G. N. SHAIKHOVA^{1*}, B. K. RAKHIMZHANOV²¹L.N. Gumilyov Eurasian national university, Nur-Sultan, Kazakhstan²JSC “National Company “Kazakhstan Gharysh Sapary”, Nur-Sultan, Kazakhstan,

E-mail: g.shaikhova@gmail.com, rahimzhanovberik@gmail.com

TRAVELING WAVE SOLUTIONS FOR THE EXTENDED MODIFIED KORTEWEG-DE VRIES EQUATION

In this paper, we study an extended modified Korteweg-de Vries equation, which contains the relevant higher-order nonlinear terms and fifth-order dispersion. This equation is the extension of the modified Korteweg-de Vries equation and described by the Ablowitz-Kaup-Newell-Segur hierarchy. The standard Korteweg-de Vries equation is the pioneer integrable model in solitary waves theory, which gives rise to multiple soliton solutions. The Korteweg-de Vries equation arises naturally from shallow water, plasma physics, and other fields of science.

To obtain exact solutions the sine-cosine method is applied. It is shown that the sine-cosine method provides a powerful mathematical tool for solving a great many nonlinear partial differential equations in mathematical physics. Traveling wave solutions are determined for extended modified Korteweg-de Vries equation. The study shows that the sine-cosine method is quite efficient and practically well suited for use in calculating traveling wave solutions for extended modified Korteweg-de Vries equation.

Keywords: extended modified Korteweg-de Vries equation, sine-cosine method, ordinary differential equation, partial differential equation, nonlinearity.

Introduction. Nonlinear evolution equations are of enormous significance and form the core of mathematical physics and engineering areas [1-5]. The studies of the nonlinear evolution equations, their solutions, and the interaction of these solutions are flourishing rapidly. Various powerful methods [6–15] have been used to find solutions to the nonlinear evolution equations.

In this paper, we study an extended modified Korteweg-de Vries (emKdV) equation in defocusing form [16,19], which is

$$u_t + u_x + \alpha(-6u^2u_x + u_{xxx}) + \alpha^2(6u^5 - 10uu_x^2 - 10u^2u_{xx} + u_{xxxx})_x = 0, \quad (1)$$

where $u(x, t)$ is a real function of the spatial coordinate x and the time t , $\alpha \ll 1$ is a nondimensional measure of the small wave amplitude relative to depth. The emKdV includes four nonlinear terms, and two linear dispersive terms, namely u_{xxx} and u_{xxxx} differently from the standard family of the fifth-order modified KdV equations. The equation (1) is modification of extended Korteweg-de Vries equation [17, 18]

$$u_t + u_x + \alpha(6u^2u_x + u_{xxx}) + \alpha^2(c_1u^2u_x + c_2u_xu_{xx} + c_3uu_{xxx} + c_4u_{xxxx}) = 0, \quad (2)$$

where the parameters c_1, c_2, c_3, c_4 , are the coefficients of the higher-order terms, and its values depend on the physical context. Equation (2) describes the evolution of steeper waves with shorter wave lengths than in the KdV equation [17, 18].

* E-mail корреспондирующего автора: g.shaikhova@gmail.com

The emKdV equation (1) is integrable, rational and periodic solutions have been also obtained by means of the N-fold Darboux transformation, meanwhile, the infinitely many conservation laws have been constructed based on the Lax pair in [16]. Multi-soliton solutions by Hirota's direct method and the Painleve test for equation (1) have been studied in [19]. Long-time asymptotic behavior has been studied in [20]. However, it is noted that the sine-cosine method [1, 13-15] for the emKdV equation (1) was not applied. So, in this research we find new kind of solutions for equation (1) by applying the sine-cosine method.

2. Description of the sine-cosine method. In this section, we describe the sine-cosine method [1, 13-15]. According to the sine-cosine method by using a wave transformation

$$u(x, t) = u(\xi), \quad \xi = (x - ct), \quad (3)$$

the partial differential equation (PDE)

$$E_1(u, u_x, u_{xx}, u_{xxx}, \dots) = 0 \quad (4)$$

can be converted to ordinary differential equation (ODE)

$$E_2(u, u', u'', u''', \dots) = 0 \quad (5)$$

Then the equation (5) is integrated as long as all terms contain derivatives where integration constants are considered zeros. The solutions of ODE (5) can be expressed in the form [1, 13-15]

$$u(x, t) = \begin{cases} \lambda \cos^\beta(\mu\xi), & |\xi| \leq \frac{\pi}{2\mu}, \\ 0, & \text{otherwise,} \end{cases} \quad (6)$$

or

$$u(x, t) = \begin{cases} \lambda \cos^\beta(\mu\xi), & |\xi| \leq \frac{\pi}{\mu}, \\ 0, & \text{otherwise,} \end{cases} \quad (7)$$

where $\xi = x - ct$ the parameters μ , λ and β will be determined, and μ is wave number and c is wave speed respectively [1]. The derivatives of (6) become

$$(u(\mu\xi))' = -\beta\mu\lambda \cos^{\beta-1}(\mu\xi) \sin(\mu\xi), \quad (8)$$

$$(u(\mu\xi))'' = -\mu^2\beta^2\lambda \cos^\beta(\mu\xi) + \mu^2\lambda\beta(\beta-1)\cos^{\beta-2}(\mu\xi), \quad (9)$$

$$(u(\mu\xi))^{(iv)} = \mu^4\beta^4\lambda \cos(\mu\xi)^\beta - 2\mu^2\lambda\beta(\beta-1)(\beta^2 - 2\beta + 2)\cos^{\beta-2} + \\ + \mu^4\beta\lambda(\beta-1)(\beta-2)(\beta-3)\cos^{\beta-4}, \quad (10)$$

and the derivatives of (7) have next forms

$$(u(\mu\xi))' = \beta\mu\lambda \sin^{\beta-1}(\mu\xi) \cos(\mu\xi), \quad (11)$$

$$(u(\mu\xi))'' = -\mu^2\beta^2\lambda \sin^\beta(\mu\xi) + \mu^2\lambda\beta(\beta-1)\sin^{\beta-2}(\mu\xi), \quad (12)$$

$$(u(\mu\xi))^{(iv)} = \mu^4\beta^4\lambda \sin(\mu\xi)^\beta - 2\mu^2\lambda\beta(\beta-1)(\beta^2 - 2\beta + 2)\sin^{\beta-2} + \quad (13)$$

$$+\mu^4\beta\lambda(\beta-1)(\beta-2)(\beta-3)\sin^{\beta-4},$$

and so on for the other derivatives. Applying (6)-(13) into the reduced ordinary differential equation (5) we obtain a trigonometric equation of $\cos^r(\mu\xi)$ or $\sin^r(\mu\xi)$ terms. Then, we determine the parameters by first balancing the exponents of each pair of cosine or sine to determine β . Next, we collect all coefficients of the same power in $\cos^r(\mu\xi)$ or $\sin^r(\mu\xi)$, where these coefficients have to vanish. The system of algebraic equations among the unknown λ and μ will be given and from that, we can determine coefficients.

3. Application of the sine-cosine method. Substituting the wave transformation

$$u(x,t) = u(\xi) = u(x - ct), \quad (14)$$

into equation (1) we get the following ordinary differential equation:

$$u'(1-c) - 2\alpha(u^3)' + \alpha u''' + \alpha^2(6u^5 - 10u(u')^2 - 10u^2u'' + u^{(iv)})' = 0. \quad (15)$$

Integrate equation (15) once, with respect to ξ , yields

$$u(1-c) - 2\alpha u^3 + \alpha u'' + \alpha^2(6u^5 - 10u(u')^2 - 10u^2u'' + u^{(iv)}) = L, \quad (16)$$

where L is constant of integration. By taking constant of integration $L = 0$ we have

$$u(1-c) - 2\alpha u^3 + \alpha u'' + \alpha^2(6u^5 - 10u(u')^2 - 10u^2u'' + u^{(iv)}) = 0, \quad (17)$$

In next subsection, we solve the equation (17) by the sine-cosine method.

3.1 The sine solution. According to method the solution of the (17) can be found by transformation

$$u(\mu\xi) = \lambda \sin^\beta(\mu\xi). \quad (18)$$

To find sine solution we use (18) and its derivatives (11)-(13). Substitute (18) and (11)-(13) into (17) we get

$$\begin{aligned} (1-c)\lambda \sin^\beta(\mu\xi) - 2\alpha\lambda^3 \sin^{3\beta}(\mu\xi) - \alpha\mu^2\beta^2\lambda \sin^\beta(\mu\xi) + \alpha\mu^2\lambda\beta(\beta-1)\sin^{\beta-2}(\mu\xi) + \\ + 6\alpha^2\lambda^5 \sin^{5\beta}(\mu\xi) - 10\alpha^2\mu^2\beta^2\lambda^3 \sin^{3\beta-2}(\mu\xi) + 10\alpha^2\mu^2\beta^2\lambda^3 \sin^{3\beta}(\mu\xi) + \\ + 10\alpha^2\mu^2\beta^2\lambda^3 \sin^{3\beta}(\mu\xi) - 10\alpha^2\mu^2\lambda^3\beta(\beta-1)\sin^{3\beta-2}(\mu\xi) + \\ + \alpha^2\mu^4\beta^4\lambda \sin^\beta(\mu\xi) - 2\alpha^2\mu^4\lambda\beta(\beta-1)(\beta^2-2\beta+2)\sin^{\beta-2}(\mu\xi) + \\ + \alpha^2\mu^4\lambda\beta(\beta-1)(\beta-2)(\beta-3)\sin^{\beta-4}(\mu\xi) = 0 \end{aligned} \quad (19)$$

Using the balance method, by equating the exponents of $\sin^i$, from (19) we determine β :

$$\begin{aligned} \beta-1 \neq 0, \quad (\beta-1)(\beta-2)(\beta-3) \neq 0, \\ \beta-4 = 5\beta \Rightarrow \beta = -1. \end{aligned} \quad (20)$$

Substitute (20) in (19) we obtain next equation

$$\begin{aligned} (1-c)\lambda \sin^{-1}(\mu\xi) - 2\alpha\lambda^3 \sin^{-3}(\mu\xi) - \alpha\mu^2\beta^2\lambda \sin^{-1}(\mu\xi) + \alpha\mu^2\lambda\beta(\beta-1)\sin^{-3}(\mu\xi) + \\ + 6\alpha^2\lambda^5 \sin^{-5}(\mu\xi) - 10\alpha^2\mu^2\beta^2\lambda^3 \sin^{-5}(\mu\xi) + 10\alpha^2\mu^2\beta^2\lambda^3 \sin^{-3}(\mu\xi) + \end{aligned}$$

$$\begin{aligned}
& +10\alpha^2\mu^2\beta^2\lambda^3\sin^{-3}(\mu\xi) - 10\alpha^2\mu^2\lambda^3\beta(\beta-1)\sin^{-5}(\mu\xi) + \\
& +\alpha^2\mu^4\beta^4\lambda\sin^{-1}(\mu\xi) - 2\alpha^2\mu^4\lambda\beta(\beta-1)(\beta^2-2\beta+2)\sin^{-3}(\mu\xi) + \\
& +\alpha^2\mu^4\lambda\beta(\beta-1)(\beta-2)(\beta-3)\sin^{-5}(\mu\xi) = 0
\end{aligned} \quad (21)$$

Equating the coefficients of each pair of the sine functions, we find the following system of algebraic equations:

$$\sin^{-1}(\mu\xi): \quad 1 - c - \alpha\mu^2 + \alpha^2\mu^4 = 0, \quad (22)$$

$$\sin^{-3}(\mu\xi): \quad -\lambda^2 + \mu^2 - 10\alpha\mu^4 + 10\alpha\mu^2\lambda^2 = 0, \quad (23)$$

$$\sin^{-5}(\mu\xi): \quad \lambda^2 - 5\mu^2\lambda^2 + 4\mu^4 = 0. \quad (24)$$

From (22)-(24) we have

$$\mu = \pm\sqrt{\frac{1}{10\alpha}}, \quad \lambda = \pm\sqrt{\frac{1}{10\alpha}}, \quad c = 1 - \alpha\mu^2 + \alpha^2\mu^4, \quad (25)$$

where α real number.

Substituting (25) into (18) we have the sine solution of the extended modified KdV equation (1)

$$u_1(x, t) = \pm\sqrt{\frac{1}{10\alpha}} \sin^{-1}\left(\sqrt{\frac{1}{10\alpha}}(x - ct)\right), \quad (26)$$

where $c = 1 - \alpha\mu^2 + \alpha^2\mu^4$.

3.2 The cosine solution. To find cosine solution we use

$$u(\mu\xi) = \lambda \cos^\beta(\mu\xi), \quad (27)$$

and its derivatives (8)-(10). Substitute (27) and (8)-(10) into (17) we get

$$\begin{aligned}
& (1-c)\lambda \cos^\beta(\mu\xi) - 2\alpha\lambda^3 \cos^{3\beta}(\mu\xi) - \alpha\mu^2\beta^2\lambda \cos^\beta(\mu\xi) + \alpha\mu^2\lambda\beta(\beta-1)\cos^{\beta-2}(\mu\xi) + \\
& + 6\alpha^2\lambda^5 \cos^{5\beta}(\mu\xi) - 10\alpha^2\mu^2\beta^2\lambda^3 \cos^{3\beta-2}(\mu\xi) + 10\alpha^2\mu^2\beta^2\lambda^3 \cos^{3\beta}(\mu\xi) + \\
& + 10\alpha^2\mu^2\beta^2\lambda^3 \cos^{3\beta}(\mu\xi) - 10\alpha^2\mu^2\lambda^3\beta(\beta-1)\cos^{3\beta-2}(\mu\xi) + \\
& + \alpha^2\mu^4\beta^4\lambda \cos^\beta(\mu\xi) - 2\alpha^2\mu^4\lambda\beta(\beta-1)(\beta^2-2\beta+2)\cos^{\beta-2}(\mu\xi) + \\
& + \alpha^2\mu^4\lambda\beta(\beta-1)(\beta-2)(\beta-3)\cos^{\beta-4}(\mu\xi) = 0
\end{aligned} \quad (28)$$

Using the balance method, by equating the exponents of $\cos^j$, from (28) we find β :

$$\begin{aligned}
& \beta - 1 \neq 0, \quad (\beta - 1)(\beta - 2)(\beta - 3) \neq 0, \\
& \beta - 4 = 5\beta \Rightarrow \beta = -1.
\end{aligned} \quad (29)$$

Substituting (29) in (28) we obtain next equation

$$(1-c)\lambda \cos^{-1}(\mu\xi) - 2\alpha\lambda^3 \cos^{-3}(\mu\xi) - \alpha\mu^2\beta^2\lambda \cos^{-1}(\mu\xi) + \alpha\mu^2\lambda\beta(\beta-1)\cos^{-3}(\mu\xi) +$$

$$\begin{aligned}
 &+6\alpha^2\lambda^5\cos^{-5}(\mu\xi)-10\alpha^2\mu^2\beta^2\lambda^3\cos^{-5}(\mu\xi)+10\alpha^2\mu^2\beta^2\lambda^3\cos^{-3}(\mu\xi)+ \\
 &+10\alpha^2\mu^2\beta^2\lambda^3\cos^{-3}(\mu\xi)-10\alpha^2\mu^2\lambda^3\beta(\beta-1)\cos^{-5}(\mu\xi)+ \\
 &+\alpha^2\mu^4\beta^4\lambda\cos^{-1}(\mu\xi)-2\alpha^2\mu^4\lambda\beta(\beta-1)(\beta^2-2\beta+2)\cos^{-3}(\mu\xi)+ \\
 &+\alpha^2\mu^4\lambda\beta(\beta-1)(\beta-2)(\beta-3)\cos^{-5}(\mu\xi)=0
 \end{aligned} \tag{30}$$

From (30) equating the coefficients of each pair of the cosine functions, we find the following system of algebraic equations:

$$\cos^{-1}(\mu\xi): 1-c-\alpha\mu^2+\alpha^2\mu^4=0, \tag{31}$$

$$\cos^{-3}(\mu\xi): -\lambda^2+\mu^2-10\alpha\mu^4+10\alpha\mu^2\lambda^2=0, \tag{32}$$

$$\cos^{-5}(\mu\xi): \lambda^2-5\mu^2\lambda^2+4\mu^4=0. \tag{33}$$

Solving system (31)-(33) leads to the results,

$$\mu=\pm\sqrt{\frac{1}{10\alpha}}, \quad \lambda=\pm\sqrt{\frac{1}{10\alpha}}, \quad c=1-\alpha\mu^2+\alpha^2\mu^4, \tag{34}$$

where α real number.

Substituting (34) into (27) we have the cosine solution

$$u_2(x,t)=\pm\sqrt{\frac{1}{10\alpha}}\cos^{-1}\left(\sqrt{\frac{1}{10\alpha}}(x-ct)\right), \tag{35}$$

where $c=1-\alpha\mu^2+\alpha^2\mu^4$.

4. Conclusion. The sine-cosine method was effectively used for the analytic treatment of the extended modified KdV equation. This equation is the generalization of the standard KdV equation and described by Ablowitz-Kaup-Newell-Segur (AKNS) hierarchy. Traveling wave solutions were derived. The obtained solutions can have an application to some practical physical problems. The applied method can be used in further works to establish more entirely new solutions for other kinds of nonlinear evolution equations.

The research work was prepared with the financial support of the Committee of Science of the Ministry of Education and Science of the Republic of Kazakhstan, IRN project AP09057947.

REFERENCES

- 1 Wazwaz A. Partial differential equations and solitary waves theory. //Springer. -2009.
- 2 Ablowitz M.J., Clarkson P.A., Solitons, nonlinear evolution equations and inverse scattering. // Cambridge University Press, New York.-1991.
- 3 Matveev V.B., Salle M.A., Darboux transformations and solitons. //Springer, Berlin.-1991.
- 4 Wadati M. The modified Korteweg-de Vries equation// Journal of the Physical Society of Japan.-1973.-Vol.34. -P.128.
- 5 Akhmediev N., Ankiewicz A., Taki M. Waves that appear from nowhere and disappear without a trace// Physics Letters A.-2009.-Vol.373.-P. 675.

- 6 Hereman W., Nuseir A. Symbolic methods to construct exact solutions of nonlinear partial differential equations // *Mathematics and Computers in Simulation*. -1997.-Vol. 43. -P.13–27.
- 7 Hirota T. *The Direct Method in Soliton Theory*. // Cambridge University Press, Cambridge. -2004.
- 8 Yesmakhanova K., Nugmanova G., Shaikhova G., Bekova G., Myrzakulov R. Coupled dispersionless and generalized Heisenberg ferromagnet equations with self-consistent sources: Geometry and equivalence // *International Journal of Geometric Methods in Modern Physics*- 2020. -Vol.17, -No 7. -P. 2050104 (19 pages).
- 9 Kutum B.B., Shaikhova G.N. q-soliton solution for two-dimensional q-Toda lattice. // *Bulletin of the Karaganda University. Physics series*. -2019. -No. 2 (95). -P. 22-26.
- 10 Shaikhova G.N., Kutum B.B., Altaybaeva A.B., Rakhimzhanov B.K. Exact solutions for the (3+1)-dimensional Kudryashov-Sinelshchikov equation// *Journal of Physics: Conference Series*. -2019. -Vol. 1416. -P. 012030(1-6).
- 11 Biswas A., Yildirimd Y., Yasar E., Zhou O., Moshokoa S., Belic M. Optical solitons for Lakshmanan-Porsezian-Daniel model by modified simple equation method // *Optik*. -2018, -Vol. 160. -P. 24-32.
- 12 Malfiet W. The tanh method: I. Exact solutions of nonlinear evolution and wave equations. *Physica Scripta*. -1996. -Vol. 54. P. 563-568.
- 13 Wazwaz A.M. The sine-cosine method for obtaining solutions with compact and noncompact structures. *Applied Mathematics and Computation*. -2004. -Vol. 159(2). P. 559-576.
- 14 Yusufoglu E., Bekir A. Solitons and periodic solutions of coupled nonlinear evolution equations by using Sine-Cosine method // *International Journal of Computer Mathematics*. -2006. -Vol. 83(12). -P. 915-924.
- 15 Wazwaz A.M. A sine-cosine method for handling nonlinear wave equations. // *Mathematical and Computer Modeling*. -2004. -No 40(5). -P. 499-508.
- 16 Wang X., Zhang J., Wang L., Conservation laws, periodic and rational solutions for an extended modified Korteweg-de Vries equation.// *Nonlinear Dynamics*. -2018.-Vol.92.-P. 1507–1516.
- 17 Marchant T.R., Smyth N.F. Soliton interaction for the extended Korteweg-de Vries equation. *IMA Journal of Applied Mathematics*. -1996.-Vol.56.-P. 157–176.
- 18 Marchant T.R., Smyth N.F. The extended Korteweg-de Vries equation and the resonant flow of a fluid over topography. *Journal of Fluid Mechanics*.-1990.-Vol.221.-P.263-288.
- 19 Wazwaz A.M., Xu G., An extended modified KdV equation and its Painlevé integrability, *Nonlinear Dynamics*. -2016.-Vol 86, -P. 1455–1460.
- 20 Liua N., Guo B., Wang D., Wang Y. Long-time asymptotic behavior for an extended modified Korteweg-de Vries equation // *Communications in Mathematical Sciences*. -2019.-Vol. 17. -P. 1877 – 1913.

Г. Н. ШАЙХОВА¹, Б. К. РАХИМЖАНОВ²

¹Евразийский национальный университет имени Л.Н. Гумилева, г. Нур-Султан, Казахстан,

²АО «Национальная компания «Казахстан Фарм Сапары», Нур-Султан, Казахстан,
e-mail: g.shaikhova@gmail.com, rahimzhanovberik@gmail.com

РЕШЕНИЯ ТИПА БЕГУЩЕЙ ВОЛНЫ ДЛЯ РАСШИРЕННОГО МОДИФИЦИРОВАННОГО УРАВНЕНИЯ КОРТЕВЕГА-ДЕ ФРИЗА

В работе исследовано расширенное модифицированное уравнение Кортевега-де Фриза, которое содержит соответствующие нелинейные члены более высокого порядка и дисперсию пятого

порядка. Это уравнение является расширением модифицированного уравнения Кортевега-де Фриза и описывается иерархией Абловица-Каупа-Ньюэлла-Сегура. Стандартное уравнение Кортевега-де Фриза является первой интегрируемой моделью в теории уединенных волн, которая приводит к многосолитонным решениям. Уравнение Кортевега-де Фриза возникает из мелкой воды, физики плазмы и других областей науки.

Для получения точных решений применен метод синус-косинусов. Показано, что метод синус-косинусов представляет собой мощный математический инструмент для решения очень многих нелинейных уравнений в частных производных математической физики. Определены решения типа бегущей волны для расширенного модифицированного уравнения Кортевега-де Фриза. Исследование показывает, что метод синус-косинусов достаточно эффективен и практически хорошо подходит для использования при вычислении решений типа бегущей волны для расширенного модифицированного уравнения Кортевега-де Фриза.

Ключевые слова: расширенное модифицированное уравнение Кортевега-де Фриза, метод синус-косинус, обыкновенное дифференциальное уравнение, дифференциальное уравнение в частных производных, нелинейность.

Г. Н. ШАЙХОВА ¹, Б. К. РАХИМЖАНОВ²

¹Л.Н. Гумилев атындағы Еуразия ұлттық университеті, Нұр-Сұлтан, Қазақстан,

²«Қазақстан Ғарыш Сапары» ұлттық компаниясы» АҚ, Нұр-Сұлтан, Қазақстан,
e-mail: g.shaikhova@gmail.com, rahimzhanovberik@gmail.com

КЕҢЕЙТІЛГЕН МОДИФИКАЦИЯЛАНҒАН КОРТЕВЕГ-ДЕ ФРИЗ ТЕҢДЕУІНІҢ ҚОЗҒАЛМАЛЫ ТОЛҚЫНДЫҚ ШЕШІМДЕРІ

Бұл жұмыста жоғары деңгейлі сызықтық емес мүшелер мен бесінші ретті дисперсияны қамтитын кеңейтілген модификацияланған Кортевег-де Фриз теңдеуі зерттеледі. Бұл теңдеу модификацияланған Кортевег-де Фриз теңдеуінің кеңейтілген түрі болып табылады және ол Абловиц-Кауп-Ньюэлл-Сегур иерархиясымен сипатталады. Стандартты Кортевег-де Фриз теңдеуі - жекеленген толқындар теориясындағы көпсолитондық шешімдерге әкелетін алғашқы интегралданатын модель. Кортевег-де Фриз теңдеуі таяз суларда, плазма физикасынан және басқа ғылым салаларынан туындайды.

Нақты шешімдер алу үшін синус-косинус әдісі қолданылады. Синус-косинус әдісі - математикалық физиканың көптеген сызықтық емес дербес туынды дифференциалдық теңдеулерін шешудің тиімді математикалық құралы екендігі көрсетілген. Кеңейтілген модификацияланған Кортевег-де Фриз теңдеуі үшін қозғалмалы толқын түріндегі шешімдер анықталды. Зерттеу көрсеткендей, синус-косинус әдісі айтарлықтай тиімді және кеңейтілген модификацияланған Кортевег-де Фриз теңдеуі үшін қозғалмалы толқындық шешімдерді есептеу үшін қолдануға өте ыңғайлы.

Түйін сөздер: кеңейтілген модификацияланған Кортевег-де Фриз теңдеуі, синус-косинус әдісі, қарапайым дифференциалдық теңдеу, дербес туынды дифференциалдық теңдеу, сызықтық емес.