Short Communication

The Burr XI Generated Family of Distribution with Illustration to Cancer Patients Data

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ABSTRACT

The Burr system of distributions appeared in (Burr, 1942). On the other hand in (Eugene et al., 2002), they pioneered the beta-generated method of creating probability distributions. Inspired by these developments, we introduce the Burr XI generated method of creating probability distributions. We show a distribution arising from this method is a good fit to real-life data, indicating the new method of creating probability distributions should be applicable in various disciplines. Finally, we ask the reader to investigate some properties and applications of a so-called Burr VI generated family of distributions of two types.

Key Words: *Burr system of distributions; Breast cancer; T-X(W) family of distributions; Beta-G*

1. Introduction and the New Family

The Burr system of distributions (Burr, 1942) arise from the differential equation

where

$$y' = y(1-y)g(x, y)$$

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$$y' = \frac{dy}{dx} = \frac{dF(x)}{dx} = f(x)$$

$$y = F(x)$$

and $g(x, y)$ is a nonnegative function for $0 \leq y \leq 1$ and x is in the range over which the solution is to be used. The Burr XI distribution has CDF, and for example, see Table 1.1 (Momanyi, 2017), given by

$$F(x) = \left(x - \frac{\sin(2\pi x)}{2\pi} \right)^r$$

where $r > 0$, and $0 < x < 1$. By differentiating the CDF above, we see the PDF of the Burr XI distribution is given by

$$f(x) = 2r \sin^2(\pi x) \left(x - \frac{\sin(2\pi x)}{2\pi} \right)^{r-1}$$

Now we introduce the Burr XI generated family of distributions via the following integral

$$\int_0^{G(x;\xi)} 2r \sin^2(\pi t) \left(t - \frac{\sin(2\pi t)}{2\pi} \right)^{r-1} dt$$

where $r > 0$ and $G(x; \xi)$ is the CDF of some baseline distribution. The parameter space of ξ depends on the chosen baseline distribution. Evaluating this integral leads to the following

Proposition 1.1. *The CDF of the Burr XI-G family of distributions is given by*

$$K(x; \xi, r) = \left(G(x; \xi) - \frac{\sin(2\pi G(x; \xi))}{2\pi} \right)^r$$

where $x \in \mathbb{R}$, $r > 0$, and $G(x; \xi)$ is the CDF of some baseline distribution. The parameter space of ξ depends on the chosen baseline distribution.

By differentiating the CDF above, we have the following

Proposition 1.2. *The PDF of the Burr XI-G family of distributions is given by*

$$k(x; \xi, r) = 2rg(x; \xi) \sin^2(\pi G(x; \xi)) \left(G(x; \xi) - \frac{\sin(2\pi G(x; \xi))}{2\pi} \right)^{r-1}$$

where $x \in \mathbb{R}$, $r > 0$, $G(x; \xi)$ and $g(x; \xi)$ are the CDF and PDF, respectively, of some base-line distribution. The parameter space of ξ depends on the chosen baseline distribution

The rest of this paper is organized as follows. In the next section we show a so-called Burr XI-Weibull distribution is a good fit to the breast cancer data (Girish and Jayakumar, 2017). The last section is devoted to the conclusions. As a further recommendation we suggest obtaining some properties and applications of a so-called Burr VI generated family of distributions of two types.

2 Practical Illustration

We assume the baseline distribution is Weibull distributed with the following CDF

$$G(x; a, b) = 1 - e^{-\left(\frac{x}{b}\right)^a}$$

where $x, a, b > 0$. By Proposition 1.1, we have the following

Corollary 2.1. *The CDF of the Burr XI-Weibull distribution is given by*

$$K(x; a, b, r) = \left(-e^{-\left(\frac{x}{b}\right)^a} + \frac{\sin(2\pi e^{-\left(\frac{x}{b}\right)^a})}{2\pi} + 1 \right)^r$$

where $x, a, b, r > 0$

Notation 2.2. We write $V \sim BXIW(a, b, r)$, if V is a Burr XI-Weibull random variable.

Figure 1. The CDF of $BXIW(1.20333, 97.7151, 0.329319)$ fitted to the empirical distribution of the breast cancer data (Girish and Jayakumar, 2017).

Remark 2.3. The PDF of the Burr XI-Weibull distribution can be obtained by differentiating the CDF

Figure 2. The PDF of $BXIW(1.20333, 97.7151, 0.329319)$ fitted to the histogram of the breast cancer data (Girish and Jayakumar, 2017).

3 Concluding Remarks and Further Recommendations

The present paper has introduced a family of distributions arising from the Burr system of distributions. In particular, a so-called Burr XI-G family of distributions have been introduced, and a member of this class of distributions is shown to be a good fit to real life data in the health sciences. The Burr VI distribution has CDF

$$Z(x; k, c, r) = (ke^{-c \sinh(x)} + 1)^{-r}$$

where $k, c, r > 0$ and $x \in \mathbb{R}$. Inspired by the $T - X(W)$ framework (Alzaatreh and Lee, 2013b), we ask the reader to investigate some properties and applications of a so-called Burr VI generated family of distributions of two types. We leave the reader with the following integral representations for their CDF's.

Definition 3.1. *The CDF of the Burr VI generated family of distributions is given by*

$$Q(x; \xi, k, c, r) = \int_{-\infty}^{W(G(x; \xi))} \frac{ckr \cosh(t)(ke^{-c \sinh(t)} + 1)^{-r}}{e^{c \sinh(t)} + k} dt$$

where $G(x; \xi)$ is the CDF of some baseline distribution. The parameter space of ξ and x depends on the chosen baseline distribution, and $k, c, r > 0$.

(a) *The Burr VI generated family of distributions is of type I, if we take*

$$W(G(x; \xi)) = \log(-\log(1 - G(x; \xi)^\alpha))$$

for some $\alpha > 0$

(b) *The Burr VI generated family of distributions is of type II, if we take*

$$W(G(x; \xi)) = \log\left(\frac{G(x; \xi)^\alpha}{1 - G(x; \xi)^\alpha}\right)$$

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for some $\alpha > 0$

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