Optimization of number and locations of discrete heaters in a two-dimensional radiant heating furnace using artificial neural networks

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ABSTRACT

In this paper, the inverse solution of determining the optimum location and number of discrete heaters in a two-dimensional radiant furnace with participating media is presented as an optimization problem and solved using an exhaustive search. The regular radiative transfer equation is replaced by a neural network-based surrogate to make the solution process faster. This surrogate model is coupled with an exhaustive search algorithm to determine the best distribution of heaters in a radiant furnace containing H_2O and CO_2 as participating gases. The RTE solution method, when replaced with ANN results in 1000 times saving in the computational time. The accuracy of the heat flux estimations for the predictions with RTE and ANN were found to be under 5%. The optimum configuration of the heaters is found to be sensitive to the medium properties and design surface temperature. This new method of using ANN in place of the RTE and coupling it with search algorithms proves to be an accurate, fast and robust tool in solving the inverse problem of optimization of heater configurations.

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1. Introduction

In a variety of industrial applications such as reheating processes in steel plants or baking of food items radiant heating furnaces are used. The primary goal of these furnaces is to heat the host specimen or material to a desired temperature in order to achieve specific material properties. For this, the material or the specimen to be heat treated is exposed to high heat loads through electrical heaters or gas/fuel fired burners mounted at the top end of the furnace. Given the high temperatures observable during operation, radiation dominates the heat transfer and if heating is performed through gas fired burners, the role of participating medium is also significant. The radiation through the heaters (or burners) is passed to the medium and ultimately to the specimen surface. Though, in such problems, one has the control over the total heat input to the heaters, the uniformity of the heating at the specimen surface is largely determined by the configuration of the heaters i.e. which heater should be kept on and which should be kept off. Considering that the important inputs to solve the problem are known apriori, then in such a case, the challenge is to estimate the best possible heater configuration to obtain a heat flux uniformity on discrete locations on the surface of the specimen. Such problems are inherently inverse in nature and primarily ill-posed because more than one boundary condition is prescribed at the specimen surface. The classical treatment of such problems is performed through regularization methods, in which their ill-posed nature is reduced by

using methods for example Tikhonov regularization, Wolfowitz algorithm and conjugate gradient. Readers are directed to refer Franca et al. [2003] for more insights on regularization methods. Howell et al. [2000] initially proposed the inverse methodology within the framework of radiation dominated furnace. This methodology has been applied further for conjugate and multi-mode heat transfer scenarios [Franca et al. 2003, Mossi et al. 2008, Moghadassian et al. 2014], participating media [Franca et al. 1998, Sarvari et al. 2003a, Sarvari et al. 2003b], and transient heating [Erturk et al. 2002, Daun et al. 2004]. For the particular case of radiant furnace with discrete heaters, due to the complexity of governing radiative transfer equation, its direct inversion is not straight-forward. However, in such cases, one can use optimization techniques where a guess to the solution can be made and improved sequentially to get close to the exact solution value. The formulation of the objective function should be such that its minimum value represents the optimal solution. In case the total solution space of the objective function is limited and approachable, an exhaustive search can be performed. Otherwise, one has to look for heuristic based search techniques, which, in general, draw their formulation from the evolution of species or their biological movement. Some of the important examples include genetic algorithm, micro-genetic algorithm and particle swarm optimization.

The use of optimization methods in solving inverse problems is not new. Sarvari et al. [2003] used micro genetic algorithm to estimate the optimum geometry of the furnace. Payan et al. [2015] used particle

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swarm optimization to determine the heat flux distribution at the heater surface in two dimensional regular and irregular geometries. Among others, Daun et al. [2003] used Wolfowitz optimization algorithm to obtain optimum design of specular enclosures with dominant radiation heat transfer. With regard to the problem of optimizing number of heaters and their respective locations in a radiant furnace, Amiri et al. [2011] used the Discrete Ordinates based solver with micro-genetic algorithm to solve a similar problem, which was extended to a participating medium by the same authors [Amiri and Coelho 2016]. Specifically, the main focus of the researchers has been to use the up to date and efficient optimization algorithm to solve these problems. However, it is to be noted that, the actual forward estimations of radiative heat fluxes from the first principles require multiple solutions of the RTE in the search algorithm which slows down the complete process. Surprisingly, attempts in order to expedite the forward calculations in optimization based inverse techniques are quite rare. Neural network modelling has appeared as a viable choice for generating such fast surrogate models. In particular to the present application in question, Howell et al. [2003] discussed a neural network based control model in transient heating application. Yuen [2009] developed an emissivity estimation model using the spectroscopic database and later applied it to estimate heat fluxes in fire resistance test furnaces [Yuen et al. 2009]. As it is observed, the inclusion of neural network as a support algorithm to facilitate the computations in a faster and better way has been addressed in literature, but its application as a full surrogate model in an inverse optimization problem has not been fully explored.

In this work, thus, the radiative heat flux calculations are accomplished by replacing radiative transfer equation with a neural network based surrogate solver. A radiant furnace, with a two-dimensional square geometry with a participating medium is considered, and the problem of achieving uniform heating at a control area on the specimen known as design surface and the corresponding best configuration of heaters is solved using this surrogate solver coupled with an exhaustive search algorithm. This replacement is expected to reduce the overall computation time significantly. The RTE based forward model is generated using the Finite Volume Method (FVM). The participation of gases is modelled with SLWSSG (Spectral line based Weighted sum of gray gases) model. This combined formulation, termed as FVM-SLW is validated against the benchmark cases available in literature before applying it to the radiant furnace problem. The outputs of the FVM-SLW based forward model are used to train a neural network using Bayesian Regularization. In what follows, a description of the test problem and solution procedure is presented followed by results and discussion thereof.

2. Formulation

2.1. RTE and Finite Volume Method

The radiative transfer equation at a given wavelength and a given direction written for an absorbing-emitting medium enclosed in a two-dimensional rectangular furnace is:

$$\mu \frac{\partial I'_\lambda}{\partial x} + \eta \frac{\partial I'_\lambda}{\partial y} = -\kappa_{g,\lambda} I'_\lambda + \kappa_{g,\lambda} I_{b,g,\lambda} \quad (1)$$

In equation (1), the term on the left-hand side represents the change in directional spectral intensity with respect to position multiplied by their respective direction cosines, and the term on right hand side incorporates attenuation due to absorption and augmentation due to emission in the radiation intensity as it moves along the space. Equation 1 is a first order differential equation and therefore can be solved by invoking one boundary condition, which can be given by wall emission [Chai et al. 1994]. The finite volume based discretization in spatial as well as angular domain has been adopted here. The detailed discussion of the discretization procedure is provided in reference [Chai et al. 1994]. The final equation in terms of the discretization coefficients of the faces and control angles is written as

$$a_{p,j}^{mn} I_{j,p}^{mn} = a_E^{mn} I_{j,E}^{mn} + a_W^{mn} I_{j,W}^{mn} + a_N^{mn} I_{j,N}^{mn} + a_S^{mn} I_{j,S}^{mn} + S_{j,p}^{mn} \quad (2)$$

In above equation, the coefficients are given by

$$a_i^{mn} = \max(-\Delta A_i D_{ci}^{mn}, 0) \quad (3)$$

and,

$$a_p^{mn} = \sum_{i=e,w,n,s} \max(\Delta A_i D_{ci}^{mn}, 0) + (\kappa_{g,j})_p \Delta V \Delta \Omega^{mn} \quad (4)$$

$$S_{j,p}^{mn} = l_{g,j}(T_g) \kappa_{g,j} I_{b,g} \quad (5)$$

where the right hand side term represents the emission from the control volume. $l_{g,j}$ is the weight pertaining to the band emission of the gas corresponding to its temperature. These weights of the bands (or gray gases) and their corresponding absorption coefficients can be readily calculated following the procedure outlined in reference [Dennison and Webb 1993]. In the present work, the gray gas approximation of SLW model is employed [Amiri and Lari 2016]. This solution strategy of combining FVM as the RTE solver and using SLW for the non gray modelling of gas mixture has been termed as the FVM-SLW method.

2.2. Artificial Neural Networks

Artificial neural networks have attracted the attention of researchers from various disciplines ever since their conception. This attraction is largely due to its applicability in a variety of engineering problems. Primarily, ANN mimics the functionality of the brain, as the fundamental entity behind making decision is a natural neuron which processes the information based on past learnings and experiences. A group of such neurons is called a network. An artificial neural network, thus in a similar way, is a collection of artificial neurons having data processing abilities as they take some information as input and process them to obtain the output using suitable functions. There are several types of architectures possible to build a neural network. The one most commonly used is a feed forward back-propagation network. These are multi-layered networks, in which the inputs, outputs and the neurons are treated as distinct layers which feed the data in a forward direction while the errors are back propagated. In a mathematical sense, given a set of inputs $I::\{x_1, x_2, x_3, \dots, x_n\}$, a neural network performs the mapping of the same to a set of outputs $O::\{O_1, O_2, O_3, \dots, O_n\}$. An artificial neuron takes the input set, and characterizes the contribution from each input by assigning a weight (w_i) to it. Each neuron also has a bias value (b_i). This weight and bias together generate the activation functions for the neurons. Using these activation functions the neuron layer obtains the output value by performing mapping (or transfer) operation. There are a variety of transfer functions available like tan-sigmoid, log-sigmoid, pure-linear etc. The mapped data is passed to the further layers for processing. If no other layers exist, the output layer, in general, uses a pure linear transfer function. Figure 1 shows a typical feed forward back propagation neural network

Figure 1: Sample feed forward back propagation neural network.

with one hidden layer and output layer, the transfer functions used here are tan-sigmoid in the hidden layer and pure-linear in the output layer. Assuming that dataset available for network generation is static (i.e. does not change with time), the neural network in its first run assigns random values to the weight and bias and calculates the output. This initially calculated output is matched with the target (actual output) and the error is calculated. Based on the magnitude of this error, the adjustments to the initial weights and bias values are made. The process goes on iteratively for all the cases in the dataset. The overall error is then calculated and process is repeated until the error reaches less than an acceptable value. Thus, this procedure trains the network over a certain range of the values of the inputs. There exist various algorithms to perform the network training. For example, the commonly used ones include the Levenberg-Marquardt [Levenberg 1944] and Bayesian Regularization [McKay 1992]. During the course of training, under the usual procedure, the input dataset is divided into three for the purposes of training, internal testing and validation. However, once the network is selected, it must be

tested with some external dataset which was not a part of training and testing data. This is accomplished to ensure the network is reasonably accurate in handling any kind of dataset introduced to it in the future.

3. Test Problem of a Radiant Furnace

Aschematic of the furnace geometry is presented in figure 2. A radiant heating furnace is approximated as a two-dimensional rectangular geometry of $1\text{ m} \times 1\text{ m}$. The top wall of the furnace is mounted with 10 discrete heaters/burners. The bottom surface hosts a specimen of size 0.4 m which has to undergo heat treatment. All other walls of the furnace are adiabatic. During the heating process, the heat gets transferred to the medium first which changes the medium temperature. Further to this, the medium attains equilibrium and all the heat attained by the medium from the heaters is transferred to the design surface. However, different configuration of the heaters would impart different profiles of the heat flux at the bottom surface. Therefore, if uniform heating of the specimen has to be achieved, one needs to optimize the heater state and locations. For the determination of local heat fluxes, the radiative transfer equation should be solved from first principles. There are 6 inputs in the problem as shown in Table 1, where, Q_h is total power of the heater, T_{des} is temperature of the surface to be heat treated, ϵ_d emissivity of the surface, Y_{H_2O} and Y_{CO_2} are H_2O and CO_2 gas concentration respectively and the configuration of heaters. Each heater state can be defined in terms of binary bits 0 and 1, where 0 would imply that heater is inactive and 1 would imply that heater is active. This type of representation helps in assigning a unique value to the configuration in terms of an integer, the binary counterpart of which actually denotes the state of the heaters. In view of this, it follows that if 10 heaters are employed and each heater can take a value either 0 or 1, total number of possible configurations would be $2^{10}=1023$. Thus, the number 1 would represent the lower bound that only the first heater is on and takes the complete heat load, while the number 1023 would signify that all heaters are on and that the heat load is distributed equally amongst all the 10 heaters. The ranges of the input variables have been shown in Table 1. The flux output has been monitored at four discrete locations as shown in figure 2.

Figure 2: Schematic of the problem of optimizing discrete heaters.

Table 1: Inputs to the RTE solver and their bounds

Variables	lower bound	upper bound	unit
Q_h	1000	5000	W
T_{des}	300	500	K
ϵ_d	0	1	-
Y_{H_2O}	0	0.25	-
Y_{CO_2}	0	0.25	-
Configuration	1	1023	-

4. Verification of the Forward Model

The FVM-SLW formulation has been tested with several benchmark cases, the results for a very general case, which involves temperature gradients and mixture of participating gases enclosed within a furnace represented by a three-dimensional geometry are considered. The furnace medium comprises 20% H_2O and 10% CO_2 on a molar basis. The geometry is rectangular ($2\text{ m} \times 2\text{ m} \times 4\text{ m}$) and the case details correspond to the one solved by Liu [1999], where the solutions for incident radiative heat fluxes are obtained through Statistical Narrow Band (SNB) as the non-gray model and the Monte Carlo (MC) method as RTE solver [Liu 1999]. The fluxes are determined at the side wall (2,1,z) and non-dimensionalized with respect to σT_{max}^4 , where $T_{max} = 1800\text{ K}$ as per [Liu 1999]. Solutions are obtained using the FVM-SLW method with 10 gray gases for a coarse ($12 \times 12 \times 24$) and fine ($24 \times 24 \times 48$) mesh. The results are compared with the SNB-MC method [Liu 1999] as shown in the figure 3. It is observed from the figure that the present solution obtained by the FVM-SLW method closely matches with the solutions from SNB-MC, which confirms the accuracy of the model to be used as a basis for the neural network development.

Figure 3: Values of non-dimensional incident radiative heat flux at the side wall of the furnace solved using present FVM-SLW and comparison with the solutions of Liu [1999].

5. Results and Discussion

5.1. Neural Network as a replacement to RTE

The problem discussed in section 3 has been solved by generating a neural network based surrogate to RTE for the aforementioned six variables in the problem. The ranges of these variables are shown in the Table 1. With these prescribed 6 inputs, an initial set of 200 samples is generated using Latin Hypercube Sampling [McKay et al. 1979]. The values of fluxes at the four monitored location on the design surface for these 200 cases have been obtained with forward solver (FVM-SLW). Further to this, the 200 input output dataset has been used to generate the ANN. The input and output variables are normalized before being input to the network as per following equations $Q_h^i = Q_h / Q_{h,max}$, $T_{des}^i = T_{des} / T_{des,max}$, and the output radiative flux is normalized as $q_d^i = q_d / q_{d,max}$, where $q_{d,max} = Q_{h,max} / \Delta A_d$, ΔA_d is the design surface area. 90% of this data is used for training through neural network tool in MATLAB [2014] and rest are stored for additional validation of the generated network. The transfer function used in neuron layer is tan-sigmoid and in the output layer is pure-linear. The training algorithm is Bayesian regularization. The network performance is tested after the training based on two parameters,

$$MSE = \frac{1}{K} \sum_{i=1}^K (q_i - q_{t,i})^2 \quad R^2 = 1 - \frac{S_E}{S_{ME}} \quad (6)$$

$$\text{and, } S_E = \sum_{i=1}^K (q_i - q_{t,i})^2 \quad S_{ME} = \sum_{i=1}^K (\bar{q}_i - q_{t,i})^2$$

Where MSE denotes the representative value of mean square error and R represents the goodness of fit. S_E is the sum of the errors squared and S_{ME} is the sum of mean errors squared. K is the total number of cases, q_d denotes exact output and $q_{e,i}$ denotes output from ANN. Initial results obtained with multiple hidden layers did not show observable improvements over those obtained with single hidden layer, so one hidden layer has been used in the present study. The MSE and R values for the training and testing data set is shown in Table 2. As can be seen from the results, sufficient accuracy in terms of mean square error and coefficient of fitness is observed for 4 neurons with both the training and testing data. The corresponding value of mean square error for the external validation dataset (10% of the total data set) was observed to be 1.3516×10^{-5} . Therefore, a network with one hidden layer containing 4 neurons has been chosen for further studies.

Table 2: Performance parameters of the network

No. of Neurons	Training		Testing	
	MSE	R	MSE	R
2	2.48E-05	0.997	3.26E-05	0.999
4	1.83E-05	0.998	4.12E-05	0.999
6	1.48E-05	0.998	3.28E-04	0.998
8	1.31E-05	0.999	4.82E-05	0.999

5.2. Comparison of solution from ANN and RTE

In order to verify the generated network, 5 random cases were chosen. The details of these cases are given in Table 3. These are termed as the test cases and the range of input variables for them corresponds to ranges shown in Table 1. For all of these cases, predictions of both the forward model and the ANN model are obtained and the values of the local net radiative heat flux on the design surface have been estimated at the monitored locations (viz. $x = 0.35, 0.45, 0.55$ and 0.65 m). An evaluation of the relative error (RE) in the prediction of local heat flux in kW/m^2 at the design surface with the RTE and the ANN has been performed. The expression for RE is given as,

$$RE = \frac{q_{i,rte} - q_{i,ann}}{q_{i,rte}} \quad (7)$$

Table 3 Five different test cases to compare the ANN solution with the RTE

Case	$Q_h(W)$	$T_{des}(K)$	ε_d	Y_{H_2O}	Y_{CO_2}	Config
1	4600	400	0.5	0.175	0.125	922
2	3000	440	0.9	0.075	0.225	307
3	2200	320	0.7	0.225	0.025	512
4	1400	480	0.3	0.025	0.175	717
5	3800	360	0.1	0.125	0.075	102

Figure 4 shows the corresponding results for this analysis. It can be clearly seen that a good accuracy is achieved with the present network as the maximum local relative errors observed are less than 5% for all the five cases. However, it is important to note that on comparing the solution time taken by the RTE and the ANN, the efficiency of the neural network is clearly visible as shown in figure 5. Here, the computational time for evaluation of heat flux by RTE model and the ANN using MATLAB has been compared on a 10 GB memory system of Dell T1650 workstation series. It can be observed that the ratio of the computational time between the two is of the order of 1000, thereby signifying huge savings in the computational cost.

5.3. Applying ANN to a two-dimensional inverse problem

The neural network based model has been applied to the discrete heater problem shown in figure 2. A sample design case is considered

(shown in Table 4) for which the input variables take the values as $T_{des} = 400$ K, $\varepsilon_d = 0.4$, $Y_{H_2O} = 0.2$, and $Y_{CO_2} = 0.1$. Based on simple energy balance, one can calculate the total heater power $Q_h(W)$ needed to be supplied to the heaters (Amiri and Coelho 2016). The goal is to then estimate the best configuration which can provide a uniform heating of 10 kW/m^2 at every location on the design surface. The objective function is written as,

$$F = \min \left(\sum_i (q_{d,i} - q_{e,i})^2 \right) \quad (8)$$

Figure 4: Relative error calculation in solutions of RTE and ANN for estimating local heat flux values at the design surface for test cases given in Table 3.

Figure 5: Calculation of computational time in estimations of local heat fluxes by RTE and ANN performed for cases given in Table 3.

Table 4 Pseudo design case for optimizing the heater configurations

Design case	$Q_h(W)$	$T_{des}(K)$	ε_d	Y_{H_2O}	Y_{CO_2}	Configuration
	4000	400	0.4	0.2	0.1	to be optimized

where $q_{d,i}$ is the desired heat flux at i^{th} location (10 kW/m^2 in this case), and $q_{e,i}$ is the estimated heat flux through ANN. As shown in figure 5, the fast calculation capability of ANN, enables one to test all possible configurations exhaustively. One can solve equation 8 with optimization algorithms such as genetic algorithm, particle swarm or any search-based methods. In the present study, a total of 10 heaters are chosen. This means that there could be 2^{10} heater configurations are possible. This is a small number and keeping in mind that the ANN model of the RTE takes less than a second to give the output, a simple exhaustive search (ES) can be performed. The flowchart of the estimation process is presented in figure 6.

Figure 6: Flow chart of ANN coupled with exhaustive search to determine optimum configuration.

5.4. Solution for design cases and corresponding optima

The optimized configuration for this design case has been shown in the figure 7 along with the value of objective function. In the subsequent step, 10 variations of such a design case have been generated by varying each of the variable at a time while keeping others constant at the design case value. Firstly, the T_{des} is increased in steps of 100 K in the range 300-500K, then ϵ_d is increased by 0.2 in the range 0.2-1, and finally the H_2O and CO_2 gas concentrations are varied in the range 0.05-0.2, by making one of them half at a time from its original value. These variations in the

Figure 7: Optimum configurations of the heaters for design case given in Table 4 (the red strip shows the heater should be ON, otherwise OFF).

Table 5 Ten different variants of design cases generated by varying the design surface temperature, design surface emissivity and gas concentrations respectively.

Case	$Q_h (W)$	$T_{des} (K)$	ϵ_d	Y_{H_2O}	Y_{CO_2}
1	4000	300	0.4	0.2	0.1
2	4000	400	0.4	0.2	0.1
3	4000	500	0.4	0.2	0.1
4	4000	400	0.2	0.2	0.1
5	4000	400	0.6	0.2	0.1
6	4000	400	0.8	0.2	0.1
7	4000	400	1	0.2	0.1
8	4000	400	0.4	0.1	0.1
9	4000	400	0.4	0.2	0.05
10	4000	400	0.4	0.1	0.05

inputs form a distinct set of 10 cases, the details of which are shown in Table 5. These cases have been taken up for the optimization study as described in previous section. The optimum configurations for all these 10 cases and the corresponding heat flux values are shown in Table 6. The obtained flux values at the monitored locations for these optimum configurations are then shown in figure 8 (for variation of design surface temperature), figure 9 (for variation of design surface emissivity) and figure 10 (for variation of gas concentrations). From the figures, it can be seen that the optimized configurations provide values very close to the desired heat flux (which is a horizontal line at 10 kW/m^2). It was found that the optimum configurations are sensitive to the variation in input parameters, which implies that each case is a unique design case and there is not a single configuration that will work for all the cases. In such a scenario, the inverse solution methodology has to be adopted for each of the cases and thus such a fast and robust ANN-ES algorithm becomes a necessity to obtain the optimum configurations.

Table 6 Results of ANN-ES for the test cases mentioned in Table 5 in terms of Optimum configurations and corresponding heat input to the heaters.

Case	Optimized configuration (index of heaters that are switched ON)	Value of objective function (F)	Heat input to each heater (kW/m^2)
1	1,2,3,4,5,6,7,8,9,10	0.00027	4.00
2	2,3,4,6,7,8	0.00022	6.67
3	1,8	0.00063	20.0
4	1,2,3,4,5,6,7,8,9,10	0.00010	4.00
5	1,2,3,4,5,6,7,8,9,10	0.00068	4.00
6	1,2,3,5,7,9	0.00060	6.67
7	1,2,3,4,7,8,9	0.00076	5.71
8	1,2,3,4,5,6,7,8,9,10	0.00123	4.00
9	1,2,3,5,9	0.00059	8.00
10	1,2,3,5,8	0.00138	8.00

5.5. Validation of optima using forward (RTE) calculations

A kind of validation of the entire procedure is to solve the RTE with the heat fluxes returned by the optimization algorithm to see if the heat fluxes at the design surface predicted by the hybrid ANN and the exhaustive search method are the same as those obtained through the full numerical solution of the RTE. For this purpose, the inputs of aforementioned 10 cases and their corresponding optimum configurations are input to the FVM-SLW solver and the solution is obtained for each of the cases. Results are represented in Table 7 and it is observed that the maximum relative error (modulus of equation 7) in local heat fluxes between the estimations of RTE and ANN are seen to be within 4%. This indicates sufficient credibility and efficacy of the procedure elaborated and results thereof.

Figure 8: Net radiative heat fluxes at monitored locations on the design surface for the optimum configuration with different values of surface temperature.

Figure 9: Net radiative heat fluxes at monitored locations on the design surface for the optimum configuration with different values of surface emissivity.

Table 7 Calculation of maximum values of relative error in the heat flux prediction with ANN and RTE for the optimum configurations.

Case	Maximum relative error (in %)
1	1.553
2	1.628
3	1.718
4	0.863
5	2.367
6	3.314
7	3.877
8	1.768
9	1.921
10	2.017

Figure 10: Net radiative heat fluxes at monitored locations on the design surface for the optimum configuration with different values of gas mole fractions ($x = Y_{H_2O}$ and $y = Y_{CO_2}$ take the value from Table 5).

6. Conclusions

Solution to the inverse problem of estimating optimum distribution of heaters in a two-dimensional radiant furnace with participating medium was presented. In the place of a traditional RTE solution method, a fast-neural network-based function was used. It was observed that the ANN based function performs 1000 times faster with good accuracy (relative errors being under 5%) in terms of local heat flux estimation. The fast ANN solution proves to be a good tool to exhaustively test all possible configurations and thus obtains the global optimum. A sample design case was considered to demonstrate the accuracy of the model. The sensitivity of the optimum configuration was performed by varying the properties of the design surface and it was observed that, the optimum configurations changed by making even slight changes to the input variables. This suggests the necessity of having such a fast inverse solution procedure to obtain quick results for any case. Overall, this methodology gives a robust, fast and accurate means of solving the inverse problem of radiant furnace design.

Nomenclature

A_i	face area of i^{th} face of control volume, m^2
a_i	discretization coefficient at i^{th} node in FVM
D_{ci}	FVM based directional weights
F	objective function
I_λ	spectral intensity, $W/m^2sr \mu m$
$I_{b\lambda}$	blackbody intensity, $W/m^2sr \mu m$
M, N	total number of directions along θ and ϕ
MSE	mean square error
Q_h	total heater power, W
q	radiative heat flux, kW/m^2
R	goodness of fit
S	source term in FVM formulation

Greek symbols

ϵ	emissivity
κ	absorption coefficient as per SLW model, m^{-1}
μ, η	direction cosines in r and z
d	properties at design surface
G	gas properties index
H	properties at heater surface
J	j^{th} gray gas
Δ	spectral quantities

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