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A Novel Approach for the Optimal PMU Placement using Binary Integer Programming Technique

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Abstract - Phasor measurement units (PMUs) are considered as a promising tool for future monitoring, protection and control of power systems. In this paper, a novel approach is proposed in order to determine the optimal number and locations of PMUs to make the power system observable. The PMU placement problem is formulated as a binary integer linear programming, in which the binary decision variables (0, 1) determine where to install a PMU, preserving the system set observability, such as full or partial. The proposed approach integrates the impacts of both existing conventional power injection/flow measurements (if any) and the possible failure of single or multiple PMU / communication line into the decision strategy of the optimal PMU allocation. The network topology and hence network connectivity matrix remains unaltered for the inclusion of conventional measurements. Program is developed using Matlab software and is tested on sample seven-bus system and IEEE 14 bus system.

Keywords : Binary integer linear programming , optimal PMU placement, maximum redundancy , failure of PMU / Communication line

I. INTRODUCTION

Synchronized Phase Measurement Unit (PMU) is a monitoring device, which was first introduced in mid-1980s. Phasor measurement units (PMU) are devices, which use synchronization signals from the global positioning system (GPS) satellites and provide the phasors of voltage and currents measured at a given substation. PMU is considered as the most promising measurement technology for monitoring the power systems. This unit is composed of a number of phasors that capture measurements of analog voltage, current waveform and the line frequency. As the PMUs become more and more affordable, their utilization will increase not only for substation applications but also at the control centers for the EMS applications. The Phasor Data Concentrator (PDC), which is located in the control center, gathers all the phasor data received from geographically distributed PMUs via the communication network. The communication network is one of the components that can be investigated and upgraded, to provide more accurate real-time information between the PMU and the control center. In this way, the observance of the power balance can be facilitated.

Wide Area Monitoring and Control (WAMC) systems based on Phasor Measurement Unit (PMU)

provides dynamic coverage of the network. Applications of the phase measuring units includes measuring frequency and magnitude of phasors, state estimation, instability prediction, adaptive relaying, and improved control. There are three basic approaches to conduct network observability analysis; namely numerical, topological, and hybrid approaches. The numerical observability approach is based on the fact that a unique solution for the state vector can be estimated if the gain matrix is nonsingular or equivalently if the measurement Jacobian matrix has a full column rank and well conditioned. Conventionally, numerical methods involve huge matrix manipulations and are computationally expensive. Also, in case of large power systems, the measurement matrix may become ill-conditioned. Many optimal PMU placement techniques, based on this concept, have been devised e.g. Simulated Annealing, particle swarm optimization [7,8], Tabu Search , Genetic Algorithm [2,10] based methods. However, all these techniques, being iterative in nature, require longer convergence time and their convergence depend upon the initial guess. The topological observability approach is based on the fact that a network is fully observable if the set of measurements can form at least one measurement spanning tree of full rank. Topological observability based approaches utilize the graph theoretic concept to find the optimal locations

and, thus, to make the system topologically observable. Few methods, based on this concept, are Depth First Search, Spanning Tree based method [6], and Integer Linear Programming based methods [4,5] etc. Due to the high cost of having a PMU at each node, some of the researcher focused on PMU placement and pseudo measurements for complete or partial observability of the power system. The placement of a minimal set of phasor measurement units at a strategic location to make the power system observable is the main objective of this paper. This paper has proposed a modified topological observability based PMU placement formulation to ensure complete system observability with and without conventional power flow and power injection measurements with one depth of unobservability and maximum redundancy. Further the formulation is extended for failure of PMU or communication line.

II. PROBLEM FORMULATION

PMUs provide two types of measurements: bus voltage phasors and branch current phasors. In this paper, it is assumed that each PMU has enough channels to record the bus voltage phasor at its associated bus and current phasors along all branches that are incident to this bus. Also the cost of all installed PMU is same.

The objective of the PMU placement problem is to render an observable system by using a minimum number of PMUs.

A. Without Conventional Measurements for Complete Observability

The optimal placement of PMU is formulated as follows

$$\text{Min } \sum x_i \sum_{i=1}^n x_i$$

Subject to the constraints

$$C X \geq b$$

Where, C is a connectivity matrix of the system, the entry of which can be defined as

$$C_{ij} = 1 \text{ if } i=j$$

$$= 1 \text{ if } i \text{ and } j \text{ are connected}$$

$$= 0 \text{ otherwise}$$

$$X = [x_1 \ x_2 \ \dots \ x_n]$$

x_i is the PMU placement variable

b is a vector of length n (no. of nodes)

An example for a sample 7-bus system is as shown in Fig. 1.

Fig. 1 Sample 7-bus system

For 7-bus sample system,

$$C = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

B. Without Conventional Measurements for Incomplete Observability (Depth of one unobservability)

The optimal placement of PMU is formulated as follows

$$\text{Min } \sum x_i \sum_{i=1}^n x_i$$

Subject to the constraints

$$A C X \geq b_1$$

Where C is a connectivity matrix of the system A is branch to node incident matrix

$X = [x_1 \ x_2 \ \dots \ x_n]$; x_i is the PMU placement variable. b_1 is a vector of length m (no. of branches)

For 7-bus sample system,

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

C. With Conventional Measurements for Complete Observability
The optimal placement of PMU is formulated as follows

$$\text{Min } \sum x_i \quad \sum_{i=1}^n x_i$$

Subject to the constraints

$$T P C X \geq b_2$$

Where, C is a connectivity matrix of the system

P is a permutation matrix

$$T = \begin{bmatrix} I_{M \times M} & 0 \\ 0 & T_{\text{meas}} \end{bmatrix}$$

Where M is the no. of buses not associated to conventional measurements

$X = [x_1 \ x_2 \ \dots \ x_n]$; x_i is the PMU placement variable b_2 is a vector defined according to cases

For Conventional measurement, following three cases needs to be analyzed in detail :

If a power flow measurement is on line ij

then the following needs to be held : $y_i + y_j \geq 1$

which means one bus voltage can be solved from this measurement and the other needs to be covered by PMU

Suppose that an injection measurement is at bus k as follows

then the following inequality needs to be held :

$$y_1 + y_p + y_k + y_q \geq 3$$

The power flow measurements and the injection measurements are associated

According to the approaches introduced in above mentioned case (1) and (2), the following two inequalities are as follows :

$$y_p + y_k \geq 1 \quad \text{and} \quad y_1 + y_p + y_k + y_q \geq 3$$

In order to satisfy $y_p + y_k \geq 1$, the first inequality needs to be subscribed from the second inequality corresponding to the injection measurement and consider that its right-hand side needs to be reduced by one due to the injection I_k as shown below :

$$(y_1 + y_p + y_k + y_q - y_p - y_k) \geq 3 - 1$$

The second inequality becomes $y_1 + y_q \geq 1$

If bus s is not associated to any conventional measurement, then the corresponding constraint of the minimization problem is still kept $y_s \geq 1$

For 7-bus sample system,

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

D. With Conventional Measurements for Incomplete Observability (Depth of one unobservability)

The optimal placement of PMU is formulated as follows

$$\text{Min } \sum x_i \quad \sum_{i=1}^n x_i$$

Subject to the constraints

$$P_1 A C X \geq P_1 b_1$$

Where, C is a connectivity matrix of the system

A is branch to node incident matrix

P_1 is a matrix that keeps the branches that are associated with zero injection measurement and removes the branches that are not associated with zero injection measurement.

$X = [x_1 \ x_2 \ \dots \ x_n]$; x_i is the PMU placement variable

b_1 is a vector of length m (no. of branches)

For 7-bus sample system,

formulation / algorithm is computationally simple, fast efficient and can be used in practice.

Table II gives the optimum no. of PMU and its location for the considered cases as follows

System	Observability / Constraint	Optimum no. of PMU		PMU Location		Remark
		Without zero injection	With zero injection	Without zero injection	With zero injection	
7-bus	Complete	2	2	2,4	2,5	No. of PMU's required is same, but location changes when zero injection is considered
	Incomplete (Depth of one unobservability)	1	1	3	4	
	Complete observability with max. redundancy as well as failure of PMU / Comm. line	4	2	1,2,4, &5	2 & 4	No. of PMU's required reduces with zero injection consideration for the same constraint and objective function
	Incomplete observability (Depth of one unobservability) with max. redundancy as well as failure of PMU / Comm. line	3	2	2,4 & 5	2 & 4	
System	Observability / Constraint	Optimum no. of PMU		PMU Location		Remark
		Without zero injection	With zero injection	Without zero injection	With zero injection	
14-bus	Complete	4	3	2,6,7,9	2,6,9	No. of PMU's required reduces, when zero injection is considered
	Incomplete (Depth of one unobservability)	2	1	4,6	6	

	Complete observability with max. redundancy as well as failure of PMU / Comm. line	9	4	2, 4, 5, 6, 7, 8, 9, 11 & 13	4, 5, 6 & 9	No. of PMU's required reduces with zero injection consideration for the same constraint and objective function
	Incomplete observability (Depth of one unobservability) with max. redundancy as well as failure of PMU / Comm. line	7	4	2, 4, 5, 6, 9, 10 & 13	3, 4, 5 & 8	

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