequent queues that can characterize their functioning. Actually, queues and delays assume the role of Measures Of Effectiveness (MOE) of the functioning of the intersection, and they are the basis of the procedures proposed by the main reference for

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the assessment of highway facilities, i.e. the American Highway Capacity Manual - HCM (TRB, 2016). In addition, these aspects of measuring the operational quality of the intersection by modelling the waiting phenomena and the delays that users spend to perform the manoeuvres require more in-depth mathematical tools than the most common queue theory formulas (Mauro and Pompigna, 2020). For these reasons, the HCM provides approximate solutions which, although with some limitations, allow to face the problem in a conventional way. For these models and procedures, both in theoretical implications and in conventional applications, the topics related to “what it is and how much the capacity for a roundabout is worth” are of great interest in highway engineering research and practice.

Here, we can say that the performance analyses of roundabouts are founded on the hypothesis that the circulatory carriageway and the exits are always undersaturated. Consequently, congestion phenomena may occur only at the entries and capacity refers commonly to the capacity of each roundabout entry, the so-called entry capacity (EC) (Mauro, 2010). Thus, if $i = 1, 2, \dots, n$ is a generic junction arm and t is a certain time instant, for a roundabout as in Fig. 1a ECs are represented by a vector $\mathbf{C}(t) = [C_i(t)]$. The methods by which the components of this vector can be calculated are constantly subject to interest and attention, and this has given rise to various approaches and models that allow to express and to evaluate each component C_i .

Speaking in general, the ECs vector $\mathbf{C}(t)$ or, more simply, every component C_i , can be considered as a function $C_i = f(\mathbf{Q}, \mathbf{G}, \mathbf{\Theta})$ of some other vectors of variables and parameters, such as: traffic demand variables $\mathbf{Q}$ (e.g., entry flow, traffic distributions for exit, opposing flow); geometric parameters $\mathbf{G}$ (e.g., legs number, diameter length, number and width of entry and ring lanes, entry radius, entry angle, etc.); and other driver/vehicle parameters $\mathbf{\Theta}$ (Macioszek, 2020; Pompigna et al., 2020). The ways in which these relationships can be studied and described are manifold, and they characterize the different models that several researches have carried out over the years.

Whatever the model, EC relates to traffic demand and, in particular, with entry and circulating flows, which can be considered as the components of the vector $\mathbf{Q}(t)$. For the generic roundabout of Figure 1a, all the possible components of the vector $\mathbf{Q}(t)$ are determined if we know:

- the entry traffic demand expressed by a vector

$$\mathbf{Q}_e(t) = [Q_{ei}(t)] \quad i = 1, 2, \dots, n \quad (1)$$

- a traffic distribution matrix (origin/destination percentage matrix) (Fig. 1b)

$$\mathbf{P}_{O/D}(t) = [P_{ij}(t)] \quad i, j = 1, 2, \dots, n \quad (2)$$

The origin/destination matrix of traffic demand $\mathbf{M}_{O/D}(t)$ from entries “ i ” to exits “ j ” of the roundabout is obtained multiplying each element of the row “ i ” of $\mathbf{P}_{O/D}(t)$ matrix (2) by the corresponding element $Q_{ei}(t)$ of vector (1):

$$\mathbf{M}_{O/D}(t) = \mathbf{P}_{O/D}(t) \cdot \mathbf{Q}_e(t) = [Q_{ij}(t)] \quad i, j = 1, 2, \dots, n \quad (3)$$

In case of steady-state system – in other words, when traffic demand does not vary with time, and, moreover, the traffic flows are served by the roundabout without congestion phenomena - the traffic demand vector and the percentage origin/destination matrix are constant: $\mathbf{Q}_e(t) = \mathbf{Q}_e$; $\mathbf{P}_{O/D}(t) = \mathbf{P}_{O/D}$. In addition, the entry flow $Q_{e,i}$ for each arm is $Q_{e,i} < C_i$, where C_i is the EC for arm “ i ”, which is also constant (Mauro, 2010).

Figure 1: (A) Roundabout with $n = 4$ arms; (B) $P_{O/D}$ matrix.

Following a widespread approach, the EC models for roundabouts can be divided into three main categories: the regression or empirical models, the gap acceptance models, and the microsimulation-based models (Mauro, 2010; Yap et al., 2013; Macioszek, 2020). Empirical models estimate entry capacity through multivariate regression models with the conflict volumes and any other variables affecting capacity, often related with the geometric characteristics of the intersection (Mauro, 2010), relying on field data. Some entry capacity models of widespread use belong to this group, including Kimber (1980), SETRA (1987), CERTU (1998), VSS/Emch (Bovy et al., 1991), Stuwe (1991), Polus and Shmueli (1997), Aakre (1997), and NRW (Brilon et al., 1993). Although these models are heavily influenced by the location and size of the survey samples, as well as by statistical analysis techniques, the parameters that have been calibrated on specific situations are often used as general cases and applied also outside the original contexts. Gap acceptance models relate roundabout capacity to the characterization of driving behaviours through microscopic traffic variables, such as critical gap and follow-up time. Gap acceptance models use different headway distributions, with parameters that can be empirically calibrated on real data. Purely by way of example, but not exhaustively, we can mention here Troutbeck (Akçelik and Troutbeck, 1991), Explorer (Van Arem, 1992), Brilon (Brilon et al., 1993), Brilon-Wu (Mauro, 2010), Vejdirektoratet (1999), HCM 2010 (TRB, 2010), HCM 2016 (TRB, 2016) (for an in-depth analysis, see (Guo et al., 2019)). It is worth noting that, although it is possible to obtain context-specific information thanks to widespread traffic data acquisition technologies, psychotechnical time values from literature are often used, without delving into the specificity of the design site. Micro-simulations models have spread enormously in recent years, with varied solutions based on commercial (e.g., Paramics (SIAS, 2019), Aimsun (TSS, 2019), and Vissim (PTV, 2019)) or open source (e.g., SUMO (Behrisch et al., 2019)) proprietary simulation programs, or using for example discrete event simulations (e.g., (Bakhsh, 2020)), game theory (e.g., (Banjanovic-Mehmedovic et al., 2016; Tian et al., 2018)), multi agent systems (e.g., (Doniec et al., 2008)), cellular automata (e.g., (Wang et al., 2005; Małeck, 2017)), and various artificial intelligence learning approaches (e.g., (Gritschneider et al., 2016; Zhao et al., 2017; Capasso et al., 2020)). Despite having a great flexibility allowing to manage an extremely large number of events, these models require significant efforts to produce case-by-case implementations, with considerable efforts in

calibration and application to real cases. Once again, however, the transferability of calibrated micro-simulation parameters is worthy of great attention (Gallelli et al., 2017).

As seen from the concise review of the models shown above, many models have been developed and detailed for specific national contexts. Therefore, transferability to different contexts is an effective issue, as in other application fields of highway engineering (Pompigna and Rupi, 2015; Pompigna and Rupi, 2017).

Thus, the aim of this work is to help identifying which model, among the most popular ones, is best suited for the analysis of a roundabout, especially in cases where there are no country-specific references. In this paper we deal with the basic case of a single lane roundabout (i.e., one lane on the ring and one lane at the entrances), focusing on the most common case of a four-arm intersection. A case study with a single-lane roundabout (external diameter $D = 40$ m) with four arms (Fig. 1a) has been considered, first by comparing EC values according with fifteen different capacity models widespread in practice.

As many researches adopt only the concept of EC for comparison purposes, without considering what can be generated in the roundabout junction as a whole, this study aims at overcoming these limitations and suggests a comparison in terms of total capacity (TC) (Mauro, 2010).

Actually, with respect to a given percentage origin/destination matrix $\mathbf{P}_{O/D}$, TC is the sum of the flows from each entry $Q_{e,i}$ that are simultaneously equal to EC ($Q_{e,i} = C_i$), that is:

$$TC = \sum_{i=1}^n Q_{e,i} \quad \text{with} \quad Q_{e,i}/C_i = 1 \quad \text{for} \quad i = 1, 2, \dots, n \quad (4)$$

Entry flows to be considered in Eq. (4) are obtained by solving a system of equations in the n unknowns $Q_{e,i}$. Each of these equations is obtained by writing the specific relation for a certain EC model $C_i = f(\mathbf{Q}, \mathbf{G}, \mathbf{\Theta})$ and replacing C_i with the unknown $Q_{e,i}$. As TC clearly depends on the shape of the $\mathbf{P}_{O/D}$ matrix, in this research a wide possibility of variation of the $\mathbf{P}_{O/D}$ matrix is explored by considering 5,000 different $\mathbf{P}_{O/D}$ cases. Through Montecarlo simulations, and then with a large number of TC values for each EC model, the probability distributions of TC can be estimated from simulations.

Then, comparing the simulated probability distributions for TC, similarities and differences emerge among the fifteen models. These results can provide a practical aid for choosing the right EC model allowing to identify a suitable capacity formulation in view of practical applications, especially in absence of country-specific reference formulas.

2. Material and Methods

In the present study the following fifteen capacity models have been compared (Tab. 1): Kimber (1980), SETRA (1987), CERTU (1998), VSS/Emch (Bovy et al., 1991), Stuwé (1991), Polus and Shmueli (1997), Aakre (1997), NRW (Brilon et al., 1993), Troutbeck (Akçelik and Troutbeck, 1991), Explorer (Van Arem, 1992), Brilon (Brilon et al., 1993), Brilon-Wu (Mauro, 2010), Vejdirektoratet (1999), HCM 2010 (TRB, 2010), and HCM 2016 (TRB, 2016). Table 2 shows the roundabout geometric parameters, the drivers' psychotechnical parameters, the respective symbols used in the EC models of Tab. 1, and the values adopted for the single-lane roundabout under study.

Table 1: Entry Capacity (EC) models.

Model	Entry Capacity
Kimber (1980)	$C = k(F - f_c \cdot Q_c) ; f = 303 \cdot x ; f_c = 0.210 \cdot t^* (1 + 0.2 \cdot x) ;$ $k = 1 - 0.00347 \cdot (\varphi - 30) - 0.978 \cdot \left(\frac{1}{r} - 0.05\right)$ $t^* = 1 + \frac{0.5}{1 + e^{\left(\frac{D-60}{10}\right)}} ; x = c + \frac{e-c}{1+2S} ; S = 1.6 \cdot \frac{e-c}{L}$
SETRA (1987)	$C = (1330 - 0.7 \cdot Q_d) \cdot [1 + 0.1 \cdot (ENT - 3.5)]$ $Q_d = (Q_c + \frac{2}{3} \cdot Q_u') \cdot [1 - 0.085 \cdot (ANN - 8)]$ $Q_u' = Q_u \cdot (1 - \frac{SEP}{15}) ; Q_u' = 0 \text{ if } SEP \geq 15m$
CERTU (1988)	$C = (1500 - \frac{5}{6} \cdot Q_d)$ $\alpha = 0.2 \text{ (typical value)} ; 0.7 \leq \beta \leq 1 \text{ (function of D and ANN)}$ $Q_d = \beta \cdot Q_c + \alpha \cdot Q_u$
VSS/Emch (1989)	$C = 1300 - 0.75 Q_c \text{ if } e \leq 1.5 \cdot c ;$ $C = 1450 - 0.85 Q_c \text{ if } e > 1.5 \cdot c$
Troutbeck (1989)	$C = 3600 \cdot \frac{\alpha \cdot Q_c \cdot e^{-\lambda(t_c - \Delta)}}{1 - e^{-\lambda \cdot t_f}} ; \lambda = \frac{\alpha \cdot Q_c}{1 - \Delta \cdot Q_c} ; \alpha = 0.8 - 0.0005 \cdot Q_c$ $t_c = t_f \cdot (3.6135 - 0.0003137 \cdot Q_c - 0.339 \cdot e - 0.2775)$ $t_f = 3.37 - 0.000394 \cdot Q_c - 0.0208 \cdot D + 0.0000889 \cdot D^2 - 0.395 + 0.388$
Stuwe (1991)	$C = 1089 \cdot \exp(-7.42 / 10000 \cdot Q_c)$
Explorer (1992)	$C = 3600 \cdot (1 - Q_d \cdot t_{\min}) \cdot \frac{Q_d \cdot e^{-Q_d(t_c - t_{\min})}}{1 - e^{-Q_d \cdot t_f}} ; Q_d = \frac{Q_c + 0.5 \cdot Q_u}{3600}$
Polus, Shmueli (1997)	$C = 394 \cdot D^{0.31} \cdot e^{-0.00095 \cdot Q_c}$
Brilon (1997)	$C = 1218 - 0.74 \cdot Q_c$
Aakre (1998)	$C = 275 \cdot x - 0.282 \cdot Q_c \cdot (1 + 0.2 \cdot x) ; x = c + \frac{e-c}{1+2S} ; S = 1.6 \cdot \frac{e-c}{L}$
NRW (1999)	$C = 1200 - (1.27 - 0.021 \cdot D) \cdot Q_c \text{ with } 13 m \leq D \leq 26 m$
Brilon-Wu (2001)	$C = 3600 \cdot (1 - t_{\min} \cdot Q_c / 3600) \cdot \frac{1}{t_f} \cdot \exp\left[-\frac{Q_c}{3600} \cdot \left(t_c - \frac{t_f}{2} - t_{\min}\right)\right]$
Vejdirektoratet (1999)	$C = \frac{Q_c \cdot e^{\frac{-Q_c \cdot t_c}{3600}}}{1 - e^{\frac{-Q_c \cdot t_f}{3600}}}$
HCM 2010	$C = 1130 \cdot e^{-0.001 \cdot Q_c}$
HCM 2016	$C = 1380 \cdot e^{-0.00102 \cdot Q_c}$

Figure 2 shows EC values according to the fifteen models in Tab. 1. For each model, as EC for a generic arm i is $C_i = f(Q, G, \Theta)$ and (G, Θ) assume values in Tab. 2, the entry capacity (pcu/h) can be expressed as a function of circulating flow Q_c (or Q_d for those

models that consider other components as well as the circulating one to consider some disturbance). The maximum EC values, obtained for a circulating flow $Q_d = 0$ pcu/h, range between a minimum of 1,080 pcu/h (Aakre model) and a maximum of 1,500 pcu/h (CERTU model).

Table 2: Parameters used in the EC models of Tab. 1 for the test roundabout.

<i>Parameter</i>	<i>Description</i>	<i>Value adopted</i>
D	Inscribed circle diameter	40 m
e = ENT	Entry width	4 m
c	Lane width	3.5 m
L	Flare mean length	10 m
ϕ	Entry angle	35°
r	Entry bend radius	15 m
C	Entry capacity	-
Q_c	circulating flow	-
Q_d	disturbance flow	-
Q_u	exiting flow	-
t_c	critical gap	4.1 s
$t_{\min}$	Min. gap between succeeding vehicles on the circle	2.1 s
t_f	follow-up time	2.9 s
SEP	splitter island	7.5 m
ANN	circulatory roadway width	7 m

It goes without saying that the values of Q_c (and then Q_d , when it is present in the model), which are taken here incrementally to test the results of each model, can be obtained through linear relationships on the basis of the $\mathbf{M}_{O/D}$ matrix in the actual traffic conditions to be analysed (Mauro, 2010).

Figure 2: EC values according to the models of Tab. 1 with values in Tab. 2

3. Results and Discussion

The main objective of the research is testing and comparing the models of Tab.1 for the test roundabout by means of total capacity (TC) evaluation. Actually, with respect to a given percentage origin/destination matrix $P_{O/D}$, TC is the sum of the flows from each entry $Q_{e,i}$ that are simultaneously equal to EC ($Q_{e,i} = C_i$), as in Eq (4) shown again below for the sake of simplicity:

$$TC = \sum_{i=1}^n Q_{e,i} \quad \text{with} \quad Q_{e,i}/C_i = 1 \quad i = 1, 2, \dots, n \quad (4)$$

Entry flows to be considered in TC estimation are obtained by solving a system of equations in the n unknowns $Q_{e,i}$. Each of these equations is obtained by writing the specific relation for the entry capacity of the model adopted (whose generic form is, as already indicated, $C_i = f(Q, G, \Theta)$) and replacing C_i with the unknown $Q_{e,i}$. Due to the variety of mathematical expressions with which C_i can be expressed, it is often appropriate to look for numerical solutions of the above system of equations, using for example iterative procedures (Mauro, 2010), as done in this research.

From what has been said, TC clearly depends on the shape of the $P_{O/D}$ matrix. Figure 3 shows four distinct limit conditions of traffic flow distribution matrix, i.e.: $P_{O/D} 1 = 100\%$ of vehicles make the U-turn manoeuvre; $P_{O/D} 2 = 100\%$ of vehicles turn left; $P_{O/D} 3$: 100 of vehicles cross the roundabout; $P_{O/D} 4 = 100\%$ of vehicles turn right.

Figure 3: Exemplification of the limit conditions of traffic flow distribution entering from the arm $i = 1$ ((A): $P_{O/D} 1$; (B): $P_{O/D} 2$; (C): $P_{O/D} 3$; (D): $P_{O/D} 4$)

As can be observed from Fig. 4 and 5, the roundabout TC values increase from matrix $P_{O/D} 1$ to matrix $P_{O/D} 4$. Higher TC values occur when, for each arm, the entry flow turns right completely (i.e. matrix $P_{O/D} 4$); in this condition, the maximum TC value is given by HCM 2016 model ($TC = 5,520$ pcu/h), which is also the most recent model. In the intermediate situation (i.e. matrix $P_{O/D} 3$), the highest value is provided by the CERTU model ($TC = 3512$ pcu/h), which is one of the oldest among the models considered. Overall, considering the four traffic limit conditions, the model that is closest to the average TC calculated considering the 15 models is the Troutbeck model (Akçelik and Troutbeck, 1991), with an average absolute percentage deviation of less than 5%.

Figure 4: TC values (traffic flow distribution matrices (A): $P_{O/D} 1$; (B): $P_{O/D} 2$; (C): $P_{O/D} 3$; (D): $P_{O/D} 4$)

In this research, in addition to the four limit conditions in Fig. 3, a wide possibility of variation of the traffic distribution is explored by considering a great number of different $P_{O/D}$ matrices. For this purpose, a Monte Carlo simulation has been performed considering 5,000 traffic distribution matrices, which have been used to process each EC model. Manoeuvring percentages have sampled from independent uniform probability distributions for each arm and then normalized. Thus, through Monte Carlo simulations we have obtained a large number of TC values for each model using again the iterative procedure in (Mauro, 2010). The results of these Monte Carlo iterations for each EC model highlight the probability distributions of TC from simulations.

More details about the sampled distribution of TC values can be observed for each model in Fig. 5 and 6. Explorer (Van Arem, 1992) and HCM 2010 (TRB, 2010) models give the lower TC values in all traffic conditions, while CERTU (1998) and NRW (Brilon et al., 1993) models offer the higher values. Instead, the values of TC distributions are very similar for the models SETRA (1987), HCM 2016 (TRB, 2016), Brilon (Brilon et al., 1993), Polus and Shmueli (1997). In each Montecarlo iteration, Troutbeck model (Akçelik and Troutbeck, 1991) shows a trend very close to the average distribution (Average Model), obtained by considering the sample average of the 15 capacity models.

Figure 5: Distributions of the TC values (box plot)

Figure 6: Sampled probability distribution for TC

Figure 7: Troutbeck – Average models comparison for sampled TC distributions

The average model (AVE), which considers for each Monte Carlo iteration (and its related traffic distribution matrix) the mean value obtained considering the TC values of the fifteen models, can represent the most suitable estimate for roundabout TC. This is actually a value that can be assumed, in the absence of other information, when there is no other way to say whether a model among the most popular ones (i.e. the fifteen considered) can be preferred (as the most appropriate for the specific context) or if someone can be definitely excluded (as surely not appropriate to the specific context).

The similarity of the probability distributions of TC obtained with the Troutbeck model (Akçelik and Troutbeck, 1991) (T) and the average of the 15 models (AVE), is qualitatively well observable in Fig. 7. Due to the high number of samples, the two distributions show an evident Gaussian trend, respectively with means $\mu_T = 2,386$ and $\mu_{AVE} = 2,395$ and with standard deviations $\sigma_T = 151$ and $\sigma_{AVE} = 141$. These values are estimated considering the 5,000 sample values of the T model and the 5,000 average values of the 15 models defining the AVE model.

A more structured test to verify the similarity of the TC distributions has been carried out using a statistical measure of dissimilarity between probability distributions. In this case we consider the Bhattacharyya Distance (BD) (Bhattacharyya, 1943).

Considering that the sampled distributions, as mentioned, are substantially Gaussian (cf. Fig. 6), an overlapping test has been carried out between the TC probability distributions estimated for each of the fifteen models (M) and that of their average (AVE) using the Bhattacharyya Distance (BD) expression for Gaussian samples (Coleman and Andrews, 1979):

$$BD(M, AVE) = \frac{1}{4} \ln \left[\frac{1}{4} \left(\frac{\sigma_M^2}{\sigma_{AVE}^2} + \frac{\sigma_{AVE}^2}{\sigma_M^2} + 2 \right) \right] + \frac{1}{4} \left[\frac{(\mu_{AVE} - \mu_M)^2}{\sigma_{AVE}^2 + \sigma_M^2} \right] \quad (5)$$

where $BD(M, AVE)$ is the Bhattacharyya distance between M and AVE TC sample distributions, μ_{AVE} and μ_M their sample mean and σ_{AVE}^2 and σ_M^2 their sample variance.

Figure 8 shows the BD values (in logarithmic scale) for each model compared to the average. The high similarity (in other words, a superimposition) between the probability distribution of TC according to the Troutbeck model (Akçelik and Troutbeck, 1991) and that according to the average of the 15 models is evident. In fact, the BD value obtained for Troutbeck model (Akçelik and Troutbeck, 1991) is extremely close to zero, as well as less than any other model.

Figure 8: BD (logarithmic scale) for TC distributions compared to the average

In these terms, the Troutbeck model (Akçelik and Troutbeck, 1991) is the one that presents the probability distribution of total capacity that is closest to the average one, identifiable with respect to the fifteen models tested. Therefore, this model emerges as the best candidate to represent, in the conditions of the experiment (i.e., a diameter of 40 m and the working assumptions for the dimensional and modeling parameters as in Table 2), the average result obtainable with the fifteen most popular EC models applied to a single-lane and four-arm roundabout.

4. Conclusion

This study suggests a method for testing roundabout capacity models in terms of total capacity (TC). Fifteen entry capacity models have been tested for a conventional single-lane roundabout with four arms (external diameter $D = 40$ m). A great number of traffic

simulations were performed, considering 5,000 traffic distribution test matrices ($\mathbf{P}_{O/D}$) obtained by a Monte Carlo simulation, including the four limit conditions of traffic distribution.

A specific statistical analysis has been carried out. From the comparison between the TC distributions obtained with the fifteen models, the relative similarities and differences emerge. The results show that Explorer and the HCM 2010 models give the lower TC values, while CERTU and NRW models offer the higher values in all traffic conditions. The values of TC distributions are very similar for the models SETRA, HCM 2016, Brilon, Polus and Shmueli.

Troutbeck model shows a probability distribution for TC that is very close to the average distribution of the fifteen models, allowing to identify a suitable capacity formulation in view of practical applications, especially in absence of country-specific reference formulas. This evidence has also been confirmed by a statistical test of distribution dissimilarity (i.e., Bhattacharyya Distance).

Actually, these results are to be considered within the conditions assumed in this research. These concern a single lane roundabout; four entry/exit arms; a diameter of 40 m and the working assumptions for the dimensional and modeling parameters (see Table 2). In the continuation of the research, the analysis can be expanded by considering different layouts and exploring the effects of any variations in the dimensional and modeling parameters.

Therefore, at least in the case tested in this paper and not considering particular contextual situations that would lead to prefer a certain model (as it results certainly more suitable) or to exclude a certain group (as it collects models surely not conform to the specific case), Troutbeck model could be preferred and adopted especially in countries where the technical guidelines do not indicate specific capacity formulations for single-lane roundabout performance analysis.

Nomenclature

EC	entry capacity
$\mathbf{C}(t)$	entry capacity vector
$C_i(t)$	entry capacity for the generic arm
$\mathbf{Q}$	traffic demand vector
$\mathbf{G}$	geometric parameters vector
$\mathbf{\Theta}$	driver/vehicle parameters vector
i	generic junction arm
n	number of junction arms
t	generic time instant
$\mathbf{Q}_e(t)$	entry traffic demand vector
$Q_{ei}(t)$	entry traffic demand for the generic arm
$\mathbf{P}_{O/D}(t)$	origin/destination percentage matrix
$P_{ij}(t)$	percentage of traffic demand $Q_{ei}(t)$ directed to exit arm j
$\mathbf{M}_{O/D}(t)$	roundabout origin/destination traffic demand matrix
$Q_{ij}(t)$	traffic demand from entry arm i to exit arm j
TC	total capacity

D	Inscribed circle diameter
e = ENT	Entry width
c	Lane width
L	Flare mean length
ϕ	Entry angle
r	entry bend radius
Q_c	circulating flow
Q_d	disturbance flow
Q_u	exiting flow
t_c	critical gap
t_{min}	minimum gap between succeeding vehicles on the circle
t_f	follow-up time
SEP	splitter island width
ANN	circulatory roadway width
pcu/h	passenger car unit per hour
AVE	Average values considering the 15 models results
T	Troutbeck model results
μ_M	mean of the TC values for the generic M model estimated by Monte Carlo simulations
σ_M	standard deviation of the TC values for the generic M model estimated by Monte Carlo simulations
μ_{AVE}	mean of the TC values for the AVE model
σ_{AVE}	standard deviation of the TC values for the AVE model
BD	Bhattacharyya Distance

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