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Rayleigh-Type Waves in a Rotating Fiber-Reinforced Half Space under the Action of Magnetic Field, Gravity and Surface Stress

Narottam Maity¹, S P Barik^{2*} and P K Chaudhuri³

¹Assistant Teacher of Mathematics, Nabagram K D Paul Vidyalaya, Serampore, India

²Associate Professor of Mathematics, Gobardanga Hindu College, West Bengal, India

³Retired Professor of Applied Mathematics, University of Calcutta, Kolkata, India

ABSTRACT

The aim of the present article is to analyze the propagation of Rayleigh waves in a rotating fiber-reinforced electrically conducting elastic solid medium under the influence of surface stress, magnetic field and gravity. The magnetic field is applied in such a direction that the problem can be considered as a two dimensional one. The wave velocity equation for Rayleigh waves has been obtained. In the absence of gravity field, surface stress, rotation and fiber-reinforcement, the frequency equation is in complete agreement with the corresponding classical results. The effects on various subjects of interest are discussed and shown graphically. Comparisons are made with the corresponding results in absence of surface stress.

*Corresponding author

S P Barik, Associate Professor of Mathematics, Gobardanga Hindu College, West Bengal, India. E-mail: spbarik1@gmail.com

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Introduction

The purpose of generating more strength to a solid without adding too much weight to it or increasing its load bearing capacity is fulfilled through the process of fibre-reinforcing. The process of fibre-reinforcing has been developing continuously with advanced technology and the products are in use in various fields; as for example, in manufacturing aircraft components fibre-reinforced material can be a good option. The characteristic property of a reinforced composite is that its components act together as single anisotropic units as long as they remain in the elastic condition. The wave propagation in a reinforced medium plays a very interesting role in civil engineering, geophysics and geodynamics. The earth may contain fibre-reinforced material like hard and soft rock. The seismic signals have to travel through various materials present in the earth in the form of layers. During earthquake and similar disturbances, a structure is excited and vibration occurs with resulting oscillatory stresses. Thus the propagation of waves in a fibre-reinforced medium depend on the physical properties of the structures. Different methods are adopted in fibre-reinforcing process, viz., long fibres are arranged in parallel in the matrix, short fibres are randomly arranged in the matrix, or fibres in the form of cloth are set in the matrix. Obviously, the strength and the load bearing capacity of the fibre-reinforced material will depend upon the fibre arrangement in the matrix. Arunakanthi and Kumar has an experimental studies on fiber reinforced concrete [1].

The importance of wave propagation in solids arises from its relevance in composite engineering, geology, seismology, seismic exploration, control system and acoustics. In the study of earthquakes, geophysics and geodynamics, surface waves play a crucial role. Well-known general surface waves i.e. Rayleigh waves, Love waves and Stoneley waves are well connected with earthquake spectrum analysis. Rayleigh waves are responsible for causing destruction to the structure owing to their slower attenuation of the energy than that of the body waves. In classical theory of elasticity, effect of gravity and surface stress is generally neglected. The effect of gravity on wave propagation in an elastic solid medium was first considered by Bromwich [2]. He has treated the force of gravity as a type of body force. Love extended the work of Bromwich and investigated the influence of gravity on superficial waves and showed that the Rayleigh wave velocity is affected by the gravity field [3]. Biot studied the influence of gravity on Rayleigh waves, assuming the force of gravity to create a type of initial stress of hydrostatic nature and the medium as incompressible [4]. De and Sengupta studied the effect of gravity on the propagation of surface waves in an elastic layer and also on Lamb's problem on a plane [5, 6, 7]. Das et.al investigated surface waves under the influence of gravity in a nonhomogeneous elastic solid medium [8]. Abd-Alla et. al. studied the surface waves propagation in a fibre-reinforced anisotropic elastic media subjected to gravity field [9]. Abd-Alla et.al also investigated the propagation of Rayleigh waves in magneto-thermo-elastic half-space of a homogeneous orthotropic material under the effect of rotation [10]. Abo-Dahab et.al. studied the effect of gravity field, initial stress and rotation on the S-waves propagation in a nonhomogeneous anisotropic medium under magnetic field

[11]. Bayones investigated propagation of Rayleigh waves in fiber-reinforced anisotropic thermo-visco elastic media under the effect of rotation [12, 13]. Abd-Alla et.al. investigated the effects of rotation and gravity field on surface waves in fibre-reinforced thermoelastic media under four theories. Maity et. al. [14]. Investigated the wave propagation in a rotating fibre-reinforced poro-elastic solid under the action of uniform magnetic field and also propagation of plane waves in a rotating magneto thermoelastic fibre-reinforced medium with voids under G-N theory[15,16]. Sethi et.al. Studied surface waves in fibre-reinforced anisotropic elastic media under the influence of gravity[17]. Abd-Alla et. al. discussed the effect of rotation, magnetic field, initial stress and gravity on Rayleigh waves in a homogeneous orthotropic elastic half-space[18]. The physical properties of bodies in the neighborhood of the surface are sensibly different from those of the interior. The surface tension in a liquid is a particular case of surface stress. The presence of surface stress on the boundary of bodies has been detected in some particular type of crystals, whose orders of magnitude agree with the predictions made by molecular theory. Compressive surface stress is involved in the case of shot peening of ductile metals and its knowledge is quite useful for the shaping of aircraft wing panels (Gurtin) [19]. The effect of surface stress on Rayleigh wave propagation has been discussed by Chandrasekharaiah [20]. The propagation of Rayleigh surface waves in a rotating fibre-reinforced medium with magnetic field, surface stress and gravity field has not been noticed so far. In the present investigation the propagation of Rayleigh waves in presence of fibre-reinforced medium, rotation, magnetic field, surface stress and gravity field has been discussed. It is assumed that the direction of the magnetic field is such that the problem can be considered as two dimensional. The effects of surface stress, density, magnetic field, rotation, fibre parameters and gravity are shown graphically.

Formulation of the problem

Let us consider an elastic medium of fibre-reinforced material occupying a half space with plane boundary. Choosing the origin 'o' of the cartesian coordinate system $oxyz$ at the plane boundary, the half space may be defined as $z \geq 0$. We shall make the following assumptions for the problem we are going to discuss:

The material of the half space is under the action of a magnetic field $\mathbf{H}_0 = (0, H_0, 0)$.

A gravitational force is acting on the medium in the direction $(0, 0, 1)$.

The half space is rotating with angular velocity $\Omega = (0, \Omega, 0)$.

The fibres in the medium are arranged parallelly in the direction $\mathbf{a} = (1, 0, 0)$.

An elastic disturbance in the form of wave is propagating along the x-axis and the disturbance is assumed to be confined to the neighborhood of the free surface $z = 0$ and vanishes as $z \rightarrow \infty$

From the assumptions made above it follows that all particles on a line parallel to y-axis are equally displaced. Therefore, all the field quantities will be independent of y coordinate and the problem can be considered as two dimensional in the x-z plane. Since the medium is assumed to be rotating with angular velocity Ω in an applied magnetic field of intensity $\mathbf{H}_0$, an induced magnetic field $\mathbf{h} = (0, h, 0)$, an induced electric field $\mathbf{E}$ and a current density $\mathbf{J}$ will be developed. If μ_0 is the magnetic permeability of the medium

then the total magnetic field in the medium is $\mathbf{B} = \mu_0 \mathbf{H}$, where $\mathbf{H} = \mathbf{H}_0 + \mathbf{h}$ is the magnetic field arising from applied magnetic field $\mathbf{H}_0$ and induced field $\mathbf{h}$. Denoting the displacement vector by $\mathbf{u} = u_i(X, t)$, the simplified system of the equations of electrodynamics for a slowly moving homogeneous electrically conducting medium, may be written as

$$\begin{aligned}\nabla \times \mathbf{h} &= \mathbf{J} + \epsilon_0 \dot{\mathbf{E}} \\ \nabla \times \mathbf{E} &= -\mu_0 \dot{\mathbf{h}} \\ \nabla \cdot \mathbf{h} &= 0 \\ \mathbf{E} &= -\dot{\mathbf{u}} \times \mathbf{B}\end{aligned}\quad (1)$$

where ϵ_0 is the electrical conductivity of the medium. Following Belfield et. al. [10] the stress-strain relations for linearly fibre-reinforced elastic medium may be expressed in tensor form as

$$\begin{aligned}\tau_{ij} &= \lambda \epsilon_{kk} \delta_{ij} + 2\mu_T \epsilon_{ij} + \alpha(a_k a_m \epsilon_{km} \delta_{ij} + a_i a_j \epsilon_{kk}) + \\ &2(\mu_L - \mu_T)(a_i a_k \epsilon_{kj} + a_j a_k \epsilon_{ki}) + \beta(a_k a_m a_i a_j \epsilon_{km})\end{aligned}\quad (2)$$

where τ_{ij} are the cartesian components of the stress tensor;

$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ are the strain components, related to the

displacement vector u_i ; λ, μ_T are elastic constants; $\alpha, \beta, (\mu_L - \mu_T)$ are reinforcement parameters and $\mathbf{a} = (a_1, a_2, a_3)$ such that

$a_1^2 + a_2^2 + a_3^2 = 1$. For a rotating elastic medium the equation of

motion, in absence of body force, can be written as

$$\tau_{ij,j} + F_i = \rho[\ddot{u}_i + \{\Omega \times (\Omega \times \mathbf{u}) + 2\Omega \times \dot{\mathbf{u}}\}_i]\quad (3)$$

In (3), ρ denotes the material density, overhead dot represents differentiation with respect to time and the suffix after second bracket represent the i th component of the vector inside and F_i are the body force components arising from the applied magnetic field and the gravitational effects.

For this problem $\mathbf{F} = \mathbf{J} \times \mathbf{B} + \rho g(w, 0, -u, x)$ and $\mathbf{u} =$

$(u_1, u_2, u_3) = (u, 0, w)$. Then utilizing relations (1) and neglecting products of $\mathbf{h}$ with $\mathbf{u}$ and its derivatives, the governing equations of motion (3) for a rotating fibre-reinforced medium under the action of applied magnetic field and gravity may be written as

$$\frac{\partial \tau_{11}}{\partial x} + \frac{\partial \tau_{13}}{\partial z} + \mu_0(\mathbf{J} \times \mathbf{H})_1 + \rho g \frac{\partial w}{\partial x} = \rho \left[\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega \frac{\partial w}{\partial t} \right]\quad (4)$$

$$\frac{\partial \tau_{21}}{\partial x} + \frac{\partial \tau_{23}}{\partial z} + \mu_0(\mathbf{J} \times \mathbf{H})_2 = \rho \left[\frac{\partial^2 v}{\partial t^2} \right]\quad (5)$$

$$\frac{\partial \tau_{31}}{\partial x} + \frac{\partial \tau_{33}}{\partial z} + \mu_0(\mathbf{J} \times \mathbf{H})_3 - \rho g \frac{\partial u}{\partial x} = \rho \left[\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega \frac{\partial u}{\partial t} \right]\quad (6)$$

Based on our assumption that the fibres are parallelly arranged in the direction $\mathbf{a} = (1, 0, 0)$ the relevant components of stress tensor are

$$\tau_{11} = (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)e_{11} + (\lambda + \alpha)e_{22} + (\lambda + \alpha)e_{33}$$

$$\tau_{22} = (\lambda + \alpha)e_{11} + (\lambda + 2\mu_T)e_{22} + \lambda e_{33}, \tau_{33} = (\lambda + \alpha)e_{11} + \lambda e_{22} + (\lambda + 2\mu_T)e_{33}$$

$$\tau_{12} = 2\mu_L e_{12}, \tau_{13} = 2\mu_L e_{13}, \tau_{23} = 2\mu_T e_{23}$$

Using (7) equation (4), (5) and (6) reduce to

$$A_{11} \frac{\partial^2 u}{\partial x^2} + B_2 \frac{\partial^2 w}{\partial x \partial z} + \mu_L \frac{\partial^2 u}{\partial z^2} - \mu_0 H_0 \frac{\partial h}{\partial x} - \epsilon_0 \mu_0^2 H_0^2 \frac{\partial^2 u}{\partial t^2} + \rho g \frac{\partial w}{\partial x} = \rho \left[\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega \frac{\partial w}{\partial t} \right] \quad (8)$$

$$\mu_L \frac{\partial^2 v}{\partial x^2} + \mu_T \frac{\partial^2 v}{\partial z^2} = \rho \frac{\partial^2 v}{\partial t^2} \quad (9)$$

$$A_{22} \frac{\partial^2 w}{\partial z^2} + B_2 \frac{\partial^2 u}{\partial x \partial z} + \mu_L \frac{\partial^2 w}{\partial x^2} - \mu_0 H_0 \frac{\partial h}{\partial z} - \epsilon_0 \mu_0^2 H_0^2 \frac{\partial^2 w}{\partial t^2} - \rho g \frac{\partial u}{\partial x} = \rho \left[\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega \frac{\partial u}{\partial t} \right] \quad (10)$$

where

$$A_{11} = \lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta, A_{22} = \lambda + 2\mu_T, B_2 = \lambda + \alpha + \mu_L$$

From equation (1) the expression for h can be written as

$$h = -H_0 \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right), \quad (11)$$

Now the dynamical equations of motion can be written as

$$(A_{11} + \mu_0 H_0^2) \frac{\partial^2 u}{\partial x^2} + (B_2 + \mu_0 H_0^2) \frac{\partial^2 w}{\partial x \partial z} + \mu_L \frac{\partial^2 u}{\partial z^2} + \rho g \frac{\partial w}{\partial x} = (\rho + \epsilon_0 \mu_0^2 H_0^2) \frac{\partial^2 u}{\partial t^2} - \rho \Omega^2 u + 2\rho \Omega \frac{\partial w}{\partial t} \quad (12)$$

$$\mu_L \frac{\partial^2 v}{\partial x^2} + \mu_T \frac{\partial^2 v}{\partial z^2} = \rho \frac{\partial^2 v}{\partial t^2} \quad (13)$$

$$(A_{22} + \mu_0 H_0^2) \frac{\partial^2 w}{\partial z^2} + (B_2 + \mu_0 H_0^2) \frac{\partial^2 u}{\partial x \partial z} + \mu_L \frac{\partial^2 w}{\partial x^2} - \rho g \frac{\partial u}{\partial x} = (\rho + \epsilon_0 \mu_0^2 H_0^2) \frac{\partial^2 w}{\partial t^2} - \rho \Omega^2 w - 2\rho \Omega \frac{\partial u}{\partial t} \quad (14)$$

For Rayleigh waves we concentrate only equation (12) and (14) which reduce to

$$(c_1^2 + c_A^2) \frac{\partial^2 u}{\partial x^2} + (c_2^2 + c_A^2) \frac{\partial^2 w}{\partial x \partial z} + c_3^2 \frac{\partial^2 u}{\partial z^2} + g \frac{\partial w}{\partial x} = (1 + c_B^2) \frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega \frac{\partial w}{\partial t} \quad (15)$$

$$(c_4^2 + c_A^2) \frac{\partial^2 w}{\partial z^2} + (c_2^2 + c_A^2) \frac{\partial^2 u}{\partial x \partial z} + c_3^2 \frac{\partial^2 w}{\partial x^2} - g \frac{\partial u}{\partial x} = (1 + c_B^2) \frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega \frac{\partial u}{\partial t} \quad (16)$$

where, $c_A^2 = \frac{\mu_0 H_0^2}{\rho}$, $c_B^2 = \frac{\epsilon_0 \mu_0^2 H_0^2}{\rho} = \frac{c_A^2}{c_0^2}$, $c_1^2 = \frac{A_{11}}{\rho}$, $c_2^2 = \frac{B_2}{\rho}$, $c_3^2 = \frac{\mu_L}{\rho}$, $c_4^2 = \frac{A_{22}}{\rho}$, $c_0^2 = \frac{1}{\epsilon_0 \mu_0}$,

c_A = alfvén velocity, c_4 = P-wave velocity, c_0 = velocity of light in vacuum. Now introducing the dimensionless variables $\bar{x}$, $\bar{z}$, $\bar{u}$, $\bar{w}$, $\bar{t}$ such that

$$x = c_1 \omega_1 \bar{x}, z = c_1 \omega_1 \bar{z}, u = c_1 \omega_1 \bar{u}, w = c_1 \omega_1 \bar{w}, t = \omega_1 \bar{t},$$

(where ω_1 is the wave number) equations (15) and (16) for the half space reduce to

$$(1 + \chi_H) \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + (\bar{c}_2^2 + \chi_H) \frac{\partial^2 \bar{w}}{\partial \bar{x} \partial \bar{z}} + \bar{c}_3^2 \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + \bar{g} \frac{\partial \bar{w}}{\partial \bar{x}} = M \frac{\partial^2 \bar{u}}{\partial \bar{t}^2} + 2\Gamma \frac{\partial \bar{w}}{\partial \bar{t}} - \Gamma^2 \bar{u} \quad (17)$$

$$(\bar{c}_4^2 + \chi_H) \frac{\partial^2 \bar{w}}{\partial \bar{z}^2} + (\bar{c}_2^2 + \chi_H) \frac{\partial^2 \bar{u}}{\partial \bar{x} \partial \bar{z}} + \bar{c}_3^2 \frac{\partial^2 \bar{w}}{\partial \bar{x}^2} - \bar{g} \frac{\partial \bar{u}}{\partial \bar{x}} = M \frac{\partial^2 \bar{w}}{\partial \bar{t}^2} - 2\Gamma \frac{\partial \bar{u}}{\partial \bar{t}} - \Gamma^2 \bar{w} \quad (18)$$

where

$$M = 1 + \chi_H \frac{c_1^2}{c_0^2}, \chi_H = \frac{c_A^2}{c_1^2}, \Gamma = \Omega \omega_1, c_4'^2 = \frac{c_4^2}{c_1^2}, c_2'^2 = \frac{c_2^2}{c_1^2}, c_3'^2 = \frac{c_3^2}{c_1^2}, \bar{g} = \frac{\omega_1}{c_1} g$$

In absence of gravity the equations (17) and (18) are in complete agreement with the equations (9) and (10) of Roy et. al.[24].

Boundary conditions

For the consideration of the surface stress one has to assumed $z = 0$ to be a material layer which adheres without slipping to the half space and is a two dimensional elastic half space. This layer is capable of supporting its own stress represented by the surface stress tensor $\Sigma_{i\alpha}$ and obeys the law Gurtin [19]

$$\Sigma_{i\alpha} = \begin{cases} \delta_{i\alpha}[\sigma + (\lambda_s + \sigma)u_{r,r}] + \mu_s u_{i,\alpha} + (\mu_s - \sigma)u_{\alpha,i} & \text{for } i, \alpha, r = 1, 2 \\ \sigma u_{3,\alpha} & \text{for } i = 3. \end{cases} \quad (19)$$

Here λ_s, μ_s are the Lamé's moduli of the material boundary and σ is the residual surface tension on the layer $z = 0$. Surface elasticity tensor has been used following Gurtin [19] forces on the bounding surface are governed by surface tension $\Sigma_{i\alpha}$ given a smooth oriented curve γ in $z = 0$ with positive unit normal n_i , $\Sigma_{i\alpha} \cdot n_\alpha$ represents the force per unit length of γ by that part of the boundary into which n_α is directed upon that part of the plane $z = 0$ away from which n_α points. λ_s, μ_s, σ have the dimensions as $n \times n$. Hence the boundary conditions are

$$\tau_{i3} + \Sigma_{i\alpha,\alpha} - \rho_s \frac{\partial^2 u_i}{\partial t^2} = 0 \quad \text{on } z = 0 \quad (20)$$

Where ρ_s is the mass per unit surface area (kgm^{-1}) of the layer and τ_{ij} are the stress tensor in the interior of the medium. The conventional stress tensor τ_{ij} has the unit of force per unit area and the new surface stress tensor $\Sigma_{i\alpha}$ has the unit of force per unit length.

Solution of the problem

For solution of the problem we take

$$(\bar{u}, 0, \bar{w}) = \{u(z), 0, w(z)\} \exp\{i\omega(x - ct)\} \quad (21)$$

in which $u(z), w(z)$ are functions of z only and $\frac{2\pi}{\omega}$ represents wave length of the harmonic waves traveling forward with speed 'c'.

Substituting (20) into the equations (17) and (18) we arrive at the following equations

$$[c_3'^2 D^2 + \{Mc^2 + \Gamma'^2 - (1 + \chi_H)\}\omega^2]u(z) + i\omega[(c_2'^2 + \chi_H)D + (\bar{g} + 2\omega c\Gamma')w(z) = 0 \quad (22)$$

$$[(c_4'^2 + \chi_H)D^2 + (Mc^2 + \Gamma'^2 - c_3'^2)\omega^2]w(z) + i\omega[(c_2'^2 + \chi_H)D - (\bar{g} + 2\omega c\Gamma')u(z) = 0 \quad (23)$$

where, $\Gamma' = \frac{\Gamma}{\omega}, D = \frac{d}{dz}$ and $\bar{g}' = \frac{\bar{g}}{\omega}$. From equation (22) and (23) we get the following equations determining $u(z)$ or $w(z)$ as

$$\begin{aligned} & \{c_3'^2(c_4'^2 + \chi_H)D^4 + [c_3'^2(Mc^2 + \Gamma'^2 - c_3'^2) + (c_4'^2 + \chi_H)\{Mc^2 + \Gamma'^2 - (1 + \chi_H)\} - (c_2'^2 + \chi_H)^2]\omega^2 D^2 \\ & + [\{Mc^2 + \Gamma'^2 - (1 + \chi_H)\}(Mc^2 + \Gamma'^2 - c_3'^2) + (\bar{g}' + 2c\Gamma')^2]\omega^4\}u(z), w(z) = 0 \end{aligned} \quad (24)$$

Let the real roots of the above equation (24) be λ_1^2 and λ_2^2 . Since $u(z); w(z)$ describe surface waves we can assume they are vanishingly small as $z \rightarrow \infty$. On this consideration we take the solutions of equations (22) and (23) in the following form

$$u(z) = (A_1 e^{-i\omega\lambda_1 z} + B_1 e^{-i\omega\lambda_2 z})e^{i\omega(x-ct)}, w(z) = (A_2 e^{-i\omega\lambda_1 z} + B_2 e^{-i\omega\lambda_2 z})e^{i\omega(x-ct)} \quad (25)$$

where, A_1, B_1, A_2, B_2 are constants. Using (25) in (22) and (23) and equating the coefficients of $e^{(-i\omega\lambda_1 z)}$ and $(-i\omega\lambda_2 z)$ to zero we obtain

$$A_2 = \frac{(1 + \chi_H) + \lambda_1^2 c_3'^2 - Mc^2 + \Gamma'^2}{(c_2'^2 + \chi_H)\lambda_1 + i(\bar{g}' + 2c\Gamma')} A_1 \text{ and } B_2 = \frac{(1 + \chi_H) + \lambda_2^2 c_3'^2 - Mc^2 + \Gamma'^2}{(c_2'^2 + \chi_H)\lambda_2 + i(\bar{g}' + 2c\Gamma')} B_1 \quad (26)$$

which can be written as $A_2 = k_1 A_1$ and $B_2 = k_2 B_1$ where

$$k_i = \frac{(1 + \chi_H) + \lambda_i^2 c_3^2 - M c^2 + \Gamma'^2}{(c_2^2 + \chi_H) \lambda_i + i(\bar{g}' + 2c\Gamma')}$$
 (27)

Since we are interested to investigate the possibility of Rayleigh waves with surface stress, in the light of Chandrasekharaiah [13] the boundary conditions may be taken as

$$\tau_{13} + \sum_{1\alpha, \alpha} -\rho_s \frac{\partial^2 u_1}{\partial t^2} = 0 \quad \text{on} \quad z = 0$$
 (28)

$$\tau_{33} + \Sigma_{3\alpha, \alpha} - \rho_s \frac{\partial^2 u_3}{\partial t^2} = 0 \quad \text{on} \quad z = 0$$
 (29)

Using boundary conditions (28) and (29) we get

$$[c_5^2(\lambda_1 - k_1) - i\omega(c_6^2 - c_R^2)]A_1 + [c_5^2(\lambda_2 - k_2) - i\omega(c_6^2 - c_R^2)]B_1 = 0$$
 (30)

$$[(c_7^2 - c_5^2 - \lambda_1 k_1 c_8^2) + i\omega k_1(c_9^2 - c_R^2)]A_1 + [(c_7^2 - c_5^2 - \lambda_2 k_2 c_8^2) + i\omega k_2(c_9^2 - c_R^2)]B_1 = 0$$
 (31)

Eliminating A_1 and B_1 from (30) and (31) we get

$$\begin{vmatrix} c_5^2(\lambda_1 - k_1) - i\omega(c_6^2 - c_R^2) & c_5^2(\lambda_2 - k_2) - i\omega(c_6^2 - c_R^2) \\ (c_7^2 - c_5^2 - \lambda_1 k_1 c_8^2) + i\omega k_1(c_9^2 - c_R^2) & (c_7^2 - c_5^2 - \lambda_2 k_2 c_8^2) + i\omega k_2(c_9^2 - c_R^2) \end{vmatrix} = 0$$
 (32)

where $c_5^2 = \frac{\mu_L}{\rho_s}$, $c_6^2 = \frac{\lambda_s + 2\mu_s}{\rho_s}$, $c_7^2 = \frac{B_2}{\rho_s}$, $c_8^2 = \frac{\lambda_s + 2\mu_T}{\rho_s}$, $c_9^2 = \frac{\sigma}{\rho_s}$, $c_5'^2 = \frac{c_5^2}{c_1^2}$, $c_6'^2 = \frac{c_6^2}{c_1^2}$, $c_7'^2 = \frac{c_7^2}{c_1^2}$, $c_8'^2 = \frac{c_8^2}{c_1^2}$, $c_9'^2 = \frac{c_9^2}{c_1^2}$,

The equation (32) represents the wave velocity equation for Rayleigh waves with surface stress in a rotating fiber-reinforced medium under the action of magnetic field and gravity. If we omit the surface stress and gravity term this frequency equation (32) agrees with the equation (5.1) obtained by Acharya [6]. In absence of surface stress, gravity, rotation and a particular consideration as $\mu_L = \mu_T = \mu$ equation (32) reduces to

$$(2 - M \frac{c^2}{c_2^2})(2 - \frac{M}{1 + \chi_H} \frac{c^2}{c_2^2}) = 4(1 - M \frac{c^2}{c_2^2})^{\frac{1}{2}}(1 - \frac{M c_2'^2}{1 + \chi_H} \frac{c^2}{c_2^2})^{\frac{1}{2}}$$
 (33)

when α, β tend to zero. Now neglecting the magnetic field we get the well known Rayleigh waves velocity equation for isotropic medium as follows

$$(2 - \frac{c^2}{c_2^2})^2 = 4(1 - \frac{c^2}{c_2^2})^{\frac{1}{2}}(1 - \frac{c^2}{c_1^2})^{\frac{1}{2}}$$
 (34)

Numerical results and discussions

The present study focuses on the effects of surface stress, magnetic field, rotation, density, gravity and fibre-reinforcing on the propagation of Rayleigh waves in a medium. To observe the effects numerically we have adopted three sets of values of relevant parameters named fiber-1 and fiber-2 respectively from the works of Maity et. al.[23], Markham[21], Zorammuana[26] as given below

$$\lambda = 5.65 \times 10^9 N \cdot m^{-2}, \mu_T = 2.46 \times 10^9 N \cdot m^{-2}, \mu_L = 5.66 \times 10^9 N \cdot m^{-2}, \rho = 2.26 \times 10^3 Kg \cdot m^{-3},$$

$$\lambda = 7.59 \times 10^9 N \cdot m^{-2}, \mu_T = 3.5 \times 10^9 N \cdot m^{-2}, \mu_L = 7.07 \times 10^9 N \cdot m^{-2}, \rho = 1.6 \times 10^3 Kg \cdot m^{-3},$$

$$\lambda_s = 7 \times 10^3 N \cdot m^{-1}, \mu_s = 8 \times 10^3 N \cdot m^{-1}, \rho_s = 7 \times 10^{-4} Kg \cdot m^{-2}, \sigma = 110 N \cdot m^{-1}$$

In our numerical analysis we shall plot the Rayleigh wave velocity in fibre-reinforced medium against the wave number under different conditions of the model. We shall examine the effects of various parameters considered separately taking other parameter values fixed, on the wave propagation velocity in the medium. We have observed that in all cases considered the wave velocity decreases asymptotically to zero as the wave number increases. Another notable feature of our analysis is that, for a fixed wave number value the wave velocity increases with the applied surface stress. The results are shown in the Fig. 2 to Fig. 6.

It is observed in Fig. 2 that increase of rigidity of the reinforced medium shows increase of velocity of propagation of Rayleigh type wave in the medium. The effect of surface stress is to increase the magnitude of velocity. The increase is larger for smaller value of wave number, and asymptotically tends to zero as wave number approaches infinity. Increase of velocity with rigidity is in agreement with the behaviour of Rayleigh wave velocity in homogeneous isotropic medium. As in homogeneous isotropic medium the wave velocity decreases with the increase of material density of the medium and the effect of surface stress is to increase magnitude of the velocity. It is clear from our numerical results that in a fibre reinforced medium the characteristic of velocity of propagation is the same as that in a homogeneous isotropic medium, only the magnitude differ. The effects of magnetic field or rotation are similar and are quite clear from the Fig. 3 and Fig. 4.

Figure 1: Geometry of the problem

Figure 2: Effect of reinforcing parameter on Rayleigh wave velocity taking $g=9.8$, $\Omega=5.0$, $\rho=1600$, $H_0=5.0 \times 10^7$

Figure 3: Effect of magnetic field on Rayleigh wave velocity taking $g=9.8$, $\Omega=5.0$, $\rho=1600$

Figure 4: Effect of Ω on Rayleigh wave velocity $g=9.8$, $\rho=1600$, $H_0=5.0 \times 10^7$

Figure 5: Effect of ρ on Rayleigh wave velocity taking $g=9.8$, $\Omega=5.0$, $H_0=5.0 \times 10^7$

Figure 6: Effect of gravity on Rayleigh wave velocity $\rho=1600$, $\Omega=5.0$, $H_0=5.0 \times 10^7$

References

1. E Arunakanthi, J D Chaitanya Kumar (2016) Experimental studies on fiber-reinforced concrete (FRC). Int J Civil Engg Tech 7: 329-336.
2. T J J A Bromwich (1898) On the influence of gravity on elastic waves and in particular on the vibrations of an elastic globe. Pro London Math Soc 30: 98-120.
3. M E Gurtin, A I M

4. Murdoch (1975) A continuum theory of elastic material surface stress. Arch Rat Mech Anal 57: 291-323.
5. MA Biot, J Willy (1965) Mechanics of incremental Deformations.
6. SN De, P R Sengupta (1974) Influence of gravity on wave propagation in an elastic layer. J Acoust Soc Am 55: 919-921.
7. SN De, PR Sengupta (1973) Plane Lamb's problem under the influence of gravity. Gerlands Beitr Geophys (Leipzig) 82: 421-426.
8. S N De and P R Sengupta (1974) Influence of gravity on wave propagation in an elastic layer. J Acoust Soc Am 55: 919-921.
9. S C Das, D P Acharya, P R Sengupta (1992) Surface waves in an inhomogeneous elastic medium under the influence of gravity. Rev Roum Des Sci Tech 37: 355-363.
10. A N Abd-Alla, T A Nofal, S M Abo-Dahab A Al-Mullise (2013) Surface waves propagation in fibre-reinforced anisotropic elastic media subjected to gravity field. In J Physical Sci 8: 574-584.
11. A M Abd-Alla, S M Abo-Dahab, F S Bayones (2013) Propagation of Rayleigh waves in magneto-thermo-elastic half-space of a homogeneous orthotropic material under the effect of rotation. J Vibr Contrl 19: 1395-1420.
12. S M Abo-Dahab, A M Abd-Alla, I H El-Sirfy (2014) Effect of gravity field, initial stress and rotation on the S-waves propagation in a non-homogeneous anisotropic medium with magnetic field. J Mech Sci Tech 28: 3003-3011.
13. F S Bayones, N S Hussein (2017) Propagation of Rayleigh waves in fiber-reinforced anisotropic solid thermo-viscoelastic media under effect of rotation. Appl Math Inf Sci 11: 1527-1535.
14. F S Bayones, N S Hussien (2018) Effect of rotation on Rayleigh waves in a fiber- reinforced solid anisotropic magneto-thermo-viscoelastic media. Mech Adv Matel and Struc 83: 1-8.
15. A N Abd-Alla, S M Abo-Dahab, A Al-Mullise (2013) Effects of rotation and Gravity field on surface waves in fibre-reinforced thermoelastic media under four theories. J Appl Math 2013 1-20-P.
16. M F. Markham (1969) Measurement of the elastic constants of fibre composites by ultra-sonics. Composites 1: 145-149.
17. N Maity, S P Barik, P K Chaudhuri (2016) Wave propagation in a rotating fibre- reinforced poro-elastic solid under the action of uniform magnetic field. Int J Comp Sci 4: 901-917.
18. I Roy, D P Acharya, S Acharya (2017) Propagation and reflection of plane waves in a rotating magneto elastic fibre-reinforced semi space with surface stress. Mech and Mech Eng 21: 1043-1061.
19. A M Abd-Alla, S M Abo-Dahab, S R Mahmoud, M I Helmy (2010) Influence of rotation, magnetic field, initial stress and gravity on Rayleigh waves in a homogeneous orthotropic elastic half-space. Appl Math Sci 4: 91-108.
20. S N De, P R Sengupta (1976) Surface waves under the influence of gravity. Gerlands Beitr Geophys (Leipzig) 85: 311-318.
21. D S Chandrasekharaiah (1987) Effect of surface stresses and voids on Rayleigh waves in an elastic solids Int J Eng Sci 25: 205-211.
22. N Maity, S P Barik, P K Chaudhuri (2018) Propagation of plane waves in a rotating magneto thermoelastic fibre-reinforced medium with voids under G-N theory. IOSR J Engg 8: 25-35.
23. M E Gurtin and A I. Murdoch (1976) Effect of surface stress on wave propagation in solids. J Appl Phys 47: 4414-4421.
24. D P Acharya, I Roy (2009) Magnetoelastic surface waves in electrically conducting fibre-reinforced media Bul Inst Math Acad Sin 4: 333-352.
25. A E H Love (1911) Some problems of geodynamic. New York: Dover.
26. M Sethi, K C Gupta, D Manisha (2013) Surface waves in fibre-reinforced anisotropic solid elastic media under the influence of gravity. Int J Appl Mech and Engg 18: 177-188.
27. A J Belfield, T G Gers, A J M Spencer (1983) Stress in elastic plates reinforced by fibres lying in concentric circles. J Mech and Phys Solids 31: 25-54.