

## The Ultimate Speed of an accelerated Electron without Infinite Mass

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### ABSTRACT

Failure to accelerate electrons beyond the speed of light, led to special relativity, where mass increases with speed, becoming infinitely large at the speed of light. Invoking aberration of electric field, this paper introduces radiative electrodynamics where an electron is accelerated to the speed of light at constant mass and with emission of radiation. The accelerating force on an electron moving in an electric field, is less than the force on a stationary one, relative to an observer. This difference is radiation reaction force, something akin to frictional force, against which work done appears as radiation. An accelerated electron moves in an electric field, with constant mass as the rest mass, emitting radiation equal to the difference between change in potential energy and change in kinetic energy. Circular revolution of an electron, in an atom, round the central force of attraction of a nucleus, as in the Rutherford's nuclear model of the hydrogen atom, is shown to be without radiation and inherently stable. Emission of radiation takes place if the electron in an atom is dislodged from a circular orbit. It then moves in an elliptic path, emitting radiation, at the frequency of revolution, with *fine structure*, before reverting to a circular orbit. Describing aberration of electric field, introducing radiative electrodynamics, obtaining the ultimate of speed of light outside special relativity, and getting radiation from accelerated electrons outside quantum mechanics, are notable results of this paper.

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### Introduction

In classical electrodynamics, Newton's second law of motion gives the force  $F$  on a body or particle of mass  $m$  equal to the rest mass  $m_0$ , moving with velocity  $v$  at time  $t$  and producing acceleration  $dv/dt$ , as vector equation [1]:

$$\mathbf{F} = m_0 \frac{d\mathbf{v}}{dt} \quad (i)$$

In relativistic electrodynamics, Force  $F$  depends on momentum  $m\mathbf{v}$ , so that Newton's law is:

$$\mathbf{F} = \frac{d(m\mathbf{v})}{dt} \quad (ii)$$

The theory of special relativity was devised to explain why an electron cannot be accelerated, by an electric field, beyond the speed of light [2, 3]. According to special relativity, mass  $m$  of a particle with its energy  $mc^2$ , increases with its speed  $v$ , relative to an observer, becoming infinitely large at the speed of light, according to mass-velocity equation:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_0 \quad (iii)$$

where  $m_0$  is the rest mass and  $\gamma$  the Lorentz factor.

Table 1 gives the speed  $v$  of an electron of charge  $-e$  and rest mass  $m_0$ , accelerated by a constant field  $E$ , from  $0$  initial speed or decelerated from speed of light  $c$ , with  $a = eE/m_0$ , in (A) classical electrodynamics and (B) relativistic electrodynamics in equations (i), (ii) & (iii).

**Table 1: Speed / Time in Classical and Relativistic Electrodynamics**

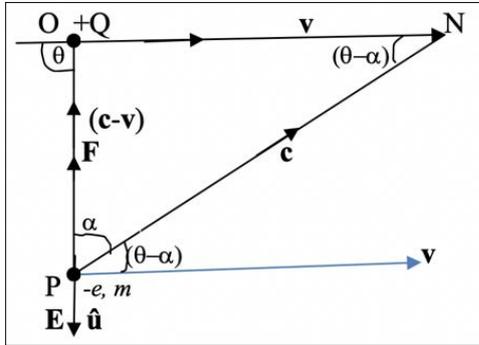
Acceleration or Deceleration	A. Classical Electrodynamics	B. Relativistic Electrodynamics
Acceleration from $0$ initial speed.	A1 $v = at$	B1 $v = at \left(1 + \frac{a^2 t^2}{c^2}\right)^{-1/2}$
Deceleration from speed of light $c$ .	A2 $v = c - at$	B2 Not available.

The kinetic energy of a particle, supposed to be  $(mc^2 - m_0c^2)$ , is accounted for by the increase in its mass. Since an infinitely large mass cannot be accelerated any faster, by any force, this explanation, of an ultimate speed, is plausible. However, the issue of infinite masses, where the speed of light  $c$  is easily attained by electrons, in high energy particle accelerators, is a difficulty. Infinitely large masses cannot be contained in the apparatus of a laboratory. This paper resolves the difficulty by invoking aberration of electric field, where accelerating force by an electric field, decreases with speed, reducing to  $0$  at speed of light  $c$ .

Aberration of light was discovered in 1728 by English astronomer and priest, James Bradley [4, 5, 6]. This, one of the most significant discoveries in science, which clearly demonstrated the relativity of speed of light, is ignored, in favor of constancy of speed of light, on Albert Einstein's theory of special relativity, against all

reasons and evidences.

Aberration of electric field is similar to aberration of light. Figure 1 depicts aberration of electric field for an electron of charge  $-e$ , moving in time  $t$ , at a point P, with velocity  $\mathbf{v}$  at angle  $\theta$  to force of electric field of magnitude  $E$ , along unit vector  $\hat{\mathbf{u}}$ , from charge  $+Q$  at O.



**Figure 1:** An electron of mass  $m = m_0$  and charge  $-e$ , at point P and time  $t$ , moving with velocity  $\mathbf{v}$  at angle  $\theta$  to force of attraction of electric field of intensity  $E$  from stationary source charge  $+Q$  at origin O

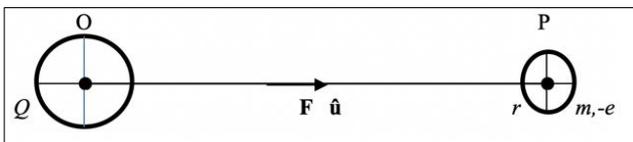
In Figure 1, with respect to the moving electron at P, the charge Q appears to be shifted (forward) from O to a point N, and the electrical force is propagated with velocity of light  $c$ , displaced by aberration angle  $\alpha$  from the instantaneous line joining P and O, such that:

$$\sin \alpha = \frac{v}{c} \sin \theta \quad (1)$$

Equation (1) is Bradley's equation, a universal formula applicable for light in space and electric field, transmitted at the velocity of light, irrespective of distance OP between the source at O and moving electron at P. The vector  $(\mathbf{c} - \mathbf{v})$  in the direction of application of the force, is the relative velocity between the electrical force transmitted with velocity of light  $c$  and the electron moving with velocity  $\mathbf{v}$ . The angle of aberration  $\alpha$  is the inclination between the vectors  $\mathbf{c}$  and  $(\mathbf{c} - \mathbf{v})$ . Aberration of electric field will feature prominently in this paper.

### Coulomb's Law and Accelerating Force

Coulomb's law of electrostatics, the most important principle in physics, was enunciated in 1785 by French physicist and engineer, Charles-Augustin de Coulomb [7, 8]. Coulomb's law is depicted in Figure 2, for force of attraction on an electron of charge  $-e$ , by a positive charge of magnitude  $Q$ . The electron can move with velocity  $\mathbf{v}$  at angle  $q$  to the instantaneous line joining the charges.



**Figure 2:** Force  $\mathbf{F}$  between two stationary electric charges  $Q$  at O and  $-e$  at P separated by distance OP

The problem of physics lies in making Coulomb's law independent of velocity of a charged particle in an electric field. Coulomb's law gives the force  $\mathbf{F}$  on a particle of charge  $-e$  stationary in an electric field  $\mathbf{E} = \hat{\mathbf{u}}E$ , due to source charge  $+Q$ , distance  $r$  apart, as:

$$\mathbf{F} = \frac{-eQ}{4\pi\epsilon_0 r^2} \hat{\mathbf{u}} = \frac{-eQ}{4\pi\epsilon_0 r^2} \frac{\mathbf{c}}{c} = -e\mathbf{E} = -eE\hat{\mathbf{u}} = \frac{-eE}{c} \mathbf{c} \quad (2)$$

where  $\mathbf{c}$  is velocity of light of magnitude  $c$ , at which an electrical force is transmitted in space.

In view of aberration of electric field, force  $\mathbf{F}$  on a particle of charge  $-e$ , and mass  $m$  equal to the rest mass  $m_0$ , moving at time  $t$  with velocity  $\mathbf{v}$ , in an electric field of magnitude  $E$ , due to charge  $+Q$ , Coulomb's law is modified, with Newton's second law of motion, to be:

$$\mathbf{F} = \frac{eE}{c} (\mathbf{c} - \mathbf{v}) = \frac{-eE}{c} \sqrt{c^2 + v^2 - 2cv \cos(\theta - \alpha)} \hat{\mathbf{u}} = m_0 \frac{d\mathbf{v}}{dt} \quad (3)$$

where  $(\theta - \alpha)$  is the angle between the vectors  $\mathbf{c}$  and  $\mathbf{v}$ . The vector  $(\mathbf{c} - \mathbf{v})$ , along PO direction of application of the force, is the relative velocity between the electrical force transmitted with velocity of light  $c$  and the particle velocity  $\mathbf{v}$ . Velocity  $c$  as a limit is implicit in equation (3).

In Figure 1 and equation (3), for the angle  $\theta$  equal to 0, there is acceleration in the direction of  $(-\hat{\mathbf{u}})$  straight line and equations (1) and (3) give the scalar differential equation:

$$-eE \left(1 - \frac{v}{c}\right) = -m_0 \frac{dv}{dt} \quad (4)$$

The solution of equation (4), a first order differential equation, for a uniform electric field of magnitude  $E$ , and with initial speed  $u$  at time  $t = 0$ , is:

$$v = c - (c - u) \exp\left(\frac{-at}{c}\right) \quad (5)$$

where  $a = eE_0/m_0$  is a constant. For acceleration from 0 initial speed, equation (5) gives:

$$v = c - c \exp\left(\frac{-at}{c}\right) \quad (6)$$

In equation (3) for  $\theta$  equal to  $\pi$  radians, there is deceleration in a straight line  $(-\hat{\mathbf{u}}$  direction) and equations (1) and (3) give the scalar differential equation:

$$-eE \left(1 + \frac{v}{c}\right) = m_0 \frac{dv}{dt} \quad (7)$$

Solving the differential equation (7) for a charged particle decelerated from speed  $u$ , by a uniform electric field of magnitude  $E$ , gives:

$$v = (c + u) \exp\left(\frac{-at}{c}\right) - c \quad (8)$$

For a particle decelerated from the speed of light  $c$ , equation (8) becomes:

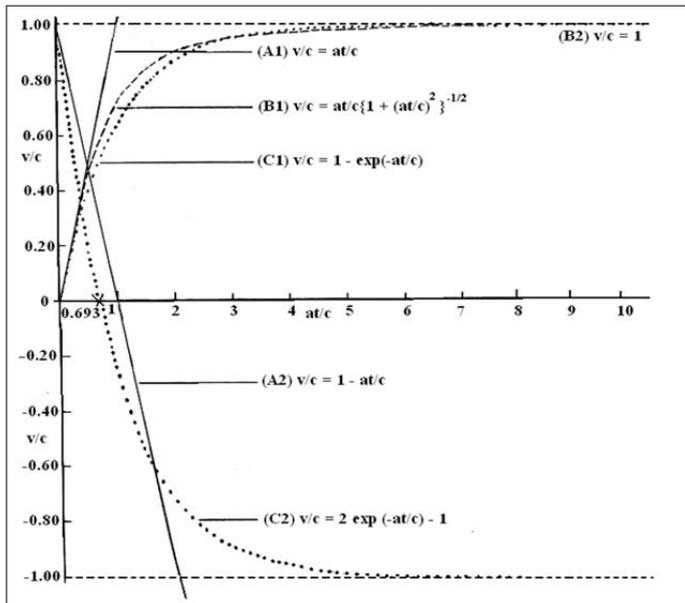
$$v = 2c \exp\left(\frac{-at}{c}\right) - c \quad (9)$$

In equation (9) the particle is decelerated to a stop in time  $t = 0.603c/a$  and then accelerated in the opposite direction to reach the speed of light  $-c$ , as the ultimate limit, with constant mass  $m$  equal to the rest mass  $m_0$ .

Figure 3 are graphs of  $v/c$  (speed in units of  $c$ ) against  $at/c$  (time in units of  $c/a$ ) for an electron of charge  $-e$  and mass  $m = m_0$  accelerated from 0 initial speed or decelerated from speed of light  $c$ , by a uniform electric field of magnitude  $E$ , where  $a =$

$eE/m$  is a constant; the lines  $A1$  and  $A2$  according to classical electrodynamics and the dashed curve  $B1$  and line  $B2$  according to relativistic electrodynamics, as presented in Table 1. The dotted curves  $C1$  and  $C2$  are according to equations (6) and (9).

Radiation makes all the difference. In (A) classical electrodynamics, potential energy lost is equal to the kinetic energy gained, there is no consideration of emission of radiation. In (B) relativistic electrodynamics there is supposed to be increase in mass of a moving particle to account for the difference between change in potential energy and change in kinetic energy. In (C), radiative electrodynamics, there is energy radiation as the difference between change in potential energy and change in kinetic energy.



**Figure 3:**  $v/c$  (speed in units of  $c$ ) against  $at/c$  (time in units of  $c/a$ ) for an electron of charge  $-e$  and mass  $m$  accelerated from 0 initial speed or decelerated from the speed of light  $c$ , by a uniform electric field of magnitude  $E$ . The the lines  $A1$  and  $A2$  are in accordance with classical electrodynamics, the dashed curve ( $B1$ ) and line ( $B2$ ) according to relativistic electrodynamics and the dotted curves  $C1$  and  $C2$  according to equations (6) and (9).

### Potential Energy in Classical Electrodynamics

Potential energy  $P$  lost by an electron in being accelerated with constant mass  $m = m_0$ , through distance  $x$  from an origin ( $x = 0$ ), to a speed  $v$  from rest, is given by the integral:

$$P = \int_0^x eE(dx) = m \int_0^v mv(dv)$$

$$P = \int_0^x eE(dx) = \frac{1}{2} mv^2$$

This is equal to the kinetic energy  $K$  gained by the electron.

$$\frac{P}{mc^2} = \frac{1}{2} \left( \frac{v}{c} \right)^2 \quad (10)$$

Potential energy  $P$  gained, equal to the kinetic energy  $K$  lost, in decelerating an electron from the speed of light  $c$  to a speed  $v$ , in a distance  $x$  in an electric field of constant magnitude  $E$ , without radiation, is:

$$-P = -\int_0^x eE(dx) = m \int_c^v v(dv) = \frac{1}{2} m(v^2 - c^2)$$

$$\frac{P}{mc^2} = \frac{1}{2} \left( 1 - \frac{v^2}{c^2} \right) \quad (11)$$

Graph of  $v/c$  against  $P/mc^2$  are  $A1$  and  $A2$ , in Figure 3 for classical equations (10) and (11).

### Potential Energy in Relativistic Electrodynamics

In relativistic electrodynamics, kinetic energy  $K$  of a particle of mass  $m$  and rest mass  $m_0$  moving with speed  $v$ , is given by the relativistic equation:

$$\int_0^x eE(dx) = K = P = mc^2 - m_0c^2$$

$$P = \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0c^2$$

$$\frac{P}{m_0c^2} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \quad (12)$$

### Potential Energy in Radiative Electrodynamics

In radiative electrodynamics, accelerating force  $F$ , in rectilinear motion, is given by:

$$F = -eE \left( 1 - \frac{v}{c} \right) \hat{u} = -m \frac{dv}{dt} \hat{u} \quad (13)$$

$$eE \left( 1 - \frac{v}{c} \right) = m \frac{dv}{dt} = mv \frac{dv}{dx} \quad (14)$$

Potential energy  $P$  lost in accelerating an electron to speed  $v$  from 0, is given by the integral:

$$P = \int_0^x eE(dx) = \int_0^v mv \frac{dv}{1 - \frac{v}{c}} \quad (15)$$

Resolving the right-hand integral into partial fractions, we obtain potential energy lost, as:

$$P = mc \int_0^v \left( \frac{1}{1 - \frac{v}{c}} - 1 \right) dv \quad (16)$$

$$P = -mc^2 \ln \left( 1 - \frac{v}{c} \right) - mcv \quad (17)$$

$$\frac{P}{mc^2} = -\ln \left( 1 - \frac{v}{c} \right) - \frac{v}{c} \quad (18)$$

Energy radiated  $R$ , difference between kinetic energy gained  $K$  and potential energy  $P$  lost, is:

$$R = P - K = -mc^2 \ln\left(1 - \frac{v}{c}\right) - mcv - \frac{1}{2}mv^2$$

$$R = -mc^2 \left\{ \ln\left(1 - \frac{v}{c}\right) + \frac{v}{c} + \frac{v^2}{2c^2} \right\} \quad (19)$$

For a decelerated electron, equations (3) and (13), with  $\theta = \pi$  radians, give:

$$\mathbf{F} = -eE\left(1 + \frac{v}{c}\right)\hat{\mathbf{u}} = m\frac{d\mathbf{v}}{dt}\hat{\mathbf{u}} \quad (20)$$

$$eE\left(1 + \frac{v}{c}\right) = -m\frac{dv}{dt} = -mv\frac{dv}{dx} \quad (21)$$

Potential energy  $P$  gained in deceleration through distance  $x$ , from speed  $c$  to  $v$ , is:

$$P = -\int_0^x eE(dx) = \int_c^v -mv\frac{(dv)}{1 + \frac{v}{c}} \quad (22)$$

Resolving the integrand into partial fractions and integrating, the potential energy gained is:

$$P = mc \int_c^v \left(1 - \frac{1}{1 - \frac{v}{c}}\right) (dv) \quad (23)$$

$$P = mc^2 \ln\left(\frac{1 + \frac{v}{c}}{2}\right) + mc^2 \left(1 - \frac{v}{c}\right) \quad (24)$$

Graphs of  $P/mc^2$  against  $v/c$  are shown as  $C1$  and  $C2$ , in Figure 3 for equations (18) and (24).

### Energy Radiated in Radiative Electrodynamics

Energy radiated  $R$ , in deceleration from the speed of light  $c$  to

speed  $v$ , is kinetic energy lost minus potential energy gained, thus:

$$R = \frac{1}{2}m(c^2 - v^2) - mc^2 \ln\left(\frac{1 + \frac{v}{c}}{2}\right) - mc^2 \left(1 - \frac{v}{c}\right)$$

$$R = -mc^2 \left\{ \frac{1}{2} + \frac{v^2}{2c^2} + \ln\left(\frac{1 + \frac{v}{c}}{2}\right) - \frac{v}{c} \right\} \quad (25)$$

Energy radiated in decelerating an electron from speed of light  $c$  to a stop is  $0.193mc^2$ . Kinetic energy lost is  $0.5mc^2$  and potential energy gained is  $0.307mc^2$ . An electron entering a retarding field with speed  $c$  is stopped and returned to the point of entry at speed  $0.594c$ .

Potential energy expressions are equations (17) and (24). Equation (17) indicates a speed limit equal to that of light  $c$ , with infinitely large loss of potential, as demonstrated by Bertozzi's experiment. Potential energy lost is supposed to be accounted for in the increase of mass with speed, in accordance with mass-velocity formula of special relativity.

### Bertozzi's Experiment

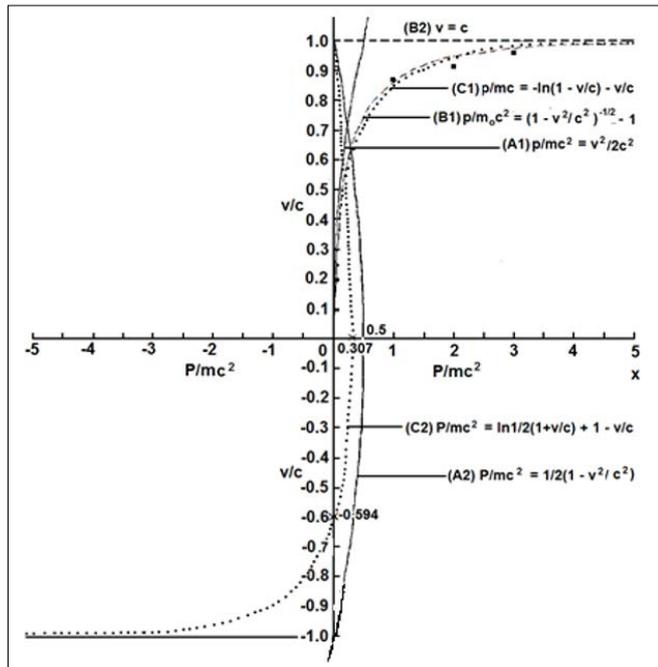
In a most remarkable experiment performed by William Bertozzi, at the Massachusetts Institute of Technology, in 1964, the speed  $v$  of high-energy electrons was determined by measuring the time  $T$  that is required for them to traverse 8.4 metres after having been accelerated through a potential energy  $P$  inside a linear accelerator [9, 10]. Bertozzi's experimental data, reproduced in Table 2, clearly demonstrated that electrons accelerated through potential energy over 15 MeV attain, for all practical purposes, the speed of light  $c$ .

A graph, with three results of Bertozzi's experiment, is shown in Figure 4 below. An experiment does not tell a lie, but interpretation of the result may be wrong. This is the case in the expression of the relativistic mass-velocity formula in a linear or cyclic accelerator, with electrons or protons moving at the speed of light.

Table 2: Results of Bertozzi's Experiments

P MeV	P/moc <sup>2</sup>	T x 10 <sup>-8</sup> sec.	V x 10 <sup>8</sup> m/sec	v/c Bertozzi's Experiment	v/c Classical Equation 4.33	v/c Relativistic Equation 4.35	v/c Radiative Equation 4.41
0.5	1*	3.23	2.60	0.87*	1.41	0.866	0.842
1.0	2*	308	2.73	0.91*	2.00	0.943	0.947
1.5	3*	2.92	2.88	0.96*	2.45	0.968	0.981
4.5	9	2.84	2.96	0.99	4.24	0.990	1.000
15.0	30	2.80	3.00	1.00	7.75	0.999	1.000

\*Inserted in Figure 4.



**Figure 4:**  $v/c$  (speed in units of  $c$ ) against  $P/mc^2$  (potential energy in units of  $mc^2$ ) for an electron of mass  $m$  accelerated from zero initial speed or decelerated from the speed of light  $c$ , the solid curves (A1 and A2) for classical electrostatics (equation 10 and 11), the dashed curve (B2) for relativistic electrostatics and the dotted curves (C1) and (C2) for equations (18) and (24). The 3 solid squares are results of Bertozzi's experiment.

An important case is an electron going around the source charge, under the attraction of a positively charged nucleus, at speed  $v$  in a circle of radius  $r$  with acceleration  $v^2/r$ . For circular motion in a radial electric field,  $\theta = \pi/2$  radians. Equations (1) and (3), with  $\cos(\theta - \alpha) = v/c$ , give:

$$-eE\sqrt{1 - \frac{v^2}{c^2}} = -\frac{m_0 v^2}{r} \quad (26)$$

$$eE = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \frac{v^2}{r} = \gamma m_0 \frac{v^2}{r} = m_v \frac{v^2}{r} \quad (27)$$

$$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_0 \quad (28)$$

where  $\gamma$  is Lorentz factor. In equation (27) it is as if the accelerating force is constant at  $-eE$ , independent of speed  $v$ , but the moving mass or relativistic mass  $m_v = \gamma m_0$ , varies as in equation (28). Equation (28) is the relativistic mass-velocity formula, a mathematical inversion of equation (26). Equation (28) is mathematically correct for circular motion only, but physically wrong in circular and rectilinear motions.

### Emission of Radiation

There must be something that prevents a charged particle from attaining a speed beyond the speed of light. It is radiation that makes all the difference. A charged particle is accelerated, by an

electric field, to the speed of light, and no faster, at constant mass, as the rest mass, and with emission of radiation. At the speed of light  $c$ , the radiation reaction force becomes equal and opposite to the accelerating force. With no force on it at the speed  $c$ , the electron moves, in time  $t$ , in the opposite direction of electric field of magnitude  $E$ , with constant mass, at constant speed  $c$ , radiating energy,  $eEct$ , equal to the potential energy lost.

From equation (3), it can be inferred that the accelerating force on a moving electron, is less than the electrostatic force  $-eE$  on the electron of charge  $-e$  when stationary in the field of intensity  $E$  and magnitude  $E$ . The difference is radiation reaction force, a vector  $\mathbf{R}_p$  thus:

$$\mathbf{R}_f = \mathbf{F} - (-e\mathbf{E}) = e\mathbf{E} + \frac{eE}{c}(\mathbf{c} - \mathbf{v}) \quad (29)$$

Radiation power, scalar product  $-\mathbf{v} \cdot \mathbf{R}_p$  with reference to angles  $\theta$  and  $\alpha$  of Figure 1, is:

$$-\mathbf{v} \cdot \mathbf{R}_f = -\mathbf{v} \cdot \left\{ e\mathbf{E} + \frac{eE}{c}(\mathbf{c} - \mathbf{v}) \right\} = eEv \left\{ \cos\theta - \cos(\theta - \alpha) + \frac{v}{c} \right\} \quad (30)$$

In equation (30), radiation power is  $eEv^2/c$  for  $\theta = 0$  or  $\pi$  radians. There is no radiation if  $\theta = \pi/2$  radians. This makes circular motion of an electron round a positively charged nucleus, without radiation and inherently stable.

### Results and Discussion

- Speed of light  $c$ , in a vacuum, is the most measured and most accurately known quantity in the universe.
- Ignoring James Bradley's discovery of aberration of light, which showed the relativity of speed of light, has been an expensive mistake in physics.
- Light speed constancy, for all observers, in special relativity, is only possible if speed of light  $c$  were infinitely large.
- Infinite mass at ultimate speed leads to zero acceleration and constant speed of light  $c$ . Zero force at ultimate speed also leads to zero acceleration and constant speed of light  $c$ . It is unfortunate that special relativity chose mass, rather than force, as the variable.
- Figure 2, equations (6) & (9), show that the speed of light,  $c$  or  $-c$ , is an ultimate limit.
- The ubiquitous Lorentz factor,  $\gamma$  in equation (28), has nothing to do with mass, but a result of motion of a charged particle perpendicular to an electric field.
- Relativistic electrostatics has no formula for radiation reaction force, but there must be radiation. So, a modified Lamor classical equation for radiation power (proportional to square of acceleration multiplied by  $g^4$ ) was adopted.
- The correctness of equation (12) for circular motion, may explain the apparent
- agreement of special relativity with observations on cyclic particle accelerators.
- Equation (14) shows that circular motion ( $\theta = \pi/2$ ) as in Rutherford's nuclear model of hydrogen atom, is without radiation. No need for Bohr's quantum theory to stabilise it.
- Equation (25), for radiation from an electron decelerated from the speed of light, is not obtainable from the point of view of classical and relativistic electrostatics
- Equation (28), the relativistic mass-velocity formula, is correct for circular motion, but as a result of accelerating force decreasing with speed, no mass increasing with speed.

- Circular motion is always with constant kinetic energy and constant potential energy, and. Therefore, no radiation. An electron dislodged from a circular orbit, revolves in an elliptic path, emitting radiation, at the frequency of revolution, before reverting to a circular orbit.
- The small spread in frequency as an excited electron revolves and reverts to a circular orbit, explains *fine structure* of emission of radiation from the atom of hydrogen gas.

### Conclusion

The paper has succeeded in introducing radiative electrodynamics and showing that a charged particle, like an electron, is accelerated at constant mass, by an electric field, to the ultimate speed of light  $c$ , with emission of radiation as the difference between change in potential energy and change in kinetic energy.

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