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Analysis of One Dimensional Inverse Heat Conduction Problem : A Review

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Abstract - A procedure to solve inverse heat conduction problem (IHCP) is to derive surface heat flux and temperature from temperature change inside a solid. The method proves to be very useful and powerful when a direct measurement of surface heat flux and temperature is difficult, owing to several working condition. The literature reviewed here discussion one dimensional inverse heat conduction problem. Procedure, criteria, methods and important results of other investigation are briefly discussed.

I. INTRODUCTION

The heat conduction problem involves determining the surface heat flux and temperature distribution in the body from transient measured temperature history at one or more internal locations. The problem is faced frequently instrumentation systems. Typical instances include the monitoring of thermal stress in thick-walled pressure parts of power plants based on temperature measurements at the element outside the surface, and predictions of heat flux and temperature from calorimeter type instrumentation. Changes in the surface condition are damped in the solids owing to the difference of heat conduction process. Conversely, as on inverse analysis procedure extrapolates from the interior to the exposed surface, the little experimental errors in the data are magnified at the surface and can lead to oscillations in the calculated surface conditions. Several procedures have been presented to lessen the sensitivity to measurement errors.

In evaluating new heat-shield materials, developing transient calorimeters and trial and testing of rocket nozzles, it is at times necessary to determine the transient surface heat flux and the surface temperature from a temperature history measured at some location inside the body. In distinction of the conventional diffusion or transient heat conduction problems this one has been termed the 'inverse problem'. If the thermal

properties are functions of temperature, the inverse problem becomes nonlinear. The classical 'direct problem' in heat conduction is to determine the interior temperature distribution of a body from the data given on its surface. However, applications need arise in which data is not available over the entire surface but is attributed at interior points. In such situations, it is necessary to calculate the surface temperature.

A procedure to solve the inverse heat transfer problem (IHTP) is extremely important in determining unknown surface temperature and heat flux from known values in the body, which are usually measured as a function of space and time, specifically under several surface conditions such as re-entry of space vehicle and accidents involving coolant breaks in the plasma facing components, a direct measurement of surface temperature change or the heat flux on the surface is almost impossible to measured, so that prediction of these values can help, depending of the solution of IHTP. Additionally numerous studies about IHTP have been carried out to conjecture the transient surface conditions during quenching of a hot body.

However, the exact solution for IHTP is mathematically examined and verified that does not exist within a contain time depending on the position at which the known value is given.

Table 1: A comparative study of procedure and method used for IHCP by several investigations

Investigators	Procedure used	Method used	Results
[1]Stolz (1960).	(a.)His procedure was unstable if the time intervals are made too small. Small time scales in the calculations were needed to obtain more information about the surface.	(a.)He used integral method but he did not report any problem with stability.	(a.)Using different approximations he found the average heat flux for a succession of short intervals.
[2]Frank (1963).	(a.)The least squares method has been used and he assumed an analytical form for time variations of the surface heat flux for the duration of the experiments.	(a.)Finite-difference method (FDM).	(a.)Least squares method was applied to the solution of heat conduction problem to get the results.
[3]Burggraf (1964).	(a.)Paper using least squares in quite different ways was written by him. (b.)He had contributed significantly to the theory and understanding of the problem, but were usually restricted to applications with constant physical properties.	(a.)The linear inverse problem in conduction heat transfer can be solved using an exact method.	(a.)He clearly found the exact solution for the investigations surface heat flux for given continuous temperature and heat flux histories at a given internal point. (b.)When Burggraf equation was utilized with discrete or experimental results.
[4]Sparrow et al. (1964).	(a.)They used another approach and had contributed to the theory and understanding of the problem, but were usually restricted to applications with constant physical properties. (b.)They introduced arbitrary function and their analysis was for one dimensional problem only.	(a.)Integral method using FDM. (b.) They used a skill fully introduced arbitrary function. The values of this function are given with time. (c.)Minimum interval of time was recommended to be 0.01sec.	(a.)They reported results of the inverse problem in transient heat conduction. They did not report any problem with stability.
[5]Back et al. (1965).	(a.)They attacked the nonlinear inverse heat conduction problem, but their method was not involved using the ideas of nonlinear.	(a.)Finite-difference method.	(a.)This was the only method sufficiently powerful to solve the non linear problem.
[6]Davies (1966).	(a.)He assumed an analytical form for the time variation of the surface heat flux for the duration of the experiment.	(a.)Finite-difference method.	(a.)They did not report any problems with stability and his method was not designed for determining heat fluxes.
[7]Beck (1968).	(a.)The method was not developed using the ideas of nonlinear estimations. (b.)Much smaller time steps were found by him to be possible by using least squares.	(a.)Integral method.	(a.)He found surface heat flux using an integral method.
[8]Beck (1970).	(a.)He presented a method which could be conveniently heat a composite body with temperature-variable thermal conductivity and specific heat. (b.)Discrete approximation by finite difference was applicable to nonlinear problems, but the success of this method was limited in many cases by the inherent ill-posed mass.	(a.)A new finite difference method is given. It was based in part upon the concepts of a general technique or solving inverse problems called nonlinear estimation. (b.)This was the most successful and consistent approach in use.	(a.)An objective of this paper was to present a method suitable for use with a digital computer for heating composite body. (b.)Another objective was to derive the methods using nonlinear estimation in order to demonstrate the applicability of nonlinear

	(c.)Although good results had been obtained in some situations, discrete methods were usually not capable of handling more complicated problems without some correction.		estimation to determine time dependent quantities at a boundary.
[9]Imber and Khan (1972).	(a.)He employed an approximate function expressed by a polynomial function to estimate the known temperature change with time in place of the discrete values which were used and applied it to the sub soldiery equation to achieve the inverse solution explicitly.	(a.)In his method a relationship between temperatures measured at two different points is assumed to avoid divergences of the inverse solution.	(a.)He obtained transient temperature distributions with embedded thermocouples.
[10]Shofi (1973).	(a.)He started from the same inverse solution in a subsidiary form as Sparrow et al. (1964) and achieved two different types of solutions as functions of time in two different ways. He verified that one of the two solutions corresponds to the solution proposed by Burggraf (1964) and the other to one obtained by Sparrow et al. (1964). He employed the data including uncertainties in the known values to evaluate the predictive accuracy of the inverse solutions. He numerically calculated the inverse saluting using a finite difference method an exhibited a relationship between the accuracy of the predicted values and the level of the uncertainties included and also between minimum predictive interval level of the uncertainties.	(a.)Finite-difference method.	(a.)He finally pointed out that Laplace transformation was promising in treating one IHTP.
[11]Garifo et al. (1975).	(a.)They described a procedure to approximate that the actual solution by solving a sequence of well-posed (b.)Success of this method was limited as narrated by Beck (1970).	(a.)Finite-difference method.	(a.)They presented solution of the inverse heat conduction problem by finite difference method (FDM).
[12]Ott and Hedrick (1977).	(a.)Finite-difference procedure was employed by them.	(a.)Finite-difference method.	(a.)They reported results of a one-dimensional implicit approach to heat conduction problem.
[13]Weber (1981).	(a.)A detailed analysis of ill-posed nature was carried out by him. The reformulation procedure was described clearly. (b.)He presented skillfully four same problems, representing slab, cylindrical, spherical, geometries as well as constant and temperature dependent physical properties. (c.)The inverse problem was closely approximated by him by a well-posed problem where solution was easily obtained. (d.)He developed clearly a complete formulation of the problem.	(a.)He used space marching difference method.	(a.)He presented the results of the analysis and solution of the ill-posed inverse heat conduction problem.

	<p>Additionally a great knowledge existed concerning the behavior of this new problem and stable, high order numerical methods which were available.</p> <p>(e.)This procedure naturally incorporated future times in the analysis. The space marching procedure utilized exact matching of the calculated temperature with the experimental temperature at a location while temperatures were measured. Thus this procedure was sensitive to measurement error. The number of past and future time steps was dependent of the number of spatial grid nodes.</p>		
[14]Henset and Hills (1986).	(a.)An initial value approach to the inverse heat conduction problems was used.	(a.)Space marching difference method was used by them.	(a.)They obtained results by an initial value approach.
[15]Raynaud and Bransier (1986).	(a.)Their procedure involved a new finite difference method for nonlinear inverse heat conduction problem.	(a.)Space marching difference method was used.	(a.)They achieved results by a new finite-difference method for the nonlinear heat conduction problem.
[16]Cheng and Chang (1990).	(a.)They showed good agreement between estimates and the exact solution for non-dimensional time of $t = 1\text{sec}$ and 5sec , although it was not mentioned that what kind of uncertainties were merged into the temperatures used. From the view point that the inverse solution for short time may be usually required during a transient heat conduction, non dimensional times $t = 1\text{sec}$ and 5sec seem to be too large because heat transfer process may reach steady state, corresponding to a change in the temperature with time.	(a.)Use of hybrid method	(a.)They showed vividly the application of hybrid method, to inverse heat conduction problem.
[17]Serra et al. (1993).	(a.)Space marching difference method naturally incorporated future times in the analysis.	<p>(a.)Space marching inverse conduction method was used.</p> <p>(b.) The equations for surface temperature and heat flux in space marching methods were numerical analogs of the exact solution [Burggraf (1964)] in which time derivatives were replaced by finite difference approximations.</p>	(a.)They presented experimental results of heat transfer in a gun barred based on a space marching inverse conduction.
[18]Taler (1996).	(a.)He described the unified mathematical procedures of transient method for measuring surface heat transfer rates.	(a.)The interior temperature measurements were converted into local instantaneous heat transfer coefficient by solving the inverse heat conduction problem for the gauge. The effect of the inaccuracies in the measurement of the interior temperature was eliminated by cubic spine smoothing of digital	(a.)The aim was to present a method for a simple and accurate determination of the time varying heat transfer coefficient (or heat flux) given an accurate temperature history of the body at a selected point beneath the surface.

		filtering of the raw interior temperature data prior to using it in the heat conduction analysis.	(b.) General case closed from equation for instantaneous surface heat flux or heat transfer coefficients were developed.
[19]Taler (1996).	(a.)He presented a simple and accurate method without sequential stepping forward in time. Additionally it was not necessary to compute all the nodal temperature at each time step. The surface heat flux and temperature distribution can be calculated at anytime without knowing of initial temperature distribution.	(a.)He used a simple method without sequential stepping forward in time.	(a.)A semi-numerical method was presented for solving the inverse heat conduction problem in homogeneous and composite bodies. The present solution did not require both the initial temperature distribution in the body and the whole temperature time history at the temperature sensors locations.
[20]Monde (2000).	(a.)This method first approximates the temperature data with a half polynomial power series in time. The resultant expression for an objective temperature or heat flux was explicitly achieved in the form of power series of time. (b.)This procedure was for one dimensional problem only. (c.)This procedure used Laplace transform technique.	(a.)A theoretical method has been developed for the inverse heat	(a.)An analytical method has been developed for the inverse heat conduction problem. (b.)The main differences from the former researchers were to employ equation expressed by a half polynomial series of time to approximate the known values. As a result inverse solution can be obtained explicitly so that no iterative calculation was required and the calculations of the solution become very quick.

II. CONCLUSIONS

An understanding of a procedure to solve inverse heat transfer problem is very important in determining unknown surface temperature and heat flux from known values in the body. The literature survey presented here is by no means complete. However, it is desired to serve as an indication of the scope of the research completed so far and to draw attention to the deficiency of our knowledge on the subject of ill posed inverse heat conduction one dimensional problem. As our understanding of this subject increases, the analytical approach will add more to our basic knowledge of ill-posed inverse heat conduction problem in single dimension.

REFERENCES:

- [1] G. Stolz (1960) Numerical solution to an inverse problem of heat conduction for simple shapes, J. Heat Transfer, 82 C: 20.
- [2] I. Frank (1963) An application of least squares method to the solution of the inverse problem of heat conduction, J. Heat Transfer, 85C: 378.
- [3] O. R. Burggraf (1964) An exact solution of the inverse problem in heat conduction theory and applications, J. Heat Transfer, 86C: 373.
- [4] E. M. Sparrow, A. H. Sheikh and T. S. Lundgren (1964) The inverse problem in transient heat conduction, J. Appl. Mechanical, 86E: 369.
- [5] J. V. Beck and H. Wolf. (1965) The nonlinear inverse heat conduction problem, ASME Paper No. 65-HT: 40.
- [6] J. M. Davies (1966) Input power determined from temperature in a simulated skin protected against thermal radiation, J. Heat Transfer, 88C: 154.
- [7] J. V. Beck (1968) Surface heat flux determination using an integral method, Nuclear Engineering, 1: 170.
- [8] J. V. Beck (1970) Nonlinear estimation applied to the nonlinear heat conduction, Int. J. Heat Transfer, 13: 703.
- [9] M. Imber and J. Khan (1972) Prediction of transient temperature distributions with embedded thermocouples; AIAA Journal, 10: 784.

- [10] M. Shoji (1973) Study of inverse problem of heat conduction (in Japanese) Trans, JSME, 44: 1633.
- [11] L. Garifo, V. E. Schrock and E. Spedicato (1975) On the solution of the inverse heat conduction problem by finite difference, Energia Nucleare, 22: 452.
- [12] L. J. Ott and R. A. Hedrick. (1977) A one dimensional implicit approach to the Inverse heat conduction problem, ORNL/NUREG – 23, Oak Ridge National Laboratory.
- [13] C. F. Weber (1981) Analysis and solution of the ill-posed inverse heat conduction problem, Int. J. Heat Transfer, 24: 1783.
- [14] E. Hensel and R.G. Hills (1986) An initial value approach to the inverse heat conduction problem, Trans. ASME, J. Heat Transfer, Page-248.
- [15] M. Raynaud and J. Bransier (1986) A new finite difference method for the non linear inverse heat conduction problem, Numerical Heat Transfer, 9: 27.
- [16] H. K. Chen and S. M. Chang (1990) Application of the hybrid method to inverse heat conduction problem, Int. J. Heat Mass Transfer, 33: 621.
- [17] J. J. Serra, J. M. Gineste, S. Serror, Y. Guilmond and M. Cantarel (1993) Experimental investigation of the heat transfer in a gun barrel based on a space marching inverse conduction method, Inverse problems and engineering, Theory and practice, New york, ASME, Page-323.
- [18] J. Taler (1996) Theory of transient experimental technique for surface heat transfer; Int. J. Heat Mass Transfer, 39: 3733.
- [19] J. Taler (1996) A semi numerical method for solving inverse heat conduction problems, Int. J. Heat Mass Transfer, 31: 105.
- [20] M. Monde (2000) Analytical method in inverse heat transfer problem using Laplace transform technique, Int. J. Heat Mass Transfer, 43: 3965.