DEPENDENCE OF WAKEFIELD EXCITATION IN PLASMA BY NON-RESONANT SEQUENCE OF ELECTRON BUNCHES ON THEIR LENGTHS

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The present paper describes the results of numerical simulation (using 2d3v code LCODE) of the regime, when the wakefield is excited at maximum growth rate in the plasma by a nonresonant sequence of relativistic electron bunches. As a result, the wakefield increases approximately in steps. The paper gives the parameters, at which this regime is achieved. It is shown that for smaller bunch radii, the amplitude of the excited wakefield is larger. At long lengths of the bunches, the amplitude of the wakefield is larger, in contrast to the excitation by the resonant sequence of bunches.

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INTRODUCTION

The PWFA has the ability to sustain an extremely high accelerating gradient [1-15]. For continuous operation of the accelerator, one needs to use a long sequence of drivers. But resonant wakefield excitation is difficult with the use of a long sequence of relativistic electron bunches because of the nonuniform and time-varying laboratory plasma. Here resonant is used to mean that the distance between the bunches in a bunch train is equal to the excited wavelength in the plasma. Then frequency of the bunches in a bunch train ω_m equals the electron plasma frequency $\omega_m = \omega_{pe}$. Also long sequence focusing and large transformer ratio obtaining [16 - 25] are difficult because of the nonuniform and time-varying laboratory plasma. Hence, the sequence of electron bunches can be nonresonant $\omega_m \neq \omega_{pe}$ for the laboratory plasma. Previously, the authors have developed the mechanism and carried out investigations [26, 27] on the excitation of the wakefield by a nonresonant sequence of electron bunches. The present paper describes the results of numerical simulation of wakefield excitation at maximum growth rate deep in the plasma under multi-bunched PWFA scheme in the nonresonant case for initially homogeneous plasma. In the plasma near the bunch injection boundary, the time chain of wakefield pulses (see Fig. 1) is formed, similarly to the case of any nonresonant excitation of oscillations. In Fig. 1 one can see that in the plasma near the bunch injection boundary the wakefield (shown in pink) is the periodical chain of wakefield pulses. The length of each wakefield pulse is equal to several excited wavelengths (Fig. 1). In the first half of each pulse the bunches get decelerated, and in the back half of each pulse the bunches get accelerated. Hence in first half of the pulse the wakefield amplitude increases and in the back half of the pulse the wakefield amplitude symmetrically decreases. Note that deep in the plasma this symmetry is not observed. In effect, a part of the bunches is defocused by the transverse wakefield. As a result, the bunches from decelerating phases are closer to the axis, and the bunches from accelerating phases are far from the axis. Hence, the bunches from the decelerating

phases interact with the wakefield stronger in comparison with the bunches from the accelerating phases, because the wakefield amplitude decreases in the transverse direction. At some depth in the plasma the sequence of bunches, which interact strongly with the wakefield, becomes resonant due to defocusing of some bunches.

In this paper the authors present the numerical simulation data on the regime of nonresonant wakefield excitation at the maximum growth rate, which is realized at some depth in the plasma. The regime is characterized by the fact that at the front of the first half of the wakefield pulse the radius of the decelerating bunch is close to the initial one. The radii of bunches from the accelerating phases are much larger due to defocusing. Therefore, the bunches from the accelerating phases interact with the wakefield much weaker than the bunch at the front of the first half of the wakefield pulse, where wakefield increases quickly. As a consequence, the wakefield amplitude grows approximately in a stepwise manner. We mean that the wakefield amplitude rapidly increases over a short period of time, and then it changes only slightly over several plasma periods, to next process of wakefield amplitude growth (Fig. 5). This stepwise growth process is repeated many times (see Fig. 5). Investigations were performed by the 2.5D numerical simulation using the code LCODE [28].

We now analyse the wakefield excitation, considering the initial density of plasma to be lower than the resonant density $n_{0e} < n_{res} (= \omega_m^2 m / (4\pi e^2))^{-1}$, ω_m is the repetition frequency of bunches. Hence the repetition frequency ω_m of the bunches is a little higher than the electron plasma frequency $\omega_m > \omega_{pe}$. We use the parameters, for which the nonresonant wakefield is excited at the highest growth rate.

We also use the cylindrical coordinate system (r, z) . The time τ is normalized to ω_{pe}^{-1} , all the distances – to $c\omega_{pe}^{-1}$, the density – to the unperturbed plasma electron density $n_{0e} = 10^{11} \text{ cm}^{-3}$, the beam current I_b – to $mc^3 / e = 17 \text{ kA}$, the fields – to $mc\omega_{pe} / e$, where m is the electron mass, e is the electron charge, c is the

speed of light, ω_{pe} is the plasma electron frequency. These normalisations are used also in the Figures.

We present the numerical simulation data on plasma wakefield excitation by a sequence of relativistic electron bunches, obtained with the 2.5D quasi-static code LCODE [28] that treats the plasma as a cold electron fluid, and the bunches as ensembles of macro-particles. The parameters are taken close to those of the plasma wakefield experiments, in which the electron beam represented by a regular sequence of electron bunches (each bunch being of energy 2.5 MeV, the rms radius $\sigma_r = 0.5$ cm, and the rms angular spread $\sigma_\theta = 0.05$ mrad) excites the wakefield in the uniform plasma of density $n_p = 10^{11}$ cm $^{-3}$. The plasma wavelength equals $\lambda = 10.6$ cm. The distribution of bunches is Gaussian in the longitudinal and transversal directions. The gamma factor of bunches is equal to $\gamma_b = 5$. The plasma ions represent the immobile background.

Spatial step equals $0.1c\omega_{pe}^{-1}$. Time step for plasma electrons equals $0.1\omega_{pe}^{-1}$. Time step for beam electrons equals $0.1\sqrt{\gamma_b}\omega_{pe}^{-1}$.

We present time dependences in selected points of observation.

The resonant and nonresonant regimes are fundamentally different. Only the 1st bunch is focused in the resonant case while the remaining heads of the bunches are defocused, and the tails of the bunches are focused. In the nonresonant case under consideration, the repetition frequency of bunches is higher than the plasma frequency $\omega_m > \omega_{pe}$, and we observe that the bunches in the fronts of the wakefield pulses approach the zero radial wakefield (see Fig. 5), while the remaining bunches are strongly defocused.

In this paper, it is shown by numerical simulation that with an increase in a certain range of bunch lengths, the wakefield amplitude increases.

RESULTS OF NUMERICAL SIMULATION

Consideration is first given to the excitation of the wakefield in the plasma by first 28 bunches with the finite length of each bunch $2\sigma_z = 1.06$ cm (see Fig. 1). One can see that the bunch is shorter than the excited plasma wavelength $\lambda = 10.6$ cm. In the plasma, at $z = 5$ cm from the bunch injection boundary, the time chain of wakefield pulses is formed (see Fig. 1), similarly to the case of any nonresonant excitation of oscillations. In Fig. 1 one can see that in the plasma near the bunch injection boundary the wakefield (shown in pink) is the periodical chain of wakefield pulses. The length of each pulse is equal to several excited wavelengths (see Fig. 1). In the first half of each pulse the bunches are decelerated, and in the back half of each pulse the bunches are accelerated. Hence, in the first half of the pulse the wakefield amplitude increases, and in the back half of the pulse the wakefield amplitude symmetrically decreases. The wakefield pulses in the plasma near the boundary of injection are symmetrical and identical (see Fig. 1). It can be assumed that there is symmetry between the bunch behaviour both at the beginning and at the end of the wakefield pulse.

However, this symmetry is not observed deep in the plasma (Figs. 2, 3). In effect, a part of bunches is defocused by the transversal wakefield. As a result, the bunches from decelerating phases are closer to the axis, whereas the bunches from accelerating phases are far from the axis. Hence, the bunches from decelerating phases interact with the wakefield stronger as opposed to the bunches from accelerating phases, by virtue of the fact that the wakefield amplitude decreases in the transversal direction. At some depth in the plasma, the sequence of bunches, which interact strongly with the wakefield, becomes resonant due to defocusing of some bunches. It can $z \geq 83$ cm be seen from Fig. 2 that the eleventh/twelfth and twenty-third/twenty-fourth bunches are approximately similar to the first bunch. That results in the increase of the wakefield amplitude (see Fig. 3).

Fig. 1. $E_{z2} = E_z|_{r=r_{b0}}$ (pink), the radial wake force F_r (dark blue), $n_{b2} = n_b|_{r=r_{b0}}$ (yellow) and the bunch radius r_b (blue) as functions of the time for $\gamma_b = 5$; $(n_e - n_{res})n_{res}^{-1} = -0.15$, $2\sigma_z = 1.06$ cm, maximum initial electron bunch current $I_b = 1.05 \cdot 10^{-3}$, $z = 5$ cm, r_{b0} is the initial radius of bunches

Fig. 2. Evolution (in time) of the beam density at $\gamma_b = 5$; $(n_e - n_{res})n_{res}^{-1} = -0.15$, $2\sigma_z = 1.7$ cm, maximum initial electron bunch current $I_b = 0.53 \cdot 10^{-3}$, $z = 1.7$ m

Fig. 3. The on-axis wakefield excitation E_z (pink) by sequence of 28 bunches in the nonresonant case $(n_e - n_{res})n_{res}^{-1} = -0.15$, $\langle E \rangle = \int dr r E_z n_b \left(\int dr r n_b \right)$ (black) and density of bunches on the axis $n_b = n_b|_{r=0}$ (yellow) as functions of the time τ for $\gamma_b=5$, $2\sigma_z=1.7$ cm, maximum initial electron bunch current $I_b=0.53 \cdot 10^{-3}$, $z=1.7$ m $2\sigma_z=1.7$ cm

Fig. 4. The E_z amplitude as a function of the coordinate along the plasma at $\gamma_b=5$; $(n_e - n_{res})n_{res}^{-1} = -0.15$

The amplitude of the excited field is maximal at the depth equal to $z \geq 83$ cm (Fig. 4). This value approximately equals 8 wavelengths.

At some depth in the plasma at the front of the first half of the wakefield pulse, the radius of the decelerating bunch is similar to the initial one. The radii of bunches from the accelerating phases are much larger due to defocusing. As a consequence, the bunches from the accelerating phases interact with the wakefield considerably weaker than the bunch at the front of the first half of the wakefield pulse. As a result, the wakefield amplitude grows approximately in steps. We mean that the amplitude of the wakefield rapidly increases over a short period of time, and for several plasma periods, to next process of wakefield amplitude growth, the ampli-

tude changes only slightly (see Fig. 5). This stepwise growth process is repeated many times (see Fig. 5).

The optimal case (for the maximal growth rate of wakefield amplitude) may be represented by the case, where the accelerated bunches are defocused from the interaction area with the wakefield; and at the front of the first half of the wakefield pulse the radius of the decelerating bunch is close to the initial one. The decelerating bunch at the front of the first half of the wakefield pulse gets into the phase of small radial wake force, and for this reason the decelerating bunch has no time to be significantly defocused.

Now we consider the dependence of the wakefield amplitude on the length and radius of the bunches (Figs. 6, 7). One can see that with an increase in a certain range of bunch length and decrease of bunch radius, the wakefield amplitude increases.

Now we demonstrate that, even with a large difference of the plasma electron density n_e and the resonance one, the effective excitation of the wakefield occurs. In Fig. 8 one can see that for $(n_e - n_{res})n_{res}^{-1} = -0.5$ growing beats of E_z contain three bunches. The 1st bunch leads to an increase of the amplitude of E_z , the 2nd hardly exchanges energy with the wakefield, because it gets into $E_z \approx 0$. The 2nd bunch simultaneously gets into the maximum F_r and it is defocused. 3rd bunch takes energy from the wakefield less than the 1st bunch gave to the wakefield.

Fig. 5. Off-axis wakefield $E_{z2} = E_z|_{r=r_{b0}}$ (pink) unlike on-axis wakefield in Figs. 3, 4, the radial wake force F_r (dark blue), $n_{b2} = n_b|_{r=r_{b0}}$ (yellow) and the bunch radius of r_b (blue) as functions of the time for $\gamma_b=5$; $(n_e - n_{res})n_{res}^{-1} = -0.15$, $2\sigma_z=1.7$ cm, maximum initial electron bunch current $I_b=0.53 \times 10^{-3}$, $z=1.7$ m. The parameters are identical to Fig. 2-4.

Fig. 6. The dependence of the wakefield amplitude on the length of the bunches

Fig. 7. The dependence of the wakefield amplitude on the radius of the bunches

Fig. 8. Off-axis wakefield $E_{z2} = E_z|_{r=r_{b0}}$ (pink), the radial wake force F_r (dark blue), $n_{b2} = n_b|_{r=r_{b0}}$ (yellow) and the bunch radius r_b (blue) as functions of the time for $\gamma_b=5$; $(n_e - n_{res})n_{res}^{-1} = -0.5$, maximum initial electron bunch current $I_b=1.05 \cdot 10^{-3}$

CONCLUSIONS

By numerical simulation the authors have demonstrated, that there are the electron bunch parameters at which the wakefield excitation by the non-resonant sequence of relativistic electron bunches gives rise to the regime, at which deep in the plasma the wakefield amplitude grows in steps at the highest growth rate. Also, it has been shown by numerical simulation that under this regime the non-resonant sequence loses some electrons due to defocusing and becomes resonant. The interaction of the wakefield with the electrons from the accelerating phases is found to be very low under this regime.

It is shown by numerical simulation that with an increase in a certain range of bunch length and decrease of bunch radius, the wakefield amplitude increases.

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ЗАВИСИМОСТЬ РЕЗОНАНСНОГО ВОЗБУЖДЕНИЯ КИЛЬВАТЕРНОГО ПОЛЯ В ПЛАЗМЕ НЕРЕЗОНАНСНОЙ ПОСЛЕДОВАТЕЛЬНОСТЬЮ ЭЛЕКТРОННЫХ СГУСТКОВ ОТ ИХ ДЛИН

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Описаны результаты численного моделирования (с использованием кода 2d3v LCODE) режима, когда кильватерное поле возбуждается с максимальной скоростью роста в плазме нерезонансной последовательностью релятивистских электронных сгустков. В результате кильватерное поле увеличивается примерно ступенчато. Приведены параметры, при которых достигается этот режим. Показано, что для меньших радиусов сгустка амплитуда возбужденного кильватерного поля больше. При больших длинах сгустков амплитуда кильватерного поля больше в отличие от режима возбуждения резонансной последовательностью сгустков.

ЗАЛЕЖНІСТЬ РЕЗОНАНСНОГО ЗБУДЖЕННЯ КІЛЬВАТЕРНОГО ПОЛЯ В ПЛАЗМІ НЕРЕЗОНАНСНОЮ ПОСЛІДОВНІСТЮ ЕЛЕКТРОННИХ ЗГУСТКІВ ВІД ЇХ ДОВЖИН

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Описані результати чисельного моделювання (з використанням коду 2d3v LCODE) режиму, коли кильватерне поле збуджується з максимальною швидкістю росту в плазмі нерезонансною послідовністю релятивістських електронних згустків. У результаті кильватерне поле збільшується приблизно поступово. Наведено параметри, при яких досягається цей режим. Показано, що для менших радіусів згустка амплітуда збудженого кильватерного поля більше. При великих довжинах згустків амплітуда кильватерного поля більше, на відміну від режиму збудження резонансною послідовністю згустків.