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Dynamic Interaction between Manipulator and Omni-Directional Platform

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Abstract – Recent advancements in mechatronics has led to tremendous development of application of robots for industrial applications involving high health hazards, rough working conditions and difficult to reach locations. Robot assisted welding is one such high risk operation for human being. The industrial arc welding robots are either mounted on the floor, table or on overhead rails. In this scenario, the jobs to be welded are brought to shop floor. However, the on-sight joining of big pipes and structures with precision and speed, demands for mobility of welding robot. In this research work, a novel concept of mounting 5-dof welding robot on the omni-directional platform (Omni-WMR) is implemented to provides immense maneuvering mobility to the manipulator. This paper presents the combined dynamic model of omnidirectional mobile robot for welding operation to study the effect of dynamic interaction between manipulator and mobile platform. Dynamic interaction is carried out to find the variation in torque at manipulator link due to the motion of platform as well as its location on the platform. In addition, the variation in torque developed at platform wheels due to positioning the manipulator at two different locations on platform is accessed. For this, two case studies have been discussed to study the effect of dynamic interaction between manipulator and omni-directional platform.

Keywords – Mobile robot; Omni-directional platform; Maneuvering; Dynamic interaction.

I. INTRODUCTION

Traditionally, robotic manipulators are fixed on the shop floor or on the ceiling or mounted on the overhead / ground rails. The workspace of such fixed-base manipulators is a limited volume of the space that can be reached by the end-effector. This aspect demands careful structuring of the tasks to be performed by manipulator so that all the facilities are within the reach of the manipulator. This is typically achieved by means of conveyor belts or other transporting devices. In the

recent years, there has been a great deal of interest in mobile robots. A mobile manipulator is typically robot mounted on a mobile platform or vehicle, equipped with a computer(s) and various sensors. Mobile manipulators have many potential applications in manufacturing, nuclear reactor maintenance, construction, planetary exploration. Basically, mobile robots are classified on the basis of maneuverability as i.e. non-holonomic and holonomic (Omni-directional drive). The conventional wheeled mobile robots (WMR) are restricted in their motion because they cannot move sideways without a preliminary maneuvering. In order to improve the maneuverability of WMR, various mechanisms like single wheeled drive, differential drive, tracked robot and synchro-drive were developed [1]. However, these efforts were not sufficient to overcome the problem of non-holonomic system associated with conventional WMR. For this reason, such robot is called 'non-holonomic'. In contrast, a holonomic robot, using omni-directional wheels, is capable of driving in any direction. In the last few years, Swedish Omni-directional wheeled (Mecanum wheeled) mobile robot (OWMR) has received growing attention in industry oriented research [2]. Along with the various advantages of omni-directional wheels, they introduce a number of challenging problems such as dynamic interaction between mobile platform and the manipulator, tip-over/roll-over stability of the system [3, 4]. Hence, dynamic modeling of mobile manipulator is the great issue to examine the effect of dynamic interaction on the torque developed at manipulator's link as well as at wheels of the platform.

II. REVIEW OF LITERATURE

In order to understand the current state of art, a detailed literature review covering various advancements in mobile manipulator design,

applications and control issues is presented in this section.

Maher and Jilani (2011) have done the kinematic and dynamic modeling of a three-center wheel omni-directional mobile robot. Using these models, they have simulated the behavior of the robot and evaluated its performance with respect to conventional mobile robots [5].

Tan et al (2008) have determined the kinematics of the mobile platform according to its mechanical structure firstly, and then deduced its unified dynamic model by Lagrangian formalism. They have proposed an adaptive hybrid controller based on RBFANN. Their controller carried out path tracking by considering kinematics of mobile platform and dynamics of robot arm [6].

Djebrani et al (2011, 2012) have developed an agent based approach which considered robot as an independent agent aimed to perform robotic tasks described in terms of displacement and force interaction with the environment. A reactive architecture and impedance control was used to ensure reliable task execution in response to environment stimuli. The control structure of the robot was built in order to reveal the main abilities in navigation and obstacle avoidance. Several simulations were performed to prove the efficiency of their concept [7, 8].

Nakazato et al (2005) developed a mobile manipulator consisting of a 6-dof manipulator and an omni-directional moving platform. As an omni-directional mechanism, four special wheels with twelve free rollers were used for each wheel and they can rotate freely around the circumference of the wheel. They developed a criterion for exploiting the redundancy to avoid the slippage of wheels, and fundamental experiments were done to show the control characteristics of the developed mobile platform [9].

Doroftei et al (2007) has introduced an omnidirectional mobile robot for educational purposes. The chapter written by these authors provided some information about conventional and special wheels designs, mechanical design aspects of the mecanum wheel and also about the robot, kinematic models, as well as electronics and control strategies: remote control, line follower and the autonomous strategy [10].

Abdessemed and Djebrani (2010) have presented the development of the complete kinematic representation of a mobile manipulator. In this case, they have considered manipulator arm and platform as a unique system. Mobile platform is exploited to solve problems like collision avoidance and joints saturations. The results obtained demonstrated the effectiveness of

their approaches for simple situations as well as for complex situations when obstacles were encountered [11].

From above literature review, it can be seen that most of researchers have limited their work towards development of either omni-directional platform, wheels of the platform or its control system in general. The attempts to exploit the benefits of robot manipulator mounted on the omni-directional platform for industrial applications like welding, material handling and service application are very few. However, after studying the overall architecture of the omni-directional platform, it is obvious that the application of omni-directional mobile manipulator for fabrication applications like arc welding is very promising. In order to provide stability to the overall structure containing manipulator, mobile platform and welding gun along with necessary equipments during hazardous welding application, there is a need to meticulously resolve vital issues like tip-over/ roll-over stability, structural complexity, and dynamic interaction between the platform and the manipulator. In view of these aspects, current research work is focused on efficient but safe implementation of a four-wheel omni-directional platform mounted with 5-dof robot for arc welding application. The kinematic and dynamic modeling of robot as well as omni-directional platform is done by Lagrange Euler formulation. Using these models, a generalized mathematical model is developed to explain the phenomenon of dynamic interaction between manipulator and mobile platform. The proposed strategy is implemented to two sample welding paths. In first case, the temporal evaluation of torque at various joints of manipulator is done by considering welding along straight path. In this case, first torque computations are done 1. Without considering mobile platform 2. Considering the mobile platform for better understanding of effect of dynamic interaction. In second case study, a zigzag welding path is considered to evaluate performance of proposed approach for variety of welding paths. The results obtained after implementation of proposed approach in the simulation environment showed the effectiveness of the developed dynamic model of the manipulator and the platform. The effect of welding parameters i.e. voltage, current, feed rate, depth of penetration, volume of the weld is not considered in both the cases because this study is limited to the dynamic coupling between the manipulator and platform.

This paper is organized as follows. In first section, detailed discussion about the omni-directional platform under consideration is presented. Then in the next session, kinematic analysis of manipulator and omni-directional platform is given. Further, dynamic modeling of manipulator and omni-directional platform

along with dynamic interaction between them is discussed. Then two case studies have been considered for two different welding paths to know the effect of dynamic interaction and lastly the observations made from it are presented in result section. The paper ends with conclusions and references used for this work.

III. OMNI-DIRECTIONAL MOBILE MANIPULATOR

The mobile manipulator considered for this work consists of a platform having four independently driven omni-directional wheels of mecanum type and a 5-dof manipulator with a welding electrode as an end-effector as shown in Figure 1. For this study, a SCORBOT-ER IV manipulator is used. This robotic system is available with the authors.

Figure 1. Omni-directional mobile manipulator considered for the study

3.1 Kinematics of SCORBOT-ER IV Manipulator

SCORBOT-ER IV is a 5-dof vertical articulated robot, with five revolute joints. It has a stationary base, shoulder, elbow, tool pitch and tool roll. The line diagram and detailed parts and joints of SCORBOT ER-IV robot are as per given in the Figure 2 and Figure 3.

Figure 2. Robot Arm Links

Figure 3. Robot Arm Joints

In order to present the structure of the manipulator, first of all the Denavit and Hartenberg (D-H) parameters were determined [12]. D-H parameters put forwards to a matrix method to describe the kinematic relationship between the contiguous links by assigning coordinate system to each link at the joint of the robot as shown in Figure 4. The relative translation and rotation between link coordinate systems $i-1$ and i can be described by a homogenous transformation matrix [13, 14], is a function of four kinematic parameters α_i (Link twist), a_i (Link length), d_i (Joint distance), and θ_i (Joint angle) as shown in Table I. These D-H parameters are directly measured from the actual SCORBOT ER-IV.

TABLE I. D-H parameters for SCORBOT-ER IV manipulator

Joint (i)	a_i	α_i	d_i	θ_i
1	a_1	$\pi/2$	d_1	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3
4	0	$\pi/2$	0	θ_4
5	0	0	d_5	θ_5

Figure 4. Frame assignment of SCORBOT-ER IV manipulator

The position and orientation of the end-effector frame relative to the base frame was found as per equation (1) by considering the n consecutive link transformation matrices relating frames fixed to adjacent links [15]

$$T_n^0 = T_1^0 T_2^1 \dots T_n^{n-1} \quad (1)$$

The composite transformation matrix between two consecutive links was obtained by using equation (2).

$$T_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\theta_i \sin\alpha_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & -\cos\theta_i \sin\alpha_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Equation (2) and D-H parameters from Table I was utilized to derive the forward kinematics expressions. These expressions represent the relationship between joint angles of the robot and corresponding Cartesian coordinates of end-effector of the robot [16]. The expression thus obtained in terms of x , y , and z coordinates of the end-effector for the SCORBOT-ER IV are as given in Equation (3), (4) and (5) respectively.

$$\begin{aligned} x = & a_1 \cos(\theta_1) + a_2 \cos(\theta_1) \cos(\theta_2) + d_5 \cos(\theta_1) \\ & \cos(\theta_2) \cos(\theta_4) \sin(\theta_3) + d_5 \cos(\theta_1) \cos(\theta_3) \cos(\theta_4) \\ & \sin(\theta_2) + d_5 \cos(\theta_1) \cos(\theta_2) \cos(\theta_3) \sin(\theta_4) - d_5 \\ & \cos(\theta_1) \sin(\theta_2) \sin(\theta_3) \sin(\theta_4) + a_3 \cos(\theta_3) \cos(\theta_1) \\ & \cos(\theta_2) - a_3 \sin(\theta_3) \sin(\theta_2) \cos(\theta_1); \end{aligned} \quad (3)$$

$$\begin{aligned} y = & a_1 \sin(\theta_1) + d_5 \cos(\theta_4) \cos(\theta_2) \sin(\theta_1) \sin(\theta_3) + d_5 \\ & \cos(\theta_4) \cos(\theta_3) \sin(\theta_1) \sin(\theta_2) + d_5 \sin(\theta_4) \cos(\theta_2) \\ & \cos(\theta_3) \sin(\theta_1) - d_5 \sin(\theta_4) \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) + \\ & a_2 \cos(\theta_2) \sin(\theta_1) + a_3 \cos(\theta_3) \cos(\theta_2) \sin(\theta_1) - a_3 \\ & \sin(\theta_3) \sin(\theta_1) \sin(\theta_2); \end{aligned} \quad (4)$$

$$\begin{aligned} z = & a_2 \sin(\theta_2) + d_1 + a_3 \sin(\theta_3) \cos(\theta_2) + a_3 \cos(\theta_3) \\ & \sin(\theta_2) - d_5 \cos(\theta_4) \cos(\theta_2) \cos(\theta_3) + d_5 \cos(\theta_4) \\ & \sin(\theta_2) \sin(\theta_3) + d_5 \sin(\theta_4) \cos(\theta_2) \sin(\theta_3) + d_5 \\ & \sin(\theta_4) \cos(\theta_3) \sin(\theta_2); \end{aligned} \quad (5)$$

Along with the kinematic analysis of manipulator, the kinematic analysis of omni-directional platform with mecanum wheels is as well necessary to understand the behavior of platform under kinematic constraints. The next section contains discussion about this analysis.

3.2 Kinematic Analysis of Omni-Directional Platform with Mecanum Wheels

Mecanum wheel is type of the omni-directional wheel invented in 1973 by Bengt Ilon, an engineer with

the Swedish company Mecanum AB. The wheel itself consists of a hub '1' carrying a number of free moving rollers '2' angled at 45° about the hub's circumference as shown in Figure 5. The angle between rollers axis and central wheel axis could have any value but in the case of conventional Swedish wheel, it is 45° . The angled peripheral rollers translate a portion of the force in the rotational direction of the wheel to a force normal to the wheel direction. Depending on each individual wheel direction and speed, the resulting combination of all these forces produces a total force vector in any desired direction. This aspect allows the platform to move freely in the direction of resulting force vector, without changing of the wheels themselves [10].

Figure 5. Components of mecanum wheel

The kinematic analysis of such omni-directional platform having mecanum wheels is about the determination of the relation between linear, angular velocity of the platform and angular velocity of the wheels with respect to the world coordinate frame. As shown in Figure 6, the X_w - Y_w - Z_w is considered as the world coordinate system.

Figure 6. Platform with mecanum wheels

This relationship is given by equation (6). The detailed discussion about it is not presented here. However, interested readers can refer [17] for further clarifications

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \omega \end{bmatrix} = \frac{R}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & 1 \\ l_a + l_b & l_a + l_b & l_a + l_b & l_a + l_b \end{bmatrix} \begin{bmatrix} \dot{\beta}_1 \\ \dot{\beta}_2 \\ \dot{\beta}_3 \\ \dot{\beta}_4 \end{bmatrix} \quad (6)$$

where, $\dot{x}$, $\dot{y}$ are linear velocity and ω is the angular velocity of the platform; R is the wheel radius; $\dot{\beta}_1$, $\dot{\beta}_2$, $\dot{\beta}_3$, $\dot{\beta}_4$ are the angular velocity of wheel 1, 2, 3 and 4 respectively; l_b is the distance between driving wheel and vertical axis of symmetry, l_a is the distance between driving wheel and horizontal axis of symmetry.

Further, the motion of the platform equipped with mecanum wheels depends on the individual direction of rotation of the wheel. The various possible motion of platform is described in the Table II. The (+) sign indicates rotation of wheel in forward direction whereas (-) sign indicates in backward direction.

TABLE II. Platform motion according to the direction of motion of wheels

Wheel No.				Wheel Direction	Direction of motion of platform
1	2	3	4		
+	+	+	+	All wheels rotate in forward direction	Forward
-	-	-	-	All wheels rotate in backward direction	Backward
-	+	-	+	Wheels 2 and 4 rotate in forward direction whereas 1 and 3 in backward direction	Right Slide
+	-	+	-	Wheels 1 and 3 rotate in forward direction whereas 2 and 4 in backward direction	Left slide
+	-	+	-	Wheels 1 and 4 rotate in forward direction whereas 2 and 3 in backward direction	Clockwise rotation

-	+	+	-	Wheels 2 and 3 rotate in forward direction whereas 1 and 4 in backward direction	Anticlockwise rotation
0	+	0	+	Wheels 1 and 3 are stationery and wheels 2 and 4 rotate in forward direction	Forward-Right
+	0	+	0	Wheels 2 and 4 are stationery and wheels 1 and 3 rotate in forward direction	Forward-Left

After performing kinematic analysis of the manipulator and platform, the dynamic analysis of both was done. The mathematical model for representation of dynamic interaction between platform and manipulator while performing welding task was developed using these dynamic relationships. The following section covers the detailed discussion about methodology and the developed model.

3.3 Dynamic Modeling of SCORBOT-ER IV Manipulator

The dynamic model of a manipulator is useful for computation of torque/ forces required for execution of a typical work cycle, which is vital information for the design of links, joints, drives, and actuators. The dynamic behavior of the manipulator provides relationship between joint actuators torques and motion of the links for simulation and design of control algorithms. The manipulator control maintains the dynamic response of the manipulator to obtain the desired performance, which directly depends on the accuracy of the dynamic model and efficiency of control algorithm. Various researchers have discovered different approaches, where in general there are two methods available i.e. Euler-Lagrange formulation and the Newton-Euler formulation. In the standard Euler-Lagrange formulation, the manipulator is treated as a whole, and the system is analyzed based on its kinetic and potential energy. The Newton-Euler formulation is quite different because each link of the manipulator is treated in turn.

3.3.1. Lagrange-Euler (LE) Formulation of SCORBOT-ER IV

The Lagrange-Euler Formulation is a systematic procedure for obtaining the dynamic model of an n -dof manipulator. The 5-dof SCORBOT-ER IV is an open kinematic chain serial link manipulator has 5 joint position or displacement variables i.e.

The LE formulation establishes the relation between the joint position, velocities, accelerations and the torque applied to the manipulator. The dynamic model based on LE formulation is obtained from the Lagrangian, as a set of equations,

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_{r,i}} \right) - \frac{\partial L}{\partial q_{r,i}} = \tau_{ri} \quad (7)$$

for $i = 1, 2, \dots, n$

$$L = K - P$$

Where, L is Lagrange function or *Lagrangian*, K is the kinetic energy and P is the potential energy of the system, torque developed at manipulator's link.

By substituting L and carrying out the differentiation, the generalized torque applied to link i of n -dof manipulator is obtained by following equation of motion [18].

$$\tau_{ri} = M_r(q) \ddot{q}_r + C_r(q, \dot{q}) + G_r(q) \quad (8)$$

Where, M_r is the inertia matrix of the manipulator, C_r is the Coriolis and centrifugal term of manipulator, G_r is the gravity loading force at joint due to link,

$\dot{q}_r$ is angular velocity vector for the manipulator, $\ddot{q}_r$ is angular acceleration vector for the manipulator, and I_p is the mass moment of inertia matrix for each link.

$$M_r = \sum_{p=\max(i,j)}^n Tr[d_{pj} I_p d_{pi}^T] \quad (9)$$

$$C_r = \sum_{p=\max(i,j,k)}^n Tr \left[\frac{\partial(d_{pk})}{\partial q_p} I_p d_{pi}^T \right] \quad (10)$$

$$G_r = - \sum_{p=i}^n m_p g d_{pi} \bar{r}_p^p \quad (11)$$

The kinematic and dynamic parameters of SCORBOT ER-IV were substituted in the above generalized equations to compute the torque for joint1, joint2, joint 3, joint 4 and joint 5 from equation (8).

After obtaining dynamic relationships for manipulator, dynamic analysis of omni-directional platform was done.

3.4 Dynamic Modeling of Omni-Directional Platform with Mecanum Wheels

Figure 7 shows omni-directional platform with manipulator mounted on it. Platform equipped with four mecanum wheels is driven independently by DC motor. Complete system was defined by four principal coordinate frames i.e. world coordinate system (X_w - Y_w - Z_w), platform frame (X - Y - Z) located at mass center, manipulator base frame (X_o - Y_o - Z_o) and end-effector frame (X_e - Y_e - Z_e).

Figure 7. Mobile manipulator with coordinate frames

The platform shown in Figure 7 has two points i.e. P_o and P_c . The point P_o is intersection of axis of symmetry (mass center of platform) and P_c is located at distance d from P_o along the vertical axis. These points indicate two different locations of the manipulator on the platform during the welding operation. This is required to study the effect of positioning the manipulator at two different locations on platform by computing the variation in torque developed at links of the manipulator as well as at the wheels of the platform [19, 20].

The omni-directional wheels impose four constraints on the platform due to their architecture and working principle. These constraints indicate that the rotational velocity of four wheels is directly related to linear velocity of platform. These constraints are as given in equation (12), (13), (14) and (15).

$$\dot{x}\cos\emptyset + \dot{y}\sin\emptyset + (la)\dot{\emptyset} = R\dot{\beta}_1 \quad (12)$$

$$\dot{x}\cos\emptyset + \dot{y}\sin\emptyset + (la)\dot{\emptyset} = R\dot{\beta}_4 \quad (13)$$

$$\dot{x}\cos\emptyset + \dot{y}\sin\emptyset + (la)\dot{\emptyset} = R\dot{\beta}_2 \quad (14)$$

$$\dot{x}\cos\emptyset + \dot{y}\sin\emptyset + (la)\dot{\emptyset} = R\dot{\beta}_3 \quad (15)$$

Where, (x, y, z) is the coordinates of point P_o in the inertial frame, $\emptyset$ is the heading angle of the mobile robot measured from X_w -axis of world coordinate system.

Generalized coordinate of the mobile platform are given as $q_p = [x \ y \ \beta_1 \ \beta_2 \ \beta_3 \ \beta_4]^T$.

The above four constraints can be written as follows,

$$A(q)\dot{q} = 0$$

Where

$$A(q) = \begin{bmatrix} \cos\emptyset & \sin\emptyset & -\frac{R}{2} & 0 & -\frac{R}{2} & 0 \\ \cos\emptyset & \sin\emptyset & 0 & -\frac{R}{2} & 0 & -\frac{R}{2} \\ \cos\emptyset & \sin\emptyset & 0 & -\frac{R}{2} & 0 & -\frac{R}{2} \\ \cos\emptyset & \sin\emptyset & -\frac{R}{2} & 0 & -\frac{R}{2} & 0 \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \end{bmatrix} = \quad (16)$$

3.4.1 Kinetic Energy of Omni-Directional Platform with Mecanum Wheels

The kinetic energy of the platform can be computed by using following equation

$$\begin{aligned} K.E. = & \frac{1}{2} M(\dot{x}^2 + \dot{y}^2) \\ & + m_c \frac{Rd}{2(la)} (\dot{\beta}_1 - \dot{\beta}_3)(\dot{y} \cos\emptyset \\ & - \dot{x} \sin\emptyset) \\ & + m_c \frac{Rd}{2(la)} (\dot{\beta}_4 - \dot{\beta}_2)(\dot{y} \cos\emptyset \\ & - \dot{x} \sin\emptyset) + \frac{1}{2} I_w (\dot{\beta}_1^2 + \dot{\beta}_3^2) \\ & + \frac{1}{2} I_w (\dot{\beta}_4^2 + \dot{\beta}_2^2) \\ & + \frac{1}{2} I C^2 (\dot{\beta}_1 - \dot{\beta}_3)^2 \\ & + \frac{1}{2} I C^2 (\dot{\beta}_4 - \dot{\beta}_2)^2 \end{aligned} \quad (17)$$

where, $I = I_c + 4m_w(la)^2 + 4I_m$ and $C = R/2 * l_a$.

M is the mass of platform ($M = m_c + 4m_w$) with driving wheels and motor including payload capacity; m_c is the mass of platform without driving wheel and motor; m_w is the mass of each driving wheel with motor; I_c is the moment of inertia of platform without driving wheels and motor about vertical axis through P_o ; I_w is the moment of inertia of each driving wheels and motor about wheel axis; and I_m is the moment of inertia of each driving wheels and motor about wheel diameter.

3.4.2 Lagrange Equation of Motion for Omni-Directional Platform with Mecanum Wheels

Lagrange formulation is used to established equation of motion for the mobile platform as follows:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_{p,i}} \right) - \frac{\partial L}{\partial q_{p,i}} = Q_i - a_{1i}\lambda_1 - a_{2i}\lambda_2 - a_{3i}\lambda_3 - a_{4i}\lambda_4 \quad (18)$$

Where, Q_i is the generalized force (torque at each wheel), $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are the Lagrange multipliers. After substituting the K. E. and carrying out the differentiation, the equation (18) can be arranged as:

$$M_p(q)\ddot{q}_p + C_p(q, \dot{q}) = E(q)\tau_w - A^T(q)\lambda \quad (19)$$

$$M_p = \begin{bmatrix} M & 0 & -2m_c C d \sin\emptyset & 0 & 2m_c C d \sin\emptyset & 0 \\ 0 & M & 0 & -2m_c C d \sin\emptyset & 0 & 2m_c C d \sin\emptyset \\ -m_c C d \sin\emptyset & m_c C d \cos\emptyset & I_w + I C^2 & 0 & 0 & 0 \\ m_c C d \sin\emptyset & -m_c C d \cos\emptyset & 0 & I_w + I C^2 & 0 & -I C^2 \\ m_c C d \sin\emptyset & -m_c C d \cos\emptyset & I C^2 & 0 & I_w - I C^2 & 0 \\ -m_c C d \sin\emptyset & m_c C d \cos\emptyset & 0 & -I C^2 & 0 & I_w + I C^2 \end{bmatrix} \quad (20)$$

$$C_p(q, \dot{q}) = \begin{bmatrix} -2m_c d \dot{\emptyset}_1^2 \cos\emptyset \\ -2m_c d \dot{\emptyset}_1^2 \sin\emptyset \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (21)$$

Where

$$\tau_w = [\tau_{w1} \ \tau_{w2} \ \tau_{w3} \ \tau_{w4}]^T$$

is the torque vector where $\tau_{w1}, \tau_{w2}, \tau_{w3}, \tau_{w4}$ are the torque

developed at wheel 1, 2, 3, and 4 respectively, M_p is the inertia matrix of the platform, C_p represents coriolis and centrifugal term of the platform, E is the constant matrix and

$$\ddot{q}_p = [\ddot{x} \ \ddot{y} \ \ddot{\beta}_1 \ \ddot{\beta}_2 \ \ddot{\beta}_3 \ \ddot{\beta}_4]^T$$

is linear and angular velocity acceleration vector for platform.

3.5 Dynamic Interaction between Mobile Platform and the Manipulator

As the manipulator is mounted on mobile platform, it is necessary to study the dynamic interaction between the mobile platform and the manipulator to carry out definite. The motion equation of the manipulator subjected to the platform motion can be written as per following equation.

$$M_r(q_r)\ddot{q}_r + C_r(q_r \ \dot{q}_r) + C_{r/p}(q_r \ \dot{q}_r \ \dot{q}_p) = \tau_{r/p} - R_r(q_r \ \dot{q}_p)\ddot{q}_p \quad (22)$$

Where, q_r denote the n -dimensional coordinate vector of the manipulator, $C_{r/p}$ denotes coriolis and centrifugal term caused by the angular motion of the mobile platform, $\tau_{r/p}$ is the input torque developed on the manipulator's link by considering the effect of platform motion, R_r is the inertia matrix which represents the effect of the platform dynamics on the manipulator, $\dot{q}_p = [\dot{x} \ \dot{y} \ \dot{\beta}_1 \ \dot{\beta}_2 \ \dot{\beta}_3 \ \dot{\beta}_4]^T$ is linear and angular velocity vector for platform.

$$\begin{aligned} C_{r/p}^i &= 2 \sum_{j=1}^m \sum_{k=1}^n \sum_{h=\max(i,k)}^n \text{trace} \left[\frac{\partial T_h}{\partial q_{r,i}} I_h \frac{\partial T_h^T}{\partial q_{p,j}} \frac{\partial T_h^T}{\partial q_{r,k}} \right] \dot{q}_{p,j} \cdot \dot{q}_{r,k} \\ &+ \sum_{j=1}^m \sum_{k=1}^n \sum_{h=i}^n \text{trace} \left[\frac{\partial T_h}{\partial q_{r,i}} I_h \frac{\partial T_h^T}{\partial q_{p,j}} \frac{\partial T_h^T}{\partial q_{p,k}} \right] \dot{q}_{p,j} \cdot \dot{q}_{p,k} \end{aligned} \quad (23)$$

$$R_r^{(ij)} = \sum_{k=i}^n \text{trace} \left[\frac{\partial T_k}{\partial q_{r,i}} I_k \frac{\partial T_k^T}{\partial q_{p,j}} \right] \quad (24)$$

$$1 \leq i \leq n$$

The first term in the R.H.S. of equation (23) characterizes coriolis effect on link i of manipulator due to the coupling of velocity of link k of the manipulator and variable $q_{p,j}$ of platform where $1 \leq j \leq m$ and $1 \leq k$

$\leq n$. Similarly, the second term represents the totality of centrifugal forces exerted on link i by $\dot{q}_{p,j}$ of platform if $j=k$, and coriolis forces exerted on link i due to the velocity coupling of two platform coordinates, i.e. $q_{p,j}$ and $q_{p,k}$ where $j \neq k$.

3.5.2 Dynamics of the Platform subject to Motion of Manipulator

The motion equation of the platform can be given in the following form,

$$\begin{aligned} M_p(q_p)\ddot{q}_p + C_p(q_p \ \dot{q}_p) + C_p^T(q_r \ \dot{q}_p \ \dot{q}_p) \\ = E\tau_{p/r} - A^T\lambda - M_{p/r}(q_r \ \dot{q}_p)\ddot{q}_p - R_p(q_r \ \dot{q}_p)\ddot{q}_r \end{aligned} \quad (25)$$

where, M_p is the inertia matrix of the platform and C_p represents coriolis and centrifugal term of the platform from equation (19), $M_{p/r}$ and $C_{p/r}$ represents the inertial term, coriolis and centrifugal terms respectively due to the presence of the manipulator, $\tau_{p/r}$ is the input torque to the platform, λ denotes the vector of Lagrange multipliers corresponding to the kinematic constraints, and R_p represents the inertia matrix which reflects the dynamic effect of the arm motion on the platform[20,21].

$$M_{p/r}^{(ij)} = \sum_{k=1}^n \text{trace} \left[\frac{\partial T_k}{\partial q_{p,i}} I_k \frac{\partial T_k^T}{\partial q_{p,j}} \right] \quad (26)$$

$$\begin{aligned} C_{p/r}^i &= 2 \sum_{j=1}^n \sum_{k=1}^m \sum_{h=j}^n \text{trace} \left[\frac{\partial T_h}{\partial q_{p,i}} I_h \frac{\partial T_h^T}{\partial q_{r,j}} \frac{\partial T_h^T}{\partial q_{p,k}} \right] \dot{q}_{r,j} \cdot \dot{q}_{p,k} \\ &+ \sum_{j=1}^n \sum_{k=1}^n \sum_{h=\max(j,k)}^n \text{trace} \left[\frac{\partial T_h}{\partial q_{p,i}} I_h \frac{\partial T_h^T}{\partial q_{r,j}} \frac{\partial T_h^T}{\partial q_{r,k}} \right] \dot{q}_{r,j} \cdot \dot{q}_{r,k} \end{aligned} \quad (27)$$

$$R_p^{(ij)} = \sum_{k=j}^n \text{trace} \left[\frac{\partial T_k}{\partial q_{p,i}} I_k \frac{\partial T_k^T}{\partial q_{r,j}} \right] \quad (28)$$

$$1 \leq i \leq m \quad 1 \leq j \leq n$$

The first term in the R.H.S. of equation (27) characterizes coriolis effect on platform coordinate due to the coupling of velocities of link j of manipulator and platform coordinate where $1 \leq k \leq m$. The second term represents the sum of centrifugal forces exerted on platform coordinate by link j of manipulator if $j=k$, and coriolis forces exerted on due to the velocity coupling of two different links of manipulator.

To study the dynamic interaction between the manipulator and the platform, the overall transformation matrix T_h (or T_k) required to determine the coriolis, centrifugal and inertia terms in equations (22) and (25) can be expressed as follows,

$$T_h = T_x \cdot T_n^0 \cdot T_y \quad (29)$$

In above equation, transformation matrix T_x expresses the kinematic relation between the manipulator and the platform with world coordinate frame. It is obtained by transforming the base frame of the manipulator to the world coordinate frame. Transformation matrix T_y expressed the kinematic relation of platform wheels with base of the manipulator which is obtained by transforming the frames assigned to the wheels as shown in Figure 8 and 9 to the base frame of the manipulator. Similarly T_n^0 defines the position and orientation of the end-effector frame relative to the base frame of the manipulator [23].

Figure 8. Mobile platform with frame assign to wheels for $P_o = P_c$

Figure 9. Mobile platform with frame assign to wheels for P_c

To obtain T_h , two different position of robot manipulator on the platform were considered i.e. firstly the manipulator was mounted exactly at the mass center of platform (P_c is coinciding with P_o , $d=0$ meter) as shown in Figure 8 and secondly it is mounted away from platform's mass center along its longitudinal axis (P_c is not coinciding with P_o , $d=0.125$ meter) as shown in Figure 9. The D-H parameters for both the position are mentioned in Table III and IV.

TABLE III. D-H Parameter for platform ($d=0$ meter)

Joint (i)	a_i	α_i	d_i	θ_i
1	$\sqrt{l_a^2 + l_b^2} = l$	$\pi/2$	0	β_1
2	$\sqrt{l_a^2 + l_b^2} = l$	$\pi/2$	0	β_2
3	$\sqrt{l_a^2 + l_b^2} = l$	$\pi/2$	0	β_3
4	$\sqrt{l_a^2 + l_b^2} = l$	$\pi/2$	0	β_4

TABLE IV. D-H Parameter for platform ($d=0.125$ meter)

Joint (i)	a_i	α_i	d_i	θ_i
1	$\sqrt{l_b^2 + (l_a - d)^2} = l_1$	$\pi/2$	0	β_1
2	$\sqrt{l_b^2 + (l_a - d)^2} = l_1$	$\pi/2$	0	β_2
3	$\sqrt{l_b^2 + (l_a + d)^2} = l_2$	$\pi/2$	0	β_3
4	$\sqrt{l_b^2 + (l_a + d)^2} = l_2$	$\pi/2$	0	β_4

IV. RESULT AND DISCUSSION

To study the dynamic interaction between the manipulator and the platform, the developed mathematical model for dynamic interaction is implemented to understand its effect when Omni-WMR used for welding at two different conditions. Figure 10 explains the complete setup of omni-WMR. The physical parameters of Omni-WMR utilized for these case problems are as per given in the Table V.

Figure 10. Setup of mobile manipulator

Case I: Dynamics of the Manipulator Subject to Platform Motion

TABLE V. Physical parameters for manipulator and platform

Manipulator Parameters				Platform Parameters			
a_1	0.016 meter	m_1 (mass of the base)	3.8 Kg	M	28.1 kg	m_c	22 kg
a_2	0.22 meter	m_2 (mass of the body)	2 Kg	m_w	1.5 kg	l_a	0.5218 meter
a_3	0.22 meter	m_3 (mass of the upper arm)	2 Kg	l_b	0.25 meter	l	0.552 meter
a_4	0 meter	m_4 (mass of forearm)	2 Kg	l_1	0.4357	l_2	0.613 meter
a_5	0 meter	m_5 (mass of end-effector)	1 Kg	I_c	2.843 kg- meter ²	I_w	0.0847 kg- meter ²
d_1	0.335 meter	d_5	0.138 meter	I_m	0.0524 kg- meter ²	R	0.055 meter

. Joint 5 was responsible for travel of the torch. The travel angle of torch is an angle relative to the torch in perpendicular position. This angle must be within 5 to 15 degrees to the normal. Any travel angle above this value would lead to spatter and instability in welding operation. Because of this constraint, the joint 5 i.e. θ_5 was kept constant. The cubic polynomial given in equation (30) was used to describe the angular motion $[\theta(t)]$ whereas first derivative of this polynomial given

In order to study effect of platform motion on dynamics of manipulator, two cases were considered. In first case, welding along straight path of 440 mm was done which doesn't require any movement of platform as it is within work volume of robot. In second case, a weldment of 860 mm was considered which demand for motion platform to complete the welding.

Problem 1: SCORBOT ER-IV manipulator was utilized to weld two plates of 440 mm length. This length of weldment along straight line can be comfortably achieved by the manipulator in its workspace. Hence, the omni-directional platform was fixed at one location while SCORBOT was performing welding operation. A cubic spline was fitted between start and end position of the weldment to describe the motion of links with respect to time. The value of θ_1 i.e. base rotation was considered to be constant. The start position on weldment was marked as 's' whereas end-position (goal) was marked as 'g'. In order to follow this straight line path, θ_2 was varied from 135° to 9° in the step of -14° , θ_3 was varied from 39° to 180° in steps of $+14^\circ$, θ_4 was varied from 113° to 130° in the step of $+1.7$ degree.

in equation (31) for angular velocity $[\dot{\theta}(t)]$ and the second derivative given in equation (32) forms expression for angular acceleration $[\ddot{\theta}(t)]$ of manipulator link and end-effector during welding.

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 \quad (30)$$

$$\dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2 \quad (31)$$

$$\ddot{\theta}(t) = 2c_2 + 6c_3 t \quad (32)$$

In order to find velocity and acceleration values, the estimation of coefficients of the splines was done by considering appropriate constraints. The time required to cover this 440 mm weldment was considered 72 second.

Time-history expressions for Joint 2 (θ_2)

The parameters for joint 2 are: $\theta_2^s = 135^\circ$ (2.356 rad), $\theta_2^g = 9^\circ$ (0.157 rad), $\dot{\theta}_2^s = \dot{\theta}_2^g = 0$ rad/sec, and $t_g = 72$ sec

To obtain the unknown coefficients of equations (33), (34) and (35), it is necessary to satisfy the constraints as, $\theta(0) = \theta^s$, $\theta(t_g) = \theta^g$, $\dot{\theta}(0) = 0$ and $\dot{\theta}(t_g) = 0$. After substituting these constraints, the coefficients can be estimated as;

$$\begin{aligned} c_0 &= \theta_2^s = 2.356, c_1 = 0 \\ c_2 &= \frac{3}{t_g^2}(\theta_2^g - \theta_2^s) = \frac{3}{72^2}(0.157 - 2.356) = -0.00127 \\ c_3 &= -\frac{2}{t_g^3}(\theta_2^g - \theta_2^s) = -\frac{2}{72^3}(90.157 - 2.356) \\ &= 0.00001178 \end{aligned}$$

The expression for joint displacement can be written as,

$$\theta(t) = 2.356 - 0.00127 * t^2 + 0.00001178 * t^3$$

The first derivative of this expression gave a parabolic velocity profile as,

$$\begin{aligned} \dot{\theta}(t) &= 2 * (-0.00127)t + 3 * (0.00001178)t^2 \\ &= -0.00254 * t + 0.00003534 * t^2 \end{aligned}$$

and the second derivative gave a linear acceleration profile as,

$$\begin{aligned} \ddot{\theta}(t) &= 2 * (-0.00127) + 6 * (0.00001178)t \\ &= -0.00254 + 0.00007068 * t \end{aligned}$$

Time-history expressions for Joint 3 (θ_3)

By adopting procedure mentioned for computation of Joint 2, the expressions for displacement, velocity and acceleration of joint 3 were determined by using the parameters.

$$\begin{aligned} \theta_3^s &= 39^\circ (0.6807 \text{ rad}), \theta_3^g = 180^\circ (3.14 \text{ rad}), \dot{\theta}_3^s = \\ \dot{\theta}_3^g &= 0 \text{ rad/sec, and } t_g = 72 \text{ sec,} \end{aligned}$$

$$\theta(t) = 0.6807 + 0.001423 * t^2 - 1.3177e - 5 * t^3$$

$$\begin{aligned} \dot{\theta}(t) &= 2 * (0.001423) * t + 3 * (-1.3177e - 5)t^2 \\ &= 0.002846 * t - 3.9533e - 5 * t^2 \end{aligned}$$

$$\begin{aligned} \ddot{\theta}(t) &= 2 * (0.001423) + 6 * (-1.3177e - 5)t \\ &= 0.002846 - 0.000079062 * t \end{aligned}$$

Time-history expressions for Joint 4 (θ_4)

On similar lines with joint 2 and joint 3, the expressions for joint displacement, velocity and acceleration are expressed as follows using parameters,

$$\begin{aligned} \theta_4^s &= 113^\circ (1.972 \text{ rad}), \theta_4^g = 130^\circ (2.268 \text{ rad}), \dot{\theta}_4^s = \\ \dot{\theta}_4^g &= 0 \text{ rad/sec, and } t_g = 72 \text{ sec} \end{aligned}$$

$$\theta(t) = 1.972 + 0.000171 * t^2 - 1.586e - 6 * t^3$$

$$\begin{aligned} \dot{\theta}(t) &= 2 * (0.000171) * t + 3 * (-1.586e - 6)t^2 \\ &= 0.000342 * t - 0.000004758 * t^2 \end{aligned}$$

$$\begin{aligned} \ddot{\theta}(t) &= 2 * (0.000171) + 6 * (-1.586e - 6)t \\ &= 0.000342 - 0.000009516 * t \end{aligned}$$

The time history thus obtained for joint 2, 3 and 4 was plotted and are given in Figure 11, 12 and 13 respectively.

Figure 11. Time history plot of position, velocity and acceleration for joint 2

Figure 12. Time history plot of position, velocity and acceleration for joint 3

Figure 13. Time history plot of position, velocity and acceleration for joint 4

The time history thus obtained was further utilized to determine torque requirements at various links. The dynamics equations derived and presented in section 3.3 of this article were used. After substituting values of various variables, the torque required at link1 (joining Joint 1 and 2), link 2 (joining Joint 2 and 3) and link 3 (joining Joint 4 and 5) was estimated and same is plotted in the Figure 14.

Figure 14. Torque variation at the links of manipulator for 440 mm weldment

Problem 2: In previous case, weldment of 440 mm was considered. It was within the work volume of SCORBOT ER-IV. Hence, there was no need to move the platform. However, in order to exploit the flexibility of OMNI-WMR, welding along straight line with weldment equal to 860 mm length was considered.

Since this weldment length is beyond the reach of the SCORBOT ER-IV, omni-directional platform was freed to move whenever required. In order to keep analysis simple, the configurations of joints was kept same as that of Problem 1. To account for the extra 420 mm weldment length, the wheels of the mobile platform were moved with angular displacement of platform wheels is 542° during the welding operation. This value of angular displacement was computed by considering length of path to be moved by platform smaller than the length of weldment. Trials revealed that the continuous movement of platform for 50 second was enough to carry out welding along 860 mm length within 72 second by the manipulator. In order to obtain time history and torque required at links of robot, analysis similar to Problem 1 was performed. The torque required at various links was obtained using equation (25) given in section 3.5.1. These equations consider the effect of platform motion on the torque requirements of the links of manipulator mounted on it. The torque at various links thus obtained is presented in Figure 15

Figure 15. Torque variation at the links of manipulator for 860 mm weldment

From the Figure 14 and 15, it is obvious that even though the joint displacement, velocity, acceleration and trajectory time were kept constant, the torque required for welding of 860 mm weldment was more as compared to 440 mm weldment. It was observed that, 15-30% more torque was required for welding 860 mm weldment at various links of the manipulator. From this observation, it can be inferred that the movement of platform shows a considerable effect on the torque requirement at the links of manipulator. It justifies the dynamic interaction between platform and the manipulator. The torque required at platform wheel during the welding is 0.326 N-m for wheel 1, 0.324 N-m for wheel 2, 0.329 N-m for wheel 3 and 0.324 N-m for wheel 4. It is obvious that the torque required is constant

at all the four wheels as the platform is moving with uniform velocity.

This study demonstrated the first part of dynamic interaction i.e. variation in torque of the manipulator when subjected to platform motion. In order to understand the second aspect of dynamic interaction i.e. dynamics of the platform subject to motion of manipulator and its location, a second case was considered and same is discussed next.

Case II: Dynamics of the Platform Subject to Motion of Manipulator

In order to understand the effect of manipulator motion and its location on platform, another weldment which requires zigzag motion of welding gun was considered. The nature of weldment is shown in Figure 16. The derivations for the dynamic interactions when two different locations were selected for mounting manipulator are presented in equation (25). In order to demonstrate this aspect, two problems were considered. In first problem, manipulator was mounted at center of gravity (C.G.) of the platform and away from C.G. in second.

Figure 16. Zigzag weldment

Figure 17. Path followed by manipulator and platform to achieve weldment shown in Fig. 16

Problem 1: Manipulator mounted at point P_o (Fig. 8)

In this problem, manipulator was mounted at point P_o which is the center of gravity of the platform ($d=0$). OMNI-WMR was moved in such a way that welding torch follows double line path (AB-BC-CD-DE) and platform follows dotted path shown in Figure 17. The total length of end-effector path was considered to be 600 mm and that of platform path 200 mm. Both paths were fitted with separate cubic splines and further

analysis was done same as shown while discussing problem 1 of case 1. The value of θ_1 i.e. base rotation was considered to be constant. The start position on weldment was marked 'A' whereas end-position was marked as 'E'. In order to follow this zigzag path, θ_2 was varied from 130° to 8° , θ_3 was varied from 100° to 132° , and θ_4 was varied from 52° to 43° . As mentioned in case 1, again the joint 5 i.e. θ_5 was kept constant and it must be within 5 to 15 degrees to the normal, otherwise it will lead to spatter and instability in welding operation. After finding out coefficients of cubic spline, expressions for joint displacement, velocity and acceleration were estimated. The total distance travelled by platform during the welding of path is 200 mm in 50 sec. However, during the welding of path i.e. AB-BC-CD-DE, the configuration of wheel rotation alters as per the each segment of path. During the segment, AB and CD all the wheel will rotate in forward direction. For segment BC, wheels 2 and 4 are stationary and wheels 1 and 3 rotate in forward direction and for DE, wheels 1 and 3 are stationary and wheels 2 and 4 rotate in forward direction. Hence piece-wise cubic polynomial trajectory is considered for each segment. As the distance travelled by the platform for each segment is 50 mm in 12.5 second, angular displacement of each wheel is 52.13° . For trajectory time of 12.5 second, the temporal analysis of torque at wheels of platform was done. The velocity profile followed by platform is shown in Figure 21.

Problem 2: Manipulator mounted at point P_c (Figure 9)

Next, the manipulator was shifted from P_o to P_c by 0.125 meter ($d=0.125$ m) as shown in Figure 9. The same path for manipulator as well as platform consider as of Problem 1. For uniformity in analysis, the boundary conditions were same as of previous problem. The temporal analysis of torque requirements at various links was done which is presented in Figure 18, 19 and 20.

Figure 18. Comparison of torque required at link 1 when manipulator mounted at P_o and P_c .

Figure 19. Comparison of torque required at link 2 when manipulator mounted at P_o and P_c

Figure 20. Comparison of torque required at link 3 when manipulator mounted at P_o and P_c

From the Figure 18, 19 and 20, it can be observed that the torque requirement does not change with change in mounting location of the manipulator on the platform.

TABLE VI. Torque developed at wheels of the platform

Path	Torque developed (N-meter)					
	AB and CD		BC		DE	
	$d=0(\text{meter})$	$d=0.125(\text{meter})$	$d=0(\text{meter})$	$d=0.125(\text{meter})$	$d=0(\text{meter})$	$d=0.125(\text{meter})$
Wheel 1	0.7459	0.8202	1.1256	1.2853	0	0
Wheel 2	0.7463	0.8213	0	0	1.1256	1.2853
Wheel 3	0.7465	0.7313	1.0858	1.1768	0	0
Wheel 4	0.7443	0.7301	0	0	1.0858	1.1768

V. CONCLUSIONS

The welding mobile omni-directional robot considered in this work is the unique kind of mobile robot. The kinematic and dynamic analysis of manipulator and omni-directional platform is performed successfully. The issue of dynamic interaction between platform and the manipulator has come up as a crucial issue in the application of such mobile robots for welding. The temporal analysis of torque required at various links of manipulator was done for straight as

This indicates that location of manipulator does not show any effect on torque requirements at any of the three links of robot. However, when torque required at omni-directional wheels was compared for both locations, different scenario can be seen. The values of torque required are given in Table VI for the wheels of the platform. From this table, it is evident that when manipulator is mounted at P_o the torque requirement is less by 8%- 15% as compared when manipulator is mounted at P_c .

Figure 21. Time history plot for angular velocity of platform wheels for Zigzag path

This shows that location of mounting manipulator on platform does have some effect on the torque required.

well as zigzag path. The study of finding the effect of location for mounting the manipulator on platform was also performed. The study revealed that the location of manipulator on platform showed variation in the torque for the platform but there was no effect on the torque requirement for the manipulator. The dynamic interaction between platform and manipulator showed huge effect on torque requirement for manipulator as well as wheels of the platform. The study presented here was done in the simulation environment using

MATLAB. The outcome of results can be efficiently implemented in real time applications such as welding, material handling, and service applications.

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