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ANALYTICAL MODELING OF PLANETARY GEAR AND SENSITIVITY OF NATURAL FREQUENCIES

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Abstract: This work develops an analytical model of planetary gears and uses it to investigate their natural frequencies and vibration modes. The model admits three planar degrees of freedom for each of the sun, ring, carrier and planets. Vibration modes are classified into rotational, translational and planet modes. The natural frequency sensitivities to system parameters are investigated for tuned (cyclically symmetric) planetary gears. Parameters under consideration include support and mesh stiffnesses, component masses, and moments of inertia. Using the well-defined vibration mode properties of tuned planetary gears, the eigen sensitivities are calculated and expressed in simple exact formulae. These formulae connect natural frequency sensitivity with the modal strain or kinetic energy and provide efficient means to determine the sensitivity to all stiffness and inertia parameters by inspection of the modal energy distribution.

1. INTRODUCTION

Planetary gears are widely used in the transmissions of helicopters, automobiles, aircraft engines, heavy machinery and marine vehicles. Figure 1.1 illustrates a single stage planetary gear set consisting of a sun gear, a ring gear, several planets, and a carrier.

Any of the carrier, ring, and sun can be selected as the input or output component, and the power is transmitted through multiple paths of the planet meshes. Planetary gears have substantial advantages over parallel shaft drives, including compactness, large torque-to weight ratio, diminished loads on shafts bearings, and reduced noise and vibration due to the relatively smaller and stiffer components.

Despite planetary gears' advantages, noise and vibration remain major concerns in their applications. In most helicopters, planetary gears are used in the last stage of gear reduction.

This planetary gear is mounted directly to the helicopter cabin, so its vibration is the main source of structure-borne cabin noise, which can exceed 100 dB (Krantz[1]). Consumers often perceive the gear noise as poor quality and mechanical problems.

Vibration reduction in planetary gears provides substantial benefits: reduced noise, improved reliability, more efficient power transfer, and reduced maintenance costs.

However, the noise and vibration issues in current planetary gear design rely mostly on empirical experience and cut-and-try methods, rather than engineering understanding. General design guidelines are needed to minimize planetary gear vibration.

The fundamental task of analytical planetary gear research is to build a dynamic model. For different analysis purposes, there are several modeling choices such as a simple dynamic factor model, compliance tooth model, torsional model, and geared rotor dynamic model (Ozguven and Houser[2]). According to the source-path-receiver relationship between the planetary gear, bearing/mounting, and the cabin, different boundaries can be selected for building the model. This study focuses on the understanding of planetary gear dynamic behavior, so a single stage gearset with discrete elements is the basic model for investigation.

The free vibration analysis calculates critical parameters such as natural frequencies and vibration modes that are essential for almost all dynamic investigations. This project reveals the unique structure of natural frequency spectra and vibration modes due to the cyclic symmetry of planetary gears (Lin and Parker[4]). All the vibration modes are classified into rotational, translational and planet

modes with distinctive properties. The free vibration properties are very useful for further analysis of planetary gear dynamics, including eigen sensitivity to design parameters, natural frequency veering, planet mesh phasing, and parametric instabilities from mesh stiffness variations.

Another key issue is how design parameters affect the natural frequencies and vibration modes. During the design process, model parameters are often altered to evaluate alternative design choices, reduce weight, and tune the system frequencies to avoid resonance. Sensitivity of the natural frequencies and vibration modes to system parameters provides important information for tuning resonances away from operating speeds, minimizing response, and optimizing the structural design. In industrial practice, planetary gear design balances many objectives in selecting component inertias and support/mesh stiffnesses. Load sharing among the planets, avoidance of resonance, and weight reduction are three important examples. The design process needs to balance how changes in key design parameters for these (or other) purposes alter the modal properties and impact dynamic response. This work identifies remarkably compact, closed-form eigen sensitivity relations that can guide these decisions and in this work derive a lumped-parameter model for spur planetary gears. The model is valid for general epicyclic gears with any number of planets and will be the fundamental tool for further research.

Analytically characterize the unique structure of the natural frequency spectra and vibration modes of general planetary gears. Also calculate the natural frequencies and vibration modes of a planetary gear used in U.S. Army Helicopter OH-58 with the help of MATLAB. And to investigate the natural frequency sensitivities to system parameters for tuned (cyclically symmetric) planetary gears. Parameters under consideration include support and mesh stiffnesses, component masses and moments of inertia.

Before 1990, the literature on analytical planetary gear dynamics is scarce. Cunliffe et al.[5] studied the eigenvalue problem for a thirteen degree of freedom system and identified the natural frequencies and vibration modes. Botman[6] investigated the vibration modes for a spur planetary gear with eighteen degrees of freedom. Frater et al.[7] extended Botman's natural frequency analysis by including unequal planet stiffness. More analytical studies were performed after 1990s. Kahraman[8,9,10,11] presented a series of papers on planetary gear dynamics. He derived a nonlinear, time-varying planar dynamic model and subsequently extended it to three-dimensions and examined the influence of planet phasing on dynamic response[8]. Using these models, Kahraman and Blankenship[12] investigated

the load sharing and mesh phasing among planets. Kahraman[13] reduced his model to a purely torsional one to predict natural frequencies and vibration modes. Other studies of planetary gear dynamics include mesh stiffness variation and load-sharing (Kasuba and August)[14], influence of some design parameters on tooth loading (Saada and Velex)[15], and mesh parametric excitation (Velex and Flamand)[16].

Agashe[17] and Parker et al.[18] used a finite element tool to investigate the dynamic response and planet phasing issues in planetary gears. Parker [19] also rigorously proved the effectiveness of using planet phasing schemes to suppress planetary gear vibration.

Lin and Parker [20,21] present a 2D rotational-translational degree of freedom spur gear model and mathematically show the unique modal properties of equally spaced and diametrically opposed planet systems. The sensitivity of natural frequencies and modes to operating speeds and various design parameters are studied by Lin and Parker[22], who also examine natural frequency veering phenomena. Mesh stiffness-induced parametric instability is studied by Lin and Parker[23].

A helical planetary gear model is formulated and the effect of mesh phasing on the dynamics of equally spaced planet systems is investigated by Kahraman[10] and Kahraman and Blankenship[13]. Parker[19] showed the effectiveness of mesh phasing in suppressing certain harmonics of planetary gear vibration modes based on self-equilibration of the dynamic mesh forces at sun-planet and ring-planet meshes.

Ambarisha and Parker[24] extended this work to derive design rules to suppress planet mode resonances. A thorough description of the relative mesh phasing between the sun-planet and ring-planet meshes in a planetary gear system is given by Parker and Lin[25].

In recent years, some researchers have used deformable gear body dynamic models. A unique finite element-contact analysis program is used by Parker et al.[26] to model nonlinear spur gear dynamics. The finite element results compare favorably with experiments. Parker et al.[27] used the same finite element method to examine planetary gear dynamics. Kahraman and Vijayakar[28] studied the effect of ring gear flexibility on the static response of planetary gears using the finite element method. A recent study by Kahraman et al.[29] dynamically analyzes a planetary gear with thin rim using the same finite element method. Yuksel and Kahraman[30] employed this finite element model to study tooth wear and its impact on the dynamic behavior of a planetary gear.

2. MODELING AND EQUATIONS

Lumped-parameter modeling is used for dynamic analysis. All gears are considered as rigid bodies and component supports are modeled by springs. A single mesh model is shown in Figure 1.2 for a pair of spur gears. All bearings/supports are modeled as two perpendicular springs. The gear tooth meshes are represented by springs acting on the line of action with parallel viscous dampers. The transmission error $e(t)$, a prescribed displacement input, is included as indicated. The tooth separation nonlinearity acts as a step function $h(t)$ where $h=1$ when the teeth are in contact and $h=0$ when the teeth lose contact. This model can be extended to planetary gears with multiple meshes.

Figure 1.2- A single mesh model

The analysis deals with planar vibration of single stage planetary gears. Excellent lubrication is assumed and tooth friction forces are neglected. No damping is included, although a viscous damper could easily be added in parallel with the mesh and bearing springs.

Several choices of coordinates may be used in the modeling. The coordinate selection does not change the physical properties of the system, but may greatly affect the analysis difficulty.

The analysis deals with planar vibration of single stage planetary gears. A lumped parameter model for spur planetary gears is shown in Figure 1.3. Each of sun, ring, carrier and N planets are treated as rigid bodies. Each component has three degrees of freedom: two translational and one rotational.

The model is similar to that used by Kahraman [9] with two distinctions: (1) the planet deflections are

described by radial and tangential coordinates, and (2) gyroscopic effects induced by carrier rotation are modeled. The radial and tangential coordinates more naturally describe the vibration modes. Gyroscopic effects in high-speed applications such as aircraft engines may dramatically alter the dynamic behavior from that at lower speeds. The coordinates illustrated in Figure 1.3 are used.

Figure 1.3-Lumped parameter model of planetary gears and system co-ordinates. (b) All translational co-ordinates $x_j, y_j, j=c, r, s$ and $\zeta_n, \eta_n, n=1, \dots, N$ are with respect to the frame $\{i, j, k\}$ rotating at constant carrier speed Ω_c .

The vibration modes can be characterized by modal strain and kinetic energy. The total modal strain energy U and kinetic energy T are related to the natural frequencies and vibration modes by

$$U = \frac{1}{2} \phi_i^T K \phi_i = U_c + U_{cu} + U_r + U_{ru} + U_s + U_{su}$$

$$\sum_1^n (U_n + U_{rn} + U_{sn}) \quad (1)$$

$$T = \frac{1}{2} \omega_i^2 \phi_i^T M \phi_i = T_c + T_{cu} + T_r + T_{ru} + T_s + T_{su} + \sum_1^n (T_n + T_{nu}) \quad (2)$$

Where

(i) $U_h, U_{hu}, h=c, r, s$ are the strain energies in the translational and rotational support springs, respectively, of the carrier, ring and sun.

(ii) $U_n, U_m, U_{sn}, n=1, \dots, N$ are the strain energies in the n -th planet bearing, ring-planet mesh and sun-planet mesh; and

(iii) $T_h, T_{hu}, h=c, s, r$ and T_n, T_{nu} are the modal translational and rotational kinetic energies of the carrier, ring, sun and planets respectively.

During the design process, system parameters are varied to evaluate alternative design choices, avoid resonances, optimize load distribution, and reduce weight. It is important to characterize the effects of parameter variations on the natural frequencies and vibration modes for effective vibration tuning. Simple, closed-form expressions are obtained to calculate eigensensitivities for the tuned system.

The eigensensitivity analysis calculates natural frequency and vibration mode derivatives with respect to stiffnesses, masses, moments of inertia and the carrier rotation speed Ω_c . Gyroscopic effects are important in high-speed applications such as aircraft engines, so in this work the Eigensensitivity to stiffness and inertia design parameters are examined in the absence of gyroscopic effects Ω_c .

The Eigen value problem for the study is in form $(K - \lambda_i M) \varphi_i = 0$ (3)

where $\lambda_i = \omega_i^2$.

The Eigen sensitivity for problems in the form (3) has been thoroughly investigated (Courant and Hilbert [36]; Adelman and Haftka [38]; Friswell [39]) and the necessary results are introduced below. The unique modal properties of planetary gears are then invoked to reduce these general results to simple expressions specific to planetary gears.

4. CALCULATION AND RESULT

From these eigenvalues λ_i natural frequency of vibrations in planetary gear model can found out as

$$\begin{aligned} \lambda_i &= \omega_i^2 & \text{or} \\ \omega_i &= \sqrt{\lambda_i} & \text{in radians per second} \\ \omega_i &= (\sqrt{\lambda_i}) / 2\pi & \text{Hertz} \end{aligned}$$

From here we get the values of Eigen values and can calculate the natural frequencies of vibration. From the values of ω_i 's we can conclude that in planetary gear system there are $3(3 + N)$ natural frequencies which is 21 in our case when $N = 4$ (number of planets). The Eigen solution properties are illustrated using the nominal system parameters listed in Table 1.

Table1- Nominal system parameters of the planetary gear

	Sun	Ring	Carrier	Planet
Mass (kg)	0.4	2.35	5.43	0.66
I_r^2 (kg)	0.39	3.00	6.29	0.61
Base diameter (mm)	77.42	275.03	177.8	100.35
Teeth number	27	99		35
Mesh stiffness (N/m)	$k_{gs} = k_{gr} = k_{gm} = 5 \times 10^8$			
Bearing stiffness (N/m)	$k_{bs} = k_{br} = k_{bc} = k_{bp} = 10^8$			
Torsional stiffness (N/m)	$k_{ts} = 10^9$ $k_{tr} = k_{tc} = k_{tp} = 0$			
Pressure angle (in degree)	$\phi_s = \phi_r = \phi_p = 24.6$			

Applying MATLAB software, For 4 number of planets eigen value Sensitivity and Modal Strain energy due to carrier, ring and sun transverse and rotational support stiffnesses k_c , k_r , k_s , k_{su} , k_{cu} , k_{ru} , k_{su} is shown in figure-1.4

Figure 1.4- Modal Strain Energy due to k_c , k_r , k_s , k_{cu} , k_{ru} and k_{su}

The graph clearly suggests that no strain energy is produced in the planet modes i.e. 7, 8 and 21 modes. We can conclude that planet modes are insensitive to all carrier, ring and sun support stiffnesses. Further no energy is stored in mode no. 12 which is the rigid body mode with $\lambda_i = 0$.

Eigenvalue Sensitivity and Modal Kinetic Energy due to carrier, ring and sun gear mass and moment of inertia T_c , T_r , T_s , T_{cu} , T_{ru} and T_{su} is shown in figure 1.5.

Figure 1.5-Modal Kinetic Energy due to m_c , m_r , m_s , I_c , I_r and I_p

The graph clearly suggests that no kinetic energy is stored in the planet modes i.e. 7, 8 and 21 modes. Thus we can conclude that planet modes are insensitive to all carrier, ring and sun gear masses and moments of inertia.

5. CONCLUSION

This work analytically investigates several key issues in planetary gear dynamics. The main results and their benefits are summarized for each specific topic. A lumped-parameter model is developed for spur planetary gears. Critical factors to gear vibration are arbitrary planet spacing and phasing, mesh stiffness variation, transmission error excitation and gyroscopic effects that are important in high-speed applications. This model is a key building block that can be expanded to couple with the housing and include multiple stages.

From the graph, it is clear that no kinetic energy is stored in the planet modes 7,8 and 21 modes. Thus we can conclude that planet modes are insensitive to all carrier, ring and sun gear masses and moments of inertia, also no strain energy is produced in the planet modes i.e. 7, 8 and 21 modes. We can conclude that planet modes are insensitive to all carrier, ring and sun support stiffnesses.

Further no energy is stored in mode no. 12 which is the rigid body mode with. This study characterizes the natural frequency spectrum and vibration mode properties in planetary gears. Planetary gears possess rich modal structure that is crucial for eigensensitivity, dynamic response, and stability analysis.

The effects of design parameter variations on planetary gear free vibration are analytically and mathematically investigated. The sensitivities of natural frequencies and vibration modes to key parameters are expressed in simple, closed-form formulae that allow one to quickly identify the parameters that most impact the modal properties.

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