Review Article

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Levy Swimmer In An Asymmetric Channel

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ABSTRACT

In this article, we numerically study the dynamics of a model Levy walker, moving in a 2 dimensional medium confined by the walls of an asymmetric channel. We show that, as a result of both asymmetric potential due to the channel and also the power law step size distribution, the Levy walker will achieve a net directed velocity in the direction preferred by the channel. Other statistical properties of the walker such as mean first passage time and the mean square displacements are also examined.

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Introduction

Extracting network from noisy motion is an old subject that goes back to the work by MV Smoluchovski [1]. A so called ratchet and pawl mechanism shows that, it is impossible to extract work from equilibrium noisy motion. In order to achieve network from noisy motion, one needs to simultaneously break the translation symmetry and also derive the system to out of equilibrium conditions.

It has been shown recently that, in a suspension of diffusing bacteria, it is possible to rotate a small scale asymmetric gear immersed in the suspension and subsequently extracts work from noisy motion of bacteria [2]. Bacteria are out of equilibrium organisms that continuously transform chemical energy to mechanical motions. There are recent experimental and theoretical indications that fluctuations affects the internal signaling pathway of *E. coli* and enable them to perform Levy walks [3, 4]. In addition to the above chemical fluctuation mechanisms, the Levy walk characteristics are also observed for bacteria moving in turbulent flows and also for swarming bacteria [5].

A Levy walk, random walk with heavy tailed power law distribution for step sizes, is a none equilibrium noisy motion. Putting a Levy walk in a medium confined by the walls of an asymmetric channel, we ask what is the long time average of the walkers velocity.

Levy walk and channel

A Levy swimmer is a random walk with two events: running events and rotating events, where the walk is a scale-invariant and characterized by a power-law run length distribution given by $P(x) \sim x^{-(1+\alpha)}$ with the Levy index α lies in the interval $0 < \alpha < 2$. We assume that a rotating event occurs after finishing each running event. In rotating events, walker chooses a new direction of motion from uniform distribution. For the case with $\alpha=2$, as the distribution function asymptotically meets the Gaussian distribution, we expect to see equilibrium behavior. Figure 1 (a), shows the geometry of a 2 dimensional asymmetric channel which characterized by two lengths L_x , L_y and an angle θ . We assume

that a very short range repulsive interaction confines the motion of walker. This short range interaction affects the trajectory of the particle as a scattering event.

To simulate the dynamics of our Levy walker, we start from a random position inside the channel and proceeds with discrete steps. In each step, two random numbers are generated. First random number is chosen from a uniform distribution in the interval $[0, 2\pi]$. We use this uniform random number, to allocate the instantaneous direction of the walker. Second random number is generated from a heavy tailed power-law distribution in the interval $[1, 3]$. The walker will continue its motion until it reaches the walls. To define the scattering from walls, we use a simplified bounce back boundary condition, depicted in figure 1(b). In this figure point A shows the particle's position just before an impact with wall. This particle is supposed to move a length L that falls it beyond the wall to point A' (outside the channel). The particle's correct position after the scattering from wall is denoted by point B in figure. We make a mirror reflection from wall in such a way that the length before scattering, L' , and the length after scattering, L'' , satisfies the condition $L' + L'' = L$. Periodic boundary condition is also applied to the trajectory of walker.

Method of inverse transform sampling is used to produce a power-law distributed numbers in the interval (a_{min}, a_{max}) [6]. Considering such small and large cut off numbers are essential because there is a minimum and maximum limit for length of steps of bacteria.

Figure1: (a) Geometry of an Asymmetric channel. (b) The prescription for the scattering from wall. Walker is supposed to

have a walk from point A to point A' that falls out of the channel. The true position after Scattering is shown by point B.

Figure 2: (a): net current as a function of time steps is shown for some different realizations of the noise. As one can see the current asymptotically reaches a steady value. (b): total current of particles in channel with different asymmetry parameter θ and different Levy indexes α . Current reduces with increasing Levy index and it decrease with asymmetry parameter. Currents tends to zero when Levy index reaches $\alpha=2$.

Results

Total net current, circulating in a channel with periodic boundary condition, is the most important quantity that we can measure. It is the net number of particles per unit time that cross the

Figure 3: Left: mean first passage time for two limiting states $\alpha=0$ and $\alpha=2$ and also along positive and negative directions of x axis. For $\alpha=2$ (Gaussian tail), there is no difference between mean first passage time in positive and negative directions of x axis and as we expected the net current vanishes. But for Levy index equals to zero, it is bigger for negative direction after a transition time. N shows negative direction and P shows positive direction. Right: mean Square displacement vs. time in log-log scale. Walker with Levy index equal to zero probes more than Walker with Levy index equal to two. Slope of fits for this two case are, respectively, 1.5 and 0.9. So walker with Levy index equal to zero and two, having super-diffusive and diffusive behavior in the x direction of the channel.

Right side of the channel and re-enter from its left side. Denoting by N_r (Nl), the number of times that the particle leave the channel from right (left) side, we can write the current as:

$$\frac{N_r - N_l}{N_r + N_l} = J \quad (1)$$

To study this current we start with a random initial position inside the channel and study the trajectory for a long time. Figure 2(a), shows the current as a function of time for different initial conditions of the walker. As one can see, after a long time the current reaches a steady state value that is positive. It is the asymmetry of channel that choose the overall direction of directed motion. In figure 2(b), we have shown the steady state current as function of Levy index α and for different values of asymmetry parameter θ . For a fixed geometry, the maximum current is associated with $\alpha=0$ and the current vanishes at $\alpha=2$. For a fixed Levy index, we see that the

current increases with increasing asymmetry parameter.

Another quantity which we have measured is the mean first passage time. Events are often started when a stochastic or random process first encounters a threshold. The threshold can be a boundary, specified state of a system or a barrier of energy. The amount of time required for a stochastic process, starting from some initial state, to encounter a threshold for the first time, is known as the first passage time and denoted by τ_{fpt} . In our system, first passage time is the smallest time in which walker goes beyond x_0 distance from beginning position, in x direction. Ensemble average on this quantity leads to mean first passage time, a quantity encompass in formation of both motion of particle and characteristics of environment. Because of asymmetry of channel in positive and negative direction of x axis, we defined mean first passage time in positive and negative directions of x axis. Figure 3(a) shows L_{fpt} for positive direction is smaller than negative direction in x axis and also, decreasing Levy index leads to reduction in mean first passage time. These two results are in agreement with the results of currents. We expect this quantity changes periodically after some transition time. This transition time is in agreement with the time in which current becomes constant. Also in this transition time, difference between negative and positive direction in mean first passage time appears. The mean squared displacement is a measure of spatial extent of random motion over time and is defined as:

$$MSD = \langle (x(t) - x(0))^2 \rangle \quad (2)$$

Where $x(t)$ are positions of the particle at time t and $\langle \dots \rangle$ shows ensemble average. We calculated this quantity for 100 Levy walkers in specially asymmetric channel. Data of horizontal and vertical axis of this figure is calculated by, respectively, averaging over times each step start in and square displacement of particles from initial position in x axis. Figure 3(b) shows MSD as a function of time. In conclusion, we show that the out of equilibrium distribution for the step size of a walker, can produce a net current for walker when it is moving in an asymmetric channel.

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