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Synchronization of Identical New Chaotic Flows Via Sliding Mode Controller and Linear Control

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Abstract— This paper deals with two methods of synchronization for two identical new 3D chaotic flows reported by Li et al. First, a sliding mode controller is proposed to synchronize the two systems, where the slave system is assumed to have a single input. Then based on the stability theory in the cascade system, a simple linear feedback law is presented to realize synchronization of chaotic systems. The asymptotic convergence to zero of the errors between the states of the master and the slave systems is shown. Simulation results are given to illustrate the effectiveness of the proposed methods.

Keywords : Chaos, synchronization, sliding mode controller, Linear feedback, Input-to-state stability (ISS)

I. INTRODUCTION

In the past several years, control and synchronization problems of chaotic systems have been extensively studied, due to their potential applications in many areas. For example, chaos has been used to monitor cardio-activity, encrypt messages, predict the dynamics of quasi-periodic and chaotic motion in the Navier–Stokes equations and synchronize highly complex nonlinear dynamical systems. The idea of synchronization is to use the output of the drive system to control the response system so that the output of the response system follows the output of the drive system asymptotically. Since Pecora and Carroll introduced the problem of synchronization between two identical chaotic systems with different initial conditions, Various effective methods have been presented to synchronize various chaotic systems, adaptive control [1,2], linear and nonlinear feedback control [3], active control [4], adaptive feedback control [5], complete synchronization back stepping design [6,7], and passive control [8] and so on.

Sliding mode control (SMC) [9,10] is a powerful and robust approach to controlling nonlinear dynamical systems that is often used in engineering and others applications. The main advantage of SMC is its insensitivity to mode errors, parametric uncertainties and other disturbances. Once the system is on the sliding manifold, the system behavior is determined by the surface of the sliding surface. This advantage gives us more freedom in designing the controller because we can modify the system model by introducing virtual disturbances to satisfy certain conditions or requirements.

The Linear feedback control [11-13] approach is based on the stability theory in cascade system. Taking some error states as virtual control inputs, input-to-state stability (ISS) theory is technically implemented to attain global asymptotical stability of the overall error system. Linear controller is designed step by step and the feedback control obtained in this way is simple.

Recently, Li et al. [14] performed a new 3D chaotic flow different from Lorenz, Chen, and Lü chaotic systems. The synchronization of this new chaotic attractor was also obtained by utilizing the sliding mode controller and linear feedback control. The rest of this paper is organized as follows. In Section 2, the new 3D chaotic flow reported by Li et al. [14] is presented. Section 3 presents the design of a sliding mode controller to synchronize two identical new 3D chaotic flows; Synchronization between two identical new chaotic systems via linear feedback control is proposed in Section 4. Finally, some concluding remarks are given in Section 5.

2. The new chaotic attractor varied the family of Lorenz systems

The new 3D chaotic flow varied Lorenz, Chen, and Lü chaotic systems was performed by Li et al. [1,4] in the recent year. This chaotic system, named as Li system in this study, is described by the following nonlinear differential equations [14].

(1)

And Qi and Chen et al. obtained a new system as follows [5]:

$$\begin{cases} \dot{x} = a(y - x) + yz \\ \dot{y} = cx - y - xz \\ \dot{z} = xy - bz \end{cases} \quad (2)$$

where x, y, z and a, b are state variables and system parameters, respectively. System (1) demonstrates a chaotic attractor as $a = 5, b = 16, c = 1$. The complex attractor with the initial conditions $(x(0), y(0), z(0)) = (0.1, 0.1, 0.1)$ is shown in Fig. 1. It exhibits that the free control Li system has bounded trajectories.

3. Synchronization via sliding mode controller

In this section, the synchronization of two identical Li chaotic systems via sliding mode controller is proposed in spite of the difference in initial conditions [9,10].

Suppose the master system is defined as the following form

$$\begin{cases} \dot{x}_1 = a(y_1 - x_1) \\ \dot{y}_1 = x_1 z_1 - y_1 \\ \dot{z}_1 = b - x_1 y_1 - c z_1 \end{cases} \quad (2)$$

and the slave system is taken as follows:

$$\begin{cases} \dot{x}_2 = a(y_2 - x_2) \\ \dot{y}_2 = x_2 z_2 - y_2 + u \\ \dot{z}_2 = b - x_2 y_2 - c z_2 \end{cases} \quad (3)$$

Note that the second ordinary differential equation (ODE) of the slave system contains the forcing term u . This term represents the controller of the system. This controller will be designed such that the master system and the slave system are synchronized after starting from different initial conditions.

Defining the errors between the states of the master and the slave systems as follows:

$$\begin{aligned} e_1 &= x_2 - x_1 \\ e_2 &= y_2 - y_1 \\ e_3 &= z_2 - z_1 \end{aligned} \quad (4)$$

Using Eqs. (2)–(4), the error dynamics can be written as follows:

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) \\ \dot{e}_2 &= e_1 z_2 + e_3 x_1 - e_2 + u \\ \dot{e}_3 &= -e_1 y_2 - e_2 x_1 - c e_3 \end{aligned} \quad (5)$$

The aim of this section is to design a controller to force the errors to asymptotically converge to zero as t tends to infinity. We will use a sliding mode control technique to force the error to converge to zero.

3.1. Design of the controller

The first step in designing a sliding mode controller is to choose the sliding surface. Since the slave system has one control input, we need to choose one sliding surface. Let the sliding surface S be such that:

$$S = e_2$$

Let β and Γ be positive scalars. Now, we have the following proposition.

Proposition 1. The sliding mode control law:

$$u = -e_1 z_2 - e_3 x_1 + e_2 - \beta e_2 - \Gamma \operatorname{sgn}(e_2) \quad (7)$$

when applied to the error system (5) guarantees the convergence of the errors e_1, e_2 and e_3 to zero as t tends to infinity.

Proof. Taking the time derivative of S in (6) and using

the dynamic model of the errors in (5) and the control law given by (7), we obtain:

$$\begin{aligned} \dot{S} &= e_1 z_2 + e_3 x_1 - e_2 + u \\ &= e_1 z_2 + e_3 x_1 - e_2 - e_1 z_2 - e_3 x_1 + e_2 - \beta e_2 \\ &\quad - \Gamma \operatorname{sgn}(e_2) \\ &= -\beta e_2 - \Gamma \operatorname{sgn}(e_2) \\ &= -\beta S - \Gamma \operatorname{sgn}(S) \end{aligned} \quad (8)$$

Now, let the Lyapunov function candidate V be such that:

$$V = \frac{1}{2} S^2 \quad (9)$$

After taking the time derivative of (9) the following expression is obtained

$$\begin{aligned} \dot{V} &= S \dot{S} \\ &= -\beta S^2 - \Gamma S \operatorname{sgn}(S) \end{aligned} \quad (10)$$

Obviously, $\dot{V}$ given by (10) is such that $\dot{V} < 0$ for $S \neq 0$. Therefore, the trajectories associated with the

discontinuous dynamics given by (8) exhibit a finite time reachability to zero from any given initial conditions provided that the gains β and Γ are chosen to be sufficiently large, strictly positive scalars. Since S is driven to zero in finite time, then the error e_2 is driven to zero in finite time. Moreover, since e_2 is driven to zero in finite time, then after such a finite time, the first ODE of (5) is such $\dot{e}_1 = -a(e_1)$. Thus, $e_1(t)$ asymptotically converges to zero as t tends to infinity. Moreover, since e_1 and e_2 converge to zero as t tends to infinity, and since y_1 and x_2 are bounded then it can be concluded from the third ODE of (5) that e_3 asymptotically converges to zero as t tends to infinity. Therefore, the errors e_1 and e_3 asymptotically converge to zero as t tends to infinity and the error e_2 converge to zero in finite time. Therefore, the states of the master and the slave chaotic systems are synchronized.

3.2. Simulation and results

The sliding mode controller given by Eq. (7) is applied to the chaotic attractor systems given by (2) and (3). The performance of the controlled systems are simulated using the MATLAB/SIMULINK software. The initial conditions of the master Chaotic system (2) are taken to be $x_1(0) = 0.1, y_1(0) = 0.1$ and $z_1(0) = 0.1$. On the other hand, the initial conditions of the slave chaotic system (3) are taken to be $x_2(0) = 1.6, y_2(0) = 5.1$ and $z_2(0) = -4.9$. The parameters of the controller are chosen such that $\beta = 100$ and $\Gamma = 10$

Fig. 2 displays the time response of states x_1, y_1, z_1 for the drive system (1) and the states x_2, y_2, z_2 for the response system (2). Fig. 3 displays the synchronization errors of systems (1) and (2). Simulation results indicate that the proposed sliding mode controller works well.

4. Synchronization via linear control

Suppose the drive system is defined as previous part

$$\begin{cases} \dot{x}_1 = a(y_1 - x_1) \\ \dot{y}_1 = x_1 z_1 - y_1 \\ \dot{z}_1 = b - x_1 y_1 - c z_1 \end{cases} \quad (11)$$

and the response system is taken as follows

$$\begin{cases} \dot{x}_2 = a(y_2 - x_2) + u_1 \\ \dot{y}_2 = x_2 z_2 - y_2 + u_2 \\ \dot{z}_2 = b - x_2 y_2 - c z_2 + u_3 \end{cases} \quad (12)$$

$$\dot{z}_2 = b - x_2 y_2 - c z_2 + u_3$$

where $u_1(t), u_2(t)$ and $u_3(t)$ are designed control functions for achieving synchronization between two new chaotic attractor systems. The state errors between systems (11) and (12) are defined as Eq. (4).

By subtracting system (11) from (12) and taking the time derivative, it gives the error dynamical system

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + u_1 \\ \dot{e}_2 = e_1 z_2 + e_3 x_1 - e_2 + u_2 \\ \dot{e}_3 = -e_1 y_2 - e_2 x_1 - c e_3 + u_3 \end{cases} \quad (13)$$

Now the synchronization problem of (11) and (12) is transformed into the stabilization problem of the error system (13), i.e., if system (13) is asymptotically stabilized by $u_j (j = 1, 2, 3)$, synchronization of new chaotic system is realized. Here presenting a linear feedback is done to achieve the stabilization. The design process of the linear controller [11-13] is mainly divided into 2 steps.

Step 1 Let $u_1 = -a e_2$, then the first equation of the error system (13) becomes

$$\dot{e}_1 = -a e_1 \quad (14)$$

Clearly, system (14) is globally uniformly asymptotically stable.

Step 2 Consider the second and third equation of error system (13) and let $u_2 = -k_1 e_2, u_3 = -k_2 e_3$, where $k_1 > -1, k_2 > -c$ are constant. Then

$$\begin{cases} \dot{e}_2 = e_1 z_2 + e_3 x_1 - e_2 - k_1 e_2 \\ \dot{e}_3 = -e_1 y_2 - e_2 x_1 - c e_3 - k_2 e_3 \end{cases} \quad (15)$$

If we regard e_1 as the virtual input of the system (15), then the systems (14) and (15) satisfy the first condition of Theorem 1 in [3]. Now we verify that they satisfy the second condition of Theorem 1 in [3]. Consider the following unforced system of system (15)

$$\begin{cases} \dot{e}_2 = e_3 x_1 - e_2 - k_1 e_2 \\ \dot{e}_3 = -e_2 x_1 - c e_3 - k_2 e_3 \end{cases} \quad (16)$$

By taking a Lyapunov function for Eq. (16) into consideration

$$V = \frac{1}{2} e_2^2 + \frac{1}{2} e_3^2 \quad (17)$$

The time derivative of V is:

(18)

where $\lambda_1, \lambda_2, \lambda_3$. From Lemma 1 in [3], the system (16) is globally exponentially stable at $(0, 0, 0)$. This means system (15) satisfies the second condition in Theorem 1 in [3]. So system (15) is globally uniformly asymptotically stable, i.e.

From the above analysis, the error system (13) is globally uniformly asymptotically stable by using the linear controller

4.1. Simulation and results

In this section, simulation results for the new chaotic is given, respectively. Fourth order Runge–Kutta integration method is used to solve the differential equations with time step length 0.0001. The parameters and initial values are chosen as section (3). Fig. 4 shows the synchronization errors of systems (11) and (12).

5. CONCLUSION

The synchronization of two new chaotic systems using a sliding mode controller and linear control scheme is tackled in this paper. In designing via sliding mode, the slave system is assumed to have a single input. As it is shown in Fig. 3, The proposed controller ensures the convergence of the errors between the states of the master and the slave systems to zero as time tends to infinity. To design a controller via linear control, the input-to-state theory in the cascade system is used step by step, in order to achieve stabilization. The linear feedback control presented is simple and easy to be implemented in real engineering process. Simulations and results illustrate the effectiveness of the proposed synchronization methods.

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Fig. 1. The chaotic attractor of Li system [18] with

Fig.2. The time response of states for drive system (1) and the response system (2) via sliding mode controller (a) signals x_1 and x_2 , (b) signals y_1 and y_2 , (c) signals z_1 and z_2 .

Fig. 3. Synchronization errors of new chaotic attractor system via sliding mode controller