An Optimal Inventory Policy for Subsequent Price Reduction Problem

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This article considers an optimal inventory policy of the retailer in response to a special price strategy on subsequent orders by the supplier. A finite horizon model with constant yearly demand is considered in this article. The closed form solution for optimal order quantities and an optimal number of orders is obtained by minimizing the total cost. Numerical and sensitivity analysis are carried out to support the theoretical results. A significant percentage difference in cost between the developed optimal policy and the EOQ policy highlights the importance of the developed optimal policy.

Keywords: Inventory, Two-Purchase Price, Three-Purchase Price, Optimal Number of Orders, Optimal Order Quantity, Optimal Cost.

1. Introduction and Literature Review

A supplier cannot build a successful business when the retailer orders items once and never orders again. For a supplier, it is important that a retailer frequently purchases from him. Hence, for the supplier, it is necessary to design a pricing strategy, which encourages the retailer to continue ordering from him. One such pricing strategy is to reasonably reduce the purchase price of the items after the first purchase. Once a new retailer purchases items with regular price, the supplier offers a price reduction from next purchase onwards in order to retain the retailer. This kind of reward to a frequent buyer is common in today's business through the loyalty programme. A loyalty programme is a strategy designed for buyers to continue to purchase items. As an example of this is the Target RED Card, which rewards its buyers 5% reduction in purchase price every time they purchase. Big Lots offers \$5 off for subsequent purchase when the buyer joins the Big Rewards customer loyalty programme. By implementing this kind of pricing strategy, a supplier attracts more retailers and hence, can generate more revenue. On the other hand, the retailer gets the benefit of the reduced purchase price.

The present paper considers the ordering policy of a retailer when the supplier reduces the purchase price of the items after the first purchase. This model is extended in which the supplier further gives discount third order onwards. These discounts are already known in advance. The yearly demand of the customers is assumed to be constant and known to the retailer. In such a scenario, the retailer has to decide how much to order, so that the total cost of the inventory is minimum. In the first model, the pricing strategy of the supplier is as follows:

For the first purchase, the purchase price of the item is P . Let Pa , ($0 < a < 1$), be the purchase price of the item for the second purchase onwards.

Corresponding to this pricing strategy, the retailer has to develop an optimal ordering policy, which is the objective of the present article. This kind of pricing strategy not only motivates the retailer to buy next time, but also attracts new retailers. This pricing strategy is a two-price inventory problem. Many researchers have shown interest in price change problems. Most of the models assume one price over an interval of time and reduced price in another interval of duration. In these durations, there may be multiple orders, unlike the situation discussed here. The price change inventory problem was first studied by Naddor (1966), who proposed the infinite horizon EOQ model with the assumption that price change occurs at the end of an EOQ cycle. Goyal (1975) studied the EOQ model when the price changes followed a straight-line rule. The optimal ordering policy when the price changes continuously due to inflation was studied by Buzacott (1975) and Erel (1992). Lev and Soyster (1979) developed a finite horizon inventory model and determined the optimal ordering policies when information about an ensuing price rise was known. An improved algorithm for determining the optimal policy for the model of Lev and Soyster (1979) was provided by Goyal (1980). Taylor and Bradley (1985) extended Naddor's (1966) model and found optimal ordering strategies for situations where the price increase did not coincide with the end of the EOQ cycle. Lev and Wiess (1990) also developed a comprehensive model that covered both finite and infinite horizons under purchase price increase and decrease. Their algorithm for solving the finite horizon EOQ model with price change was modified by Andre Gascon (1995). Ghosh (2003) and Huang and Kulkarni (2003) discussed the infinite horizon deterministic inventory model under an announced price increase. Khouja and Park (2003) developed a closed form approximate solution for an inventory model with continuous decrease in purchase price. They made a restrictive assumption of the equal cycle time. Their model was improved by Khouja and Goyal (2006) by considering cycle times of varying lengths. Arcelus et al. (2006) modelled the retailer's response to special sales characterized by uncertain termination dates, but with the ending date known at the time of its realization. An EOQ model with known price increase and partial back-ordering was developed by Taleizadeh and Pentico (2013). Pakkala and Babitha (2013) developed an optimal inventory policy for the two-price problem for a situation in which the discount price is announced and will remain for a random duration.

Even though there are many articles on price change problem, as far as is known no attempt has been made to develop a model for subsequent price reduction by the supplier. The solution discussed in the earlier literature of price change is different as the price change occurs after a certain point of time, which may be known or uncertain, and hence, the price change is time dependent. But the present article considers the price change problem, where the price change occurs after the first purchase, and after the second purchase, and hence, the discount is not directly time dependent. This leads to the derivation of a new solution procedure for the problem mentioned here.

The paper is organized as follows. Assumptions and notations used in the article is considered in Section 2. Section 3 presents the model development. In Section 4, the two-price model is extended to three-price problem. Numerical and sensitivity analysis is considered in Section 5. Section 6 presents the comparison of the optimal policy with the EOQ policy. Section 7 concludes the paper.

2. Assumptions and Notation

First, the two- price problem is discussed, that is, the situation of price discount is offered after the first order. The following assumptions and notations are used in the article.

Assumptions

1. Planning horizon is finite and known;
2. Demand Is Constant and Uniform Over the Planning Horizon;
3. Shortages are not allowed;
4. Initial inventory is zero; and
5. Lead time is zero.

Notation

1. P is the actual purchase price of the item,
2. T is the planning horizon (in number of years),
3. α is the discount rate on the purchase price for first purchase onwards of the retailer,
4. D is the demand for one year,
5. K is the ordering cost,
6. F is the carrying cost fraction per unit amount of money per year,
7. m is the number of orders after the first purchase, and
8. n is the total number of orders throughout the planning horizon.

3. Model Development

Let Q' be the order quantity corresponding to the first purchase. Let Q be the quantity of items ordered by the retailer from the first purchase onwards. Let $t_1, t_2, t_3, \dots, t_n$ be the time points at which n orders are placed. The inventory position is represented in Figure 1.

Figure 1 Inventory Position

Since the total number of orders is $m+1$ with the order cost of K per order, the total set-up cost is $(m+1)K$. The total items' cost over the planning horizon T is $Q'P + mQP\alpha$, since Q' and Q (m times) items are ordered at price P and $P\alpha$, respectively. The average inventory for the first order cycle is $Q'/2$, which is kept for the time of Q'/D , hence, the holding cost for this cycle is $\frac{Q'^2 PF}{2D}$. Similarly, the holding cost for the remaining order cycles is given by $\frac{mQ^2 P\alpha F}{2D}$. The different costs of the model are listed in the following Table.

Table 1 Various costs of the Inventory Model

Type of the cost	Cost
Total set-up cost	$(m + 1)K$
Items' cost	$Q'P + mQP\alpha$
Holding cost	$\frac{Q'^2 PF}{2D} + \frac{mQ^2 P\alpha F}{2D}$

Hence, the total cost is given by

$$TC = (m + 1)K + Q'P + mQP\alpha + \frac{Q'^2 PF}{2D} + \frac{mQ^2 P\alpha F}{2D} \quad (1)$$

Since the yearly demand is D , we have $Q' + mQ = TD$. Hence, Q' can be written as $Q' = TD - mQ$.

Therefore, the total cost can be written as function $C_m()$ of a single variable Q .

$$C_m(Q) = (m + 1)K + (TD - mQ)P + mQP\alpha + \frac{(TD - mQ)^2 PF}{2D} + \frac{mQ^2 P\alpha F}{2D} \quad (2)$$

The optimal value of order quantity Q for a given value of m is derived in the following theorem.

Theorem 1

The optimal order quantity corresponding to purchase price $P\alpha$ for a given value of m is given by

$$Q_m^* = \frac{D}{F} \left(\frac{1 - \alpha + TF}{m + \alpha} \right) \quad (3)$$

Proof

$$\text{We have } C_m(Q) = (m + 1)K + (TD - mQ)P + mQP\alpha + \frac{(TD - mQ)^2 PF}{2D} + \frac{mQ^2 P\alpha F}{2D}$$

$$C_m(Q) = (m + 1)K + TDP + \frac{PFT^2 D}{2} - mQP(1 - \alpha + FT) + \frac{mQ^2 PF(m + \alpha)}{2D}$$

After equating the first derivative of C_m with respect to Q to zero yields

$$\frac{dC_m}{dQ} = -mP(1 - \alpha + FT) + \frac{mQPF(m + \alpha)}{D} = 0,$$

$$\text{which implies that } Q_m^* = \frac{D}{F} \left(\frac{1 - \alpha + TF}{m + \alpha} \right).$$

It can be observed that the optimal order quantity depends on the number of orders m , which is also a decision variable. After the substitution of Q_m^* in (2), the optimal cost for a given m , denoted by $C^*(m)$, can be found and is given by

$$C^*(m) = (m+1)K + TDP + \frac{PFT^2D}{2} - \frac{mPFL^2}{2D(m+\alpha)} \quad (4)$$

Where $L = \frac{D}{F}(1 - \alpha + TF)$.

It can be noted that $C^*(m)$ is independent of optimal Q' and optimal Q . This is because optimal Q depends on m and other parameters, and Q' is $TD-mQ$ expressed in terms of m and parameters only. The optimal order quantity corresponding to purchase price Pa and the optimal cost of the model can be found by obtaining optimal number of orders after the first purchase, denoted by m^* , which minimizes (4). Hence, the determination of the optimal number of orders requires special attention. The derivation of the optimal number of orders is based on the fact that function $C^*(\cdot)$ is a convex function of m , which is proved in the following theorem.

Theorem 2

Function C^* is a discrete convex function of m .

Proof

The function C^* is the convex function of m , if $C^*(m-1) + C^*(m+1) > 2C^*(m)$, for all $m \geq 1$. (5)

Let $m (\geq 1)$ be an arbitrary number. Assume $C^*(m-1) + C^*(m+1) - 2C^*(m) > 0$.

Which implies, $-\frac{(m-1)}{(m-1+\alpha)} - \frac{(m+1)}{(m+1+\alpha)} + \frac{2m}{(m+\alpha)} > 0$.

After simplification, the above inequality yields $\alpha > 0$, which is always true. Hence, (5) holds for all $m \geq 1$.

3.1 Determination of Optimal Number of orders after the first order

The optimal number of orders after the first purchase can be found by the search method. But if it is very large, it may be difficult to find the optimal m through the search procedure. In the following theorem, the closed form solution to the optimal number of orders after the purchase is derived

Theorem 3

The optimal number of orders after the first purchase, denoted by m^* , is given by

$$m^* = \left\lceil \frac{1}{2} + \sqrt{\frac{PFL^2\alpha}{2DK} + \frac{1}{4}} - \alpha \right\rceil. \quad (6)$$

Where, $[x]$ is the largest integer less than or equal to x .

Proof

Function C^* can be written as $C^*(m) = (m+1)K + C_0 - \frac{mC'}{(m+\alpha)}$, where $C_0 = TDP + \frac{PFT^2D}{2}$, $C' = \frac{PFL^2}{2D}$.

Since C^* is a convex function of m , there exists m^* such that $C^*(m^*) \leq C^*(m^* - 1)$ and $C^*(m^*) \leq C^*(m^* + 1)$.

This m^* will be the optimal number of orders after the first purchase. It can also be seen that $m^* = \max\{m \geq 1: C^*(m) \leq C^*(m - 1)\}$.

Consider $C^*(m) \leq C^*(m - 1)$

$$\Rightarrow (m + 1)K + C_0 - \frac{mC'}{(m + \alpha)} \leq mK + C_0 - \frac{(m - 1)C'}{(m - 1 + \alpha)}$$

$$\Rightarrow m'(m' - 1) \leq \frac{C'\alpha}{K}, \text{ where } m' = (m + \alpha)$$

$$\Rightarrow m'^2 - m' - \frac{C'\alpha}{K} \leq 0.$$

First let us find the value of m (need not be an integer), which satisfies $m'^2 - m' - \frac{C'\alpha}{K} = 0$.

The feasible solution of m is $m = \frac{1}{2} + \sqrt{\frac{PFL^2\alpha}{2DK} + \frac{1}{4}} - \alpha$. Hence, for

$m = \frac{1}{2} + \sqrt{\frac{PFL^2\alpha}{2DK} + \frac{1}{4}} - \alpha$ we have $m'^2 - m' - \frac{C'\alpha}{K} = 0$. Therefore, for

$$m = \left[\frac{1}{2} + \sqrt{\frac{PFL^2\alpha}{2DK} + \frac{1}{4}} - \alpha \right] \leq \frac{1}{2} + \sqrt{\frac{PFL^2\alpha}{2DK} + \frac{1}{4}}, \text{ we have } (m + \alpha)^2 - (m + \alpha) - \frac{C'\alpha}{K} \leq 0.$$

It can be observed that $m = \left[\frac{1}{2} + \sqrt{\frac{PFL^2\alpha}{2DK} + \frac{1}{4}} - \alpha \right]$ is maximum among the integers, which satisfies $m'^2 - m' - \frac{C'\alpha}{K} \leq 0$. Hence, the optimal number of orders

$$m^* = \left[\frac{1}{2} + \sqrt{\frac{PFL^2\alpha}{2DK} + \frac{1}{4}} - \alpha \right].$$

It may be noted that for a single price in a finite horizon problem, the closed form solution to the number of orders is given by Lev and Weiss (1990) and Arcelus et al. (2015).

The algorithm to obtain the optimal number of orders, optimal order quantities, and optimal cost when $m \geq 1$ is given below, however, $m=0$ could be optimal in rare cases, especially when the finite horizon is small. In such cases, the cost for a single order for the entire planning horizon with price P is also computed and compared to determine optimality.

3.2 Algorithm to Obtain Optimal Number of Orders, Optimal Order Quantities, and Optimal Cost

The following steps are used to obtain the optimal number of orders, optimal order quantities and optimal cost.

Step 1

Compute $m^* = \left\lceil \frac{1}{2} + \sqrt{\frac{PFL^2\alpha}{2DK} + \frac{1}{4}} - \alpha \right\rceil$. This will be the optimal number of orders after the first purchase.

Step 2

Compute $\frac{D}{F} \left(\frac{1-\alpha+TF}{m^*+\alpha} \right)$, this gives the optimal quantity, Q^* to be ordered after the first purchase.

Step 3

Obtain an optimal order quantity corresponding to first purchase Q'^* using the formula $Q'^* = TD - m^*Q^*$.

Step 4

Optimal cost can be obtained by substituting $m^* = \left\lceil \frac{1}{2} + \sqrt{\frac{PFL^2\alpha}{2DK} + \frac{1}{4}} - \alpha \right\rceil$ in (4).

4. Extension of the Model to Three Prices

The above two- price model can be extended to the three- price problem. One of the pricing strategies, which leads to the three- price inventory problem is considered below.

For the first purchase, the purchase price is P . Let $P\alpha_1$, ($0 < \alpha_1 < 1$), be the purchase price of the item for the second purchase. For 2nd purchase onwards, let $P\alpha_2$, ($0 < \alpha_2 < 1$, $\alpha_2 < \alpha_1$) be the purchase price. Let Q_1 and Q_2 be the order quantity corresponding to the first and second purchase. Let Q_3 be the quantity of items ordered by the retailer from the 2nd purchase onwards. Let m be the number of orders after the second purchase (note that for two- price problem, m is the number of orders after the first purchase). The total cost for the planning horizon T is given by

$$C_m(Q_1, Q_2, Q_3) = (m+2)K + Q_1P + Q_2P\alpha_1 + mQ_3P\alpha_2 + \frac{Q_1^2PF}{2D} + \frac{Q_2^2P\alpha_1F}{2D} + \frac{mQ_3^2P\alpha_2F}{2D} \quad (7)$$

Since the yearly demand is D , we have $Q_1 + Q_2 + mQ_3 = TD$.

The optimal values of Q_1, Q_2 , and Q_3 for a given value of m is derived in the following theorem.

Theorem 4

The optimal order quantities for a given value of m is given by:

$$Q_1^* = \frac{D}{F} \left(\left(\frac{TF+m+2}{1+\frac{1}{\alpha_1}+\frac{m}{\alpha_2}} \right) - 1 \right), Q_2^* = \frac{D}{F} \left(\left(\frac{TF+m+2}{\alpha_1(1+\frac{1}{\alpha_1}+\frac{m}{\alpha_2})} \right) - 1 \right) \text{ and}$$

$$Q_3^* = \frac{D}{F} \left(\left(\frac{TF+m+2}{\alpha_2 \left(1 + \frac{1}{\alpha_1} + \frac{m}{\alpha_2} \right)} \right) - 1 \right)$$

Proof

We use Lagrange's multiplier technique to minimize $C_m(Q_1, Q_2, Q_3)$ subject to the constraint $Q_1 + Q_2 + mQ_3 = TD$.

Lagrange's multiplier function is given by

$$L_m(Q_1, Q_2, Q_3) = (m+2)K + Q_1P + Q_2P\alpha_1 + mQ_3P\alpha_2 + \frac{Q_1^{2PF}}{2D} + \frac{Q_2^{2P\alpha_1F}}{2D} + m\frac{Q_3^{2P\alpha_2F}}{2D} - \lambda(g_m(Q_1, Q_2, Q_3) - TD) \quad (8)$$

Where, $g_m(Q_1, Q_2, Q_3) = Q_1 + Q_2 + mQ_3$ and λ is Lagrange's multiplier.

Equating the partial derivative of Lagrange's function with respect to each variable and Lagrange's multiplier to zero yields the following set of equations

$$\frac{\partial L_m}{\partial Q_1} = P + \frac{Q_1^{PF}}{D} - \lambda = 0 \quad (9)$$

$$\frac{\partial L_m}{\partial Q_2} = P\alpha_1 + \frac{Q_2^{P\alpha_1F}}{D} - \lambda = 0 \quad (10)$$

$$\frac{\partial L_m}{\partial Q_3} = mP\alpha_2 + \frac{mQ_3^{P\alpha_2F}}{D} - m\lambda = 0 \quad (11)$$

$$Q_1 + Q_2 + mQ_3 - TD = 0 \quad (12)$$

Solving the first three equations yields

$$Q_1 = \frac{D}{PF}(\lambda - P), Q_2 = \frac{D}{P\alpha_1F}(\lambda - P\alpha_1), \text{ and } Q_3 = \frac{D}{P\alpha_2F}(\lambda - P\alpha_2).$$

Substituting these values in (12), we get $\lambda = \left(\frac{TF+m+2}{1 + \frac{1}{\alpha_1} + \frac{m}{\alpha_2}} \right) P$.

Hence, we have $Q_1 = \frac{D}{F} \left(\left(\frac{TF+m+2}{1 + \frac{1}{\alpha_1} + \frac{m}{\alpha_2}} \right) - 1 \right)$, $Q_2 = \frac{D}{F} \left(\left(\frac{TF+m+2}{\alpha_1 \left(1 + \frac{1}{\alpha_1} + \frac{m}{\alpha_2} \right)} \right) - 1 \right)$ and

$$Q_3 = \frac{D}{F} \left(\left(\frac{TF+m+2}{\alpha_2 \left(1 + \frac{1}{\alpha_1} + \frac{m}{\alpha_2} \right)} \right) - 1 \right)$$

In order to prove that the cost function $C_m(\cdot)$ attains its minimum at these stationary points, one needs to prove that the determinant of the bordered Hessian matrix at these stationary points is negative. The bordered Hessian matrix, denoted by H , is given by

$$H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & \alpha_1 \end{bmatrix}.$$

The determinant of this matrix is $-1-\alpha_1$ which is always negative.

$$\text{Hence, } Q_1 = \frac{D}{F} \left(\left(\frac{TF+m+2}{1+\frac{1}{\alpha_1}+\frac{m}{\alpha_2}} \right) - 1 \right), Q_2 = \frac{D}{F} \left(\left(\frac{TF+m+2}{\alpha_1(1+\frac{1}{\alpha_1}+\frac{m}{\alpha_2})} \right) - 1 \right) \text{ and}$$

$$Q_3 = \frac{D}{F} \left(\left(\frac{TF+m+2}{\alpha_2(1+\frac{1}{\alpha_1}+\frac{m}{\alpha_2})} \right) - 1 \right) \quad (13)$$

will minimize the cost function $C_m(Q_1, Q_2, Q_3)$ for a given value of m . After the substitution of optimal values of Q_1, Q_2 , and Q_3 , the cost function becomes the function of the single variable m . This function is denoted by C^* , which is given by

$$C^*(m) = (m+2)K + \frac{PD}{F} \phi_1(m) + \frac{PD}{2F} \phi_2(m) \quad (14)$$

Where $\phi_1(m) = \Psi(m)(m+2) - (1 + \alpha_1 + m\alpha_2)$,

$$\phi_2(m) = (\Psi(m) - 1)^2 + \frac{(\Psi(m) - \alpha_1)^2}{\alpha_1} + m \frac{(\Psi(m) - \alpha_2)^2}{\alpha_2}; \Psi(m) = \frac{TF+m+2}{1+\frac{1}{\alpha_1}+\frac{m}{\alpha_2}}.$$

It can be proved that C^* is a discrete convex function of m (the proof is similar to that of the two-price case). It is difficult to find the closed form solution for optimal m value, since it involves a complicated expression. We use the following algorithm to find the optimal number of orders after the second purchase. Once the optimal number of orders is found, the optimal order quantities and the optimal cost can be found. The optimal number of orders $n = \left\lceil \sqrt{\frac{DPFT^2\alpha}{2K}} + \frac{1}{4} - \frac{1}{2} \right\rceil$ for single price (finite horizon) case is considered as an initial value for $n(=m+2)$. Where, $\lceil \cdot \rceil$ indicates upper integer part.

Hence, the initial value for m will be $m = \left\lceil \sqrt{\frac{DPFT^2\alpha}{2K}} + \frac{1}{4} - \frac{1}{2} \right\rceil - 2$. This may not be optimal m , but close to it. When the planning horizon is small, we need to compare the optimal cost corresponding to a single order with price P alone for the entire planning horizon and two-order with prices P and $P\alpha_1$ for optimality. In both these cases, $m=0$.

4.1 Algorithm to Find Optimal Number of Orders

Step 1

Compute $m = \left\lceil \sqrt{\frac{DPFT^2\alpha}{2K}} + \frac{1}{4} - \frac{1}{2} \right\rceil - 2$. Check whether this m satisfies the condition

$$C^*(m-1) > C^*(m) \text{ and } C^*(m) < C^*(m+1). \quad (15)$$

If it satisfies condition (15), then the optimal number of orders after the second purchase will be $\left\lceil \sqrt{\frac{DPFT^2\alpha}{2K} + \frac{1}{4}} - \frac{1}{2} \right\rceil - 2$. If this m does not satisfy condition (15), then it will satisfy either one of the following conditions

$$1. \quad C^*(m+1) < C^*(m) < C^*(m-1) \quad (16)$$

$$2. \quad C^*(m-1) < C^*(m) < C^*(m+1) \quad (17)$$

Step 2

Case 1

If (16) is satisfied by m in Step 1, then this m will be less than the optimal m . Hence, the m value of Step 1 has to be increased by 1 and has to verify condition (15) again for $m+1$. This process will continue until we arrive at optimal m , which satisfies (15). This process will terminate quickly.

Case 2

If (17) is satisfied by m in Step 1, then this m will be greater than optimal m . Hence, the m value of Step 1 has to be decreased by 1 and has to verify condition (15) again for $m-1$. This process will continue until we arrive at optimal m , which satisfies (15).

Step 3

Once the optimal value of m (denoted by m^*) is determined, the optimal order quantities can be found by substituting this optimal value of m in (13). By substituting the optimal order quantities in (7), the optimal cost can be found. When $\alpha_1=1$ and $\alpha_1=\alpha_2$, the three-price problem reduces to the two-price problem.

5. Numerical and Sensitivity Analysis

To support the theoretical results, a numerical illustration was considered. Suppose a retailer orders items from a supplier with the purchase price for the 1st purchase as $P=$ Rs. 100. The set-up cost per order is taken as Rs. 1200. In order to make a new retailer to buy again, the supplier reduces the purchase price by 2% ($\alpha_1=0.98$) for the second purchase, and by 5% from the second purchase onwards ($\alpha_2=0.95$). The yearly demand and planning horizon is considered to be 1500 items and 5 years, respectively. The carrying cost fraction per year per Rs is assumed to be 0.4. These parameters are said to be the basic parameters.

The optimal number of orders is found to be $n^*=25$ and hence, $m^*=23$. According to the optimal policy, quantity to be ordered for the first purchase and the second purchase is $Q_1^* = 109.9464$ units and $Q_2^* = 188.7208$ units. Then, from the second purchase onwards, the retailer has to order $Q_3^* = 313.1014$ units, which results in minimum cost of Rs. 772802.6 for the planning horizon of 5 years. For this set of parameters, the convex property of C^* with respect to the number of orders m is displayed in Figure 2.

Figure 2 Total cost vs. Number of Orders after Second Purchase

Table 2 and Table 3 show the sensitivity analysis for the three-price and two-price problems, respectively. The first two columns represent the parameters and its value. The parameter values are changed by -40%, -20%, 20%, and 40% with respect to the basic parameters. The basic parameter for the two- price problem is $(K, F, D, T, P, \alpha) = (1200, 0.4, 1500, 5, 100, 0.95)$. Column3 of Table 2 represents the optimal number of orders after the second order. Similarly, Column 3 of Table 3 represents the optimal number of orders after the first order. The optimal order quantities are considered in Columns4, 5, and 6 of Table 2 for the three-price problem, and in Columns 4 and 5 of Table 3 for the two-price problem. Column 7 of Table 2 and Column 6of Table 3 give the optimal cost of model C^* for the three-price and two-price problems, respectively. The last column of both the Tables gives the percentage change in the optimal cost relative to the optimal cost corresponding to the base parameter. The following observations can be drawn from Table 2 and Table 3.

1. The retailer thinks of ordering less quantity during the first purchase since he can buy more items at a lower price after the first purchase. But when the ordering cost increases, the retailer has to increase the order quantity of the first order, which can be observed from the Table.
2. As expected, the number of orders decreases with the increase in set-up cost.
3. The optimal number of orders increases with the carrying cost fraction. The reason for this is that it is better to have less inventory when the holding cost is more. Hence, the optimal order quantity for each order is less. But in such a scenario, the retailer must order a greater number of times since the total order quantity for the planning horizon is fixed.
4. The purchase price and demand will have the same impact on optimal cost.

Table 2 Sensitivity Analysis for Three-Price Problem

Parameters	Parameter value	m^*	Q_1^*	Q_2^*	Q_3^*	$C^*(Rs.)$	Percentage Difference
D	900	18	110.7653	158.9442	235.0161	474341.9	-38.6206
	1200	21	108.7218	172.1651	272.3387	624002.2	-19.2546
	1800	26	93.58211	187.3287	335.3496	920973.2	19.1731
	2100	28	84.33375	193.1977	365.0882	1068684	38.2868
K	720	31	37.66157	114.9608	237.0122	759042.8	-1.7805
	960	26	77.98509	156.1072	279.458	766357.6	-0.8339
	1440	21	135.9022	215.2064	340.4234	778622.8	0.7531
	1680	20	150.6564	230.2616	355.9541	783997.7	1.4486

<i>F</i>	0.24	18	69.22033	198.184	401.8109	759873.5	-1.6730
	0.32	21	92.15974	189.7038	343.7208	766737.7	-0.7848
	0.48	26	107.5207	173.4906	277.6534	778281.5	0.7089
	0.56	28	111.0674	167.9988	257.8905	783329.1	1.3621
<i>P</i>	60	18	184.6089	264.907	391.6936	474341.9	-38.6206
	80	21	135.9022	215.2064	340.4234	624002.2	-19.2546
	120	26	77.98509	156.1072	279.458	920973.2	19.1731
	140	28	60.2384	137.9984	260.7773	1068684	38.2868
<i>T</i>	3	14	99.08019	177.6328	301.6634	464409	-39.9057
	4	19	98.7025	177.2475	301.2658	618608.8	-19.9525
	6	28	107.8664	186.5983	310.912	926993.5	19.9522
	7	33	106.382	185.0837	309.3495	1081187	39.9047

Table 3 Sensitivity Analysis for Two-Price

Parameters	Parameter Value	<i>m</i>	<i>Q</i> *	<i>Q</i> *	<i>C</i> *(Rs.)	Percentage Difference
<i>D</i>	900	18	118.7335	243.4037	473720.2	-38.6400
	1200	21	116.1731	280.1822	623307.7	-19.2642
	1800	26	100.1855	342.3006	920169.0	19.1876
	2100	29	78.8815	359.3489	1067835.0	38.3145
<i>K</i>	720	31	41.0798	240.6103	758500.7	-1.7529
	960	27	73.7925	275.0447	765693.7	-0.8212
	1440	22	130.7190	334.9673	777768.7	0.7428
	1680	20	161.0979	366.9451	783042.5	1.4259
<i>F</i>	0.24	19	59.5238	391.604	758970.2	-1.6921
	0.32	22	85.7843	337.0098	765927.2	-0.7910
	0.48	26	113.6364	284.0909	777570.5	0.7171
	0.56	28	116.5803	263.6936	782662.7	1.3767
<i>P</i>	60	18	197.8892	405.6728	473720.2	-38.6400
	80	21	145.2164	350.2278	623307.7	-19.2642
	120	26	83.4879	285.2505	920169.0	19.1876
	140	29	56.3439	256.6778	1067835.0	38.3145
<i>T</i>	3	14	110.3679	313.5452	463648.0	-39.9446
	4	19	107.1429	310.1504	617839.3	-19.9725
	6	29	103.9232	306.7613	926230.6	19.9727
	7	34	103.0043	305.7940	1080428.0	39.9456

6. Comparison with EOQ Policy

The optimal policy obtained above yields lower cost compared with any other policy. A business manager may use the EOQ policy because it is easy to implement. However, there will be significant loss, if the optimal policy is not implemented. This significant difference in the case of the three-price problem is displayed in Table 4. The parameters considered are K =Rs.2400, F =0.2, α_1 =0.98, α_2 =0.95, P =Rs.100, and D =1000. The cost associated with the EOQ policy and optimal cost is represented in Column 2 and Column 3 of the Table. The planning horizon is considered in the first column of the Table. The percentage difference is considered in the last column. In the EOQ policy, the economic order quantity will be ordered as many times as possible, and any remaining demand is met by the remainder order. The economic order quantity

for the first order, second order, and after the second order is given by $Q_1 = \sqrt{\frac{2KD}{PF}}$, $Q_2 = \sqrt{\frac{2KD}{P\alpha_1 F}}$ and $Q_3 = \sqrt{\frac{2KD}{P\alpha_2 F}}$. It can be observed that since the holding cost is proportional to the purchase cost, the economic order quantity for the first order will be different from the next orders. From Table 4, it can be observed that as the planning horizon increases, the percentage difference in the cost of the optimal policy and the EOQ policy decreases. This implies that the optimal policy performs better than the EOQ policy for a smaller horizon compared with a larger one.

Table 4 Comparison of Optimal Policy with the EOQ Policy

T	EOQ Cost (Rs.)	Optimal cost (Rs.)	Percentage difference in cost
1	110912.5	107751.4	2.9337
1.5	163205.9	160023.7	1.9886
2	215499.4	212297.1	1.5084
2.5	267793.0	264571.1	1.2178
3	320086.7	316845.3	1.0230
3.5	372380.4	369119.8	0.8833
4	424674.2	421394.4	0.7783
4.5	476968.1	473669.0	0.6965
5	529262.1	525943.7	0.6309

7. Conclusion and Suggestion for Future Research Work

The present article considers an optimal inventory policy of the retailer in response to a special price strategy on subsequent orders by the supplier. The demand of the customers is assumed to be constant and known to the retailer. A closed form solution to the optimal number of orders and optimal order quantities are developed for the two-price problem by minimizing the total cost of the model. The two-price model is extended to the three-price problem. In order to highlight the importance of the derived optimal policy, a comparison is made with the EOQ policy. It is found that a large percentage difference exists for a smaller planning horizon. Any other similar pricing strategy of the supplier can be considered for further research work. One such similar pricing strategy is the discount only for the first purchase.

8. References

1. Arcelus, F.J., Pakkala, T.P.M., and Srinivasan, G. (2006). "On the interaction between retailer's inventory policies and manufacturer trade deals in response to supply-uncertainty occurrences". *Annals of Operations Research*, Vol. 143, pp. 45–58.
2. Arcelus, F.J., Pakkala, T.P.M., and Srinivasan, G. (2015). "Inventory replenishment for profit maximization over a finite horizon". *International Journal of Operations and Quantitative Management*, Vol. 21, pp. 127-139.

3. Buzacott, J. A. (1975). "Economic order quantities with inflation". *Operational Research Quarterly*, Vol. 26, No. 3, pp. 553-558.
4. Erel, E. (1992). "The effect of continuous price change in the EOQ". *Omega*, Vol. 21, pp. 593-594.
5. Gascon, A. (1995). "On the Finite Horizon EOQ Model with cost changes". *Operations Research*, Vol. 43(No. 4), pp. 716-717.
6. Ghosh, A.K. (2003). "On some inventory models involving shortages under an announced". *International Journal of Systems Science*, Vol. 34, No. 2, pp. 129-137.
7. Goyal, S. K. (1980). "A note on the paper: An inventory model with finite horizon and price changes". *Journal of the Operational Research Society*, Vol. 30, No. 9, pp. 839-840.
8. Goyal, S. K. (1975). "An inventory model for a product for which price fluctuates". *Operations Research*, Vol. 3, No. 2, pp. 112-117.
9. Huang, W., and Kulkarni, V.G. (2003). "Optimal EOQ for announced price increases in infinite horizon". *Operations Research*, Vol. 51, No. 2, pp. 336-339.
10. Khouja, M., and Goyal, S. K. (2006). "Single item optimal lot sizing under continuous unit cost decrease". *International Journal of Production Economics*, Vol. 102, pp. 87-94.
11. Khouja, M., and Park, S. (2003). "Optimal lot sizing under continuous price decrease". *Omega*, Vol. 31, pp. 539-545.
12. Lev, B., and Soyster, A. L. (1979). "An inventory model with finite horizon and price changes". *Journal of the Operational Research Society*, Vol. 30, No. 1, pp. 43-53.
13. Lev, B., and Weiss, H.J. (1990). "Inventory models with cost changes". *Operations Research*, Vol. 38, No. 1, pp. 53-63.
14. Naddor, E. (1966). *Inventory systems*, New York: Wiley.
15. Pakkala, T.P.M., and Babitha (2013). "Study of dynamic ordering strategy under one-time discount offer". *The Proceedings of the International Conference on Frontiers of Statistics and its Applications*, 284-290.
16. Taleizadeh, A.A., and Pentico, D.W. (2013). "An economic order quantity model with a known price increase and partial backordering". *European Journal of Operational Research*, Vol. 228, No. 3, 516-525.
17. Taylor, S.G., and Bradley, C.E. (1985). "Optimal ordering strategies for announced price increases". *Operations Research*, Vol. 33, No. 2, 312-325.

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