July 2014

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Recommended Citation

MISHRA, SONALI; BHARDWAJ, NITISH; and JAIN, DR. RITA (2014) "THEORETICAL CONCEPTS & APPLICATIONS OF INDEPENDENT COMPONENT ANALYSIS," *International Journal of Computer Science and Informatics*: Vol. 4 : Iss. 1 , Article 3.

DOI: 10.47893/IJCSI.2014.1166

Available at: <https://www.interscience.in/ijcsi/vol4/iss1/3>

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THEORETICAL CONCEPTS & APPLICATIONS OF INDEPENDENT COMPONENT ANALYSIS

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Abstract- This paper deals with the study of Independent Component Analysis. Independent Component Analysis is basically a method which is used to implement the concept of Blind Source Separation. Blind Source Separation is a technique which is used to extract set of source signal from set of their mixed source signals. The various techniques which are used for implementing Blind Source Separation totally depends upon the properties and the characteristics of original sources. Also there are many fields nowadays in which Independent Component Analysis is widely used. This paper deals with the theoretical concepts of Independent Component Analysis, its principles and its widely used applications.

Keywords- Independent Component Analysis, Blind Source Separation, Scatter Plot, Decorrelation, Non- Gaussian.

I. INTRODUCTION

Independent Component Analysis (ICA) is one of the major technique used to implement Blind Source Separation. As the various methods used for BSS basically distinguished on the basis of some peculiar properties of original sources or the input sources, so the basic property which we used for ICA is statistical independence between the sources. Uncorrelatedness and non-gaussianity between the sources are also used for developing the algorithm the ICA.

II. THEORETICAL CONCEPT OF ICA

Independent Component Analysis is a computational method of separating a multivariate signal into additive sub components supposing mutual statistical independence of the source Signals. Here independent components means that all the observed signal should be statistically independent, i.e. no component of one signal must be related to any component of other signal. ICA is a special case of blind source separation. The most important concept which is used to implement ICA is the property of statistical independence which must be present between the original sources. By using this technique a linear transformation of original independent sources is observed. Independent Component Analysis can be divided into noiseless and noisy case. Noiseless ICA can be considered as a special case of noisy ICA.

Fig 1: Original sources before mixing

Fig 2: Observed mixed sources

Fig 3: Original sources at the output after applying ICA

III. STATISTICAL INDEPENDENCE

In probability theory two events are said to be statistically independent if the probability of occurrence of one event does not affect the probability of occurrence of other event. In the similar way we can say for random variables that the two random variables are independent if the occurrence of one event does not affect the probability distribution of other event. Condition of statistical independence for two events Two events A and B are independent if and only if their joint probability equals the product of their probabilities:

$$P(A \cap B) = P(A)P(B)$$

IV. DEFINITION OF ICA

“We have given a set of observations of random variables $x_1(t)$, $x_2(t)$... $x_n(t)$, where t is the time or

sample index, assume that they are generated as a linear mixture of independent components: $X=A*S$, where A is some unknown mixing matrix and S are the independent components. Independent component analysis now consists of estimating both the matrix A and the s , when we only observe the $x_i(t)$ [1].”

V. ICA MODELS

Linear Noiseless ICA: The components x_i of the observed random vector are generated as a sum of the independent Components

$$x_i = a_{i1}s_1 + \dots + a_{ik}s_k + \dots + a_{in}s_n \quad (1)$$

so for two source speakers s_1 & s_2 the emitted speech signals is recorded and can be expressed as a linear equation:

$$x_1(t) = a_{11}s_1 + a_{12}s_2 \quad (2)$$

$$x_2(t) = a_{21}s_1 + a_{22}s_2 \quad (3)$$

where a_{11} , a_{12} , a_{21} & a_{22} are some parameters which depend on the distance between speaker & microphones. These coefficients form the mixing matrix A . The task is to estimate both the mixing matrix A and the sources. The original sources S can be recovered by multiplying the observed signals X with the inverse of the mixing matrix $W=A^{-1}$, also known as the unmixing matrix. Here it is assumed that the mixing matrix is square.

Linear Noisy ICA: With the added assumption of zero-mean and uncorrelated Gaussian noise, the ICA model takes the form

$$X = A*S + n \quad (4)$$

VI. DEMONSTRATION OF ICA

We have been given a set of observations of random variables ($x_1(t)$, $x_2(t)$,..... $x_n(t)$) where t is the time or sample index .Now assume that they are generated as a linear mixture of independent components.

$$X=A*S \quad (5)$$

This can be represented in matrix form as given below:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = A \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$

Where A is some unknown matrix. Independent Component Analysis now consist of methods of estimating both the matrix A and sources S when we only observes X . Here we assume that numbers of independent components are equals to the number of observed signals. Now there is a fundamental requirement which is necessary to include which says that the original signals can only be recovered successfully if and only if the components ‘ s ’ are non-Gaussian. This assumption makes this method of ICA different from other methods in which the concept of non gaussianity is not considered. In fact

ICA could also be called as non-Gaussian factor analysis.

VII. PRINCIPLE

ICA basically works on two principles:

Principle1_Uncorrelatedness: This is one of the important assumptions which explain the first principle of ICA. This principle states that if we want to recover original independent signals than there will restriction on input sources is that they must be uncorrelated to each other.

Principle2_Maximum non-gaussianity: The second principle if ICA states that to get our original sources there is one more restriction on the input sources is that they must be non-gaussian.

Independent Components are Non Gaussian Components. The basic concept of non-gaussianity can be implemented with the help of is Central Limit Theorem which states that “sum of non-Gaussian random variables are closer to Gaussian in comparison with the original individual ones”.

So by using these two principles and concept of statistical independence between the original sources the method of ICA widely used in many applications which will be discussed in detail below

VIII. CONCEPT OF SCATTER PLOT

To check the precise results at the output we can make use of scatter plot or scatter diagram. A scatter diagram is a tool for analysing relationships between two variables. One variable is plotted on the horizontal axis and the other is plotted on the vertical axis. The pattern of their intersecting points can graphically show relationship patterns. Most often a scatter diagram is used to prove or disprove cause-and-effect relationships.

While the diagram shows relationships, it does not by itself prove that one variable causes the other. In addition to showing possible cause- and-effect relationships, a scatter diagram can show that two variables are from a common cause that is unknown or that one variable can be used as a surrogate for the other. We can use scatter plot for various purposes to examine theories about cause-and-effect relationships and to search for root causes of an identified, to design a control system to ensure that gains from quality improvement efforts are maintained. How we can interpret our data with the help of scatter plot. Scatter diagrams will generally show one of six possible correlations.[7]

- Strong Positive Correlation: The value of Y clearly increases as the value of X increases

Fig 4: Scatterplot showing strong positive correlation

- Strong Negative Correlation: The value of Y clearly decreases as the value of X increases.

Fig 5: Scatterplot showing weak positive correlation

- Weak Positive Correlation: The value of Y increases lightly as the value of X increases.

Fig 6: Scatterplot showing strong negative correlation

- Weak Negative Correlation The value of Y decreases slightly as the value of X increases.

Fig 7: Scatter plot showing weak negative correlation

- Complex Correlation: The value of Y seems to be related to the value of X, but the relationship is not easily determined.

Fig 8: Scatterplot showing complex correlation

- No-Correlation: There is no demonstrated connection between the two variables.

Fig 9: Scatterplot showing no correlation

IX. APPLICATIONS OF ICA

- Cocktail Party Problem

The Cocktail Party Effect also known as selective attention is the phenomenon of being able to focus one's auditory attention on a particular stimulus while filtering out a range of other stimuli. The Cocktail Party Problem is illustrated in Fig.9. If each of the J voices you can hear at a party is recorded by N microphones, the recordings will be a matrix composed of a set of N vectors, each of

Fig 10: Diagram showing concept of Cocktail party problem

which is a (weighted) linear superposition of the J voices. For a discrete set of M samples, we can denote the sources by an $J \times M$ matrix, Z, and the N recordings by an $N \times M$ matrix X. Z is therefore transformed into the observables X (through the propagation of sound waves through the room) by multiplying it by a $N \times J$ mixing matrix A such that $X^T = AZ$ Figure.(10) illustrates this paradigm where sound waves from $J = 3$ independent speakers ($z_1, z_2, \text{And } z_3$, left) are superimposed (centre), and recorded as three mixed source vectors with slightly different phases and volumes at three spatially separated but otherwise identical microphones. In order for us to 'pick out' a voice from an ensemble of voices in a crowded room, we must perform some type of BSS to recover the original sources from the observed mixture. Mathematically, we want to find a demixing matrix W, which when multiplied by the Recordings X^T , produces an estimate Y^T of the sources Z^T . Therefore W is a set of Weights (approximately 16) equal to A inverse. One of the key methods for performing BSS is known as ICA.[5]

- ICA for Astronomical data

ICA for removing artefacts from astronomical telescope images. To use ICA for the artefact detection and removal. This is motivated by the fact that for astrophysical data, the independence of the artefacts is often theoretically guaranteed, and also the linear mixing model holds exactly. This is an almost ideal application for ICA. In the astrophysical data, we have a number of digital images, recorded over consequent nights when the conditions are favourable, and carefully calibrated for geometrical and photometric alignments. ICA is then used for this image set to reveal independent components that might be artefacts.[9]

- ICA in CDMA communications

Nowadays CDMA technique is widely used for communication purpose. As CDMA has more advantages as compared to FDMA and TDMA. CDMA systems require more advanced signal processing methods, and correct reception of CDMA signals is more difficult because of several disturbing phenomena. A serious problem is multipath propagation. In case of direct sequence CDMA data can be cast in form of linear ICA.

The transmitted symbols have a number of possible values, and the spreading code of the desired user is known. Also multipath propagation and time delays make separation of desired user symbol a challenging task for linear ICA. In the first stage of this project, we applied ICA and BSS methods to various problems in multiuser detection, trying to take into account the available prior information. In particular, we have considered estimation of the desired user's time delay, estimation of fading channels in , and detection of the desired user's symbol sequence . The results are very good showing that ICA based methods can yield better performances than more conventional methods.

In the second stage, independent component analysis is applied to blind suppression of interference caused by bit-pulsed jamming in a direct sequence CDMA communication system. This jamming problem is important in practical CDMA communication systems. We have taken into account both data modulation and temporally uncorrelated jamming is taken into account, improving and extending earlier previous work on the same problem.[9]

- A method for making group inferences from functional MRI data using Independent Component Analysis.

ICA is a promising analysis method that is being increasingly applied to fMRI data. A principal advantage of this approach is its applicability to cognitive paradigms for which detailed models of brain activity are not available. ICA has been successfully utilized to analyse single-subject fMRI

data sets, and an extension of this work would be to provide for group inferences.[8]

- Evolution of multivariate statistical process control: application of independent component analysis and external analysis

Univariate and multivariate statistical process control (USPC and MSPC) methods have been widely used in process industries for fault detection. However, their practicability and achievable performance are limited due to the assumptions that a continuous process is operated in a particular steady state and that variables are normally distributed. In the present work, external analysis is proposed to distinguish faults from normal changes in operating conditions. In addition, to further improve the monitoring performance, a new MSPC method based on ICA is used. The simulation results of a linear multivariable system and a CSTR process have clearly shown the superiority of ICA-based SPC over USPC and PCA-based SPC, and also the usefulness of the proposed external analysis.

- Feature Extraction

In the feature extraction the columns of mixing matrix A represents features and s_i gives the coefficient of the i^{th} feature of an observed data vector x . Theory of redundancy reduction motivates to use ICA for feature extraction.[1]

- Electroencephalogram

Electroencephalogram (EEG) is the recording of electrical activity along the scalp. An EEG data forms by recordings electrical potential of various points on the scalp. These potentials are presumably generated by some underlying components of brain activity [1]. This phenomenon is somewhat similar to as cocktail party problem. It is expected that only original components of brain activity should be recorded, but observation contains mixtures of components. ICA gives important information about the independent components of brain activity by giving access to it.

- Reducing Noise in Natural images

An ICA algorithm finds wide application for removing noise from the images. This feature if ICA widely known as image denoising i.e. removal of white gaussian noise from the natural images.

Fig 11: Image denoising using ICA

X. SHORTCOMINGS OF ICA

- We cannot determine the variances (energies) of the independent components.
- We cannot determine the order of the independent components. The reason is that, again both s and A being unknown, we can freely change the order of the terms in the sum in, and call any of the independent components the first one.
- ICA assumes a linear stationary mixing model (the mixing matrix is a set of constants independent of the changing structure of the data over time). However, for many applications this is only true from certain observation points or for very short lengths of time. For example, consider the earlier case of noise on the ECG. [4]
- The sources mixed by A are assumed statistically independent. That is, they are generated from some underlying processes that are unrelated. In the cocktail party problem, this is trivially obvious; each speaker is not modulating their words as a function of any other words being spoken at the same time. However, in the case of the ECG noise/artefact removal, this is sometimes not true. [4]

XI. CONCLUSION

In this paper we have given the theoretical concept of Independent Component Analysis. We have discussed the basic definition of ICA along with its various models, and principle on which ICA works.

Then we have explained the concept of scatter plot which is basically used to prove the second principle

i.e. presence of correlation between two variables or signals. And at last the various areas of areas are discussed in which ICA is widely used like that in image denoising, feature extraction, face recognition, Telecommunication and many more.

REFERENCES

- [1] Shivi Chaturvedi, Ajay Somkuwar, "Survey of Independent Component Analysis and its Application," Proc. of VLSI Embedded Systems Signal Processing & Communication Technologies Conference NCVESCOM - 2010, April 2010.
- [2] Alex Favaro Aaron Lewis Garrett Schlesinger, "ICA for Musical Signal Separation," CS 229 Machine Learning Final Projects, Autumn 2011. [Online]. Available <http://cs229.stanford.edu/projects2011>.
- [3] Hyvärinen, Karhunen, Oja, "Independent Component Analysis," John Wiley & Sons, March 2001, Chapter 1 : Introduction, pp. 1-2.
- [4] G.D. Clifford, "Blind Source Separation : Principal & Independent Component Analysis," Biomedical Signal and Image Processing Spring 2008, Chapter 15. [Online]. Available www.mit.edu/~gari/teaching/6.555/LECTURE_NOTES/ch15_bss.pdf.
- [5] Fabian J. Theis, "A short tutorial on blind source separation," Institute of Biophysics University of Regensburg, Germany, TUAT, Tokyo, 28-3-2005.
- [6] Independent Components Analysis. [Online]. Available http://courses.washington.edu/matlab2/matlab/Lesson_17.asv.
- [7] <http://web2.concordia.ca/Quality/tools/25scatter.pdf>.
- [8] V. Calhoun, T. Adali and G. Pearlson, "Independent component analysis applied to fMRI data : a generative model for validating results," Journal of VLSI Signal Processing Systems, Volume 37 Issue 2/3, June – July 2004, pp. 281-291.
- [9] <http://research.ics.aalto.fi/ica/biennial2001-3.pdf>