Dynamic Model of TRMS Based on Homogeneous Transformation and Euler-Lagrange Equation

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Abstract— The aerodynamic experimental set TRMS (Twin Rotor MIMO System) is a strong nonlinear system, which has been taken by many scientists as an object to test modern control algorithms. The paper built a complete and explicit dynamic model for the TRMS based on dissociating TRMS into 3 subsystems, using homogeneous transformations in orthogonal coordinate systems to calculate the mass point's position and velocity in component sub-systems; Euler - Lagrange equation was applied to model the dynamics for the object.

Keywords—TRMS, dynamics, homogeneous transformation, mechanical system dissociation, Euler - Lagrange.

I. INTRODUCTION

Feedback Instrument Ltd's Twin Rotor MIMO System (TRMS) is an open-structured experimental set that has installed PID control algorithms, visually describing a twin-rotor helicopter flying instrument [1], [2]. This is a strong nonlinear object with interleaving, so although it was designed and manufactured in 1998, so far TRMS has been used by many researchers as a control object for testing control algorithms. At the forefront of TRMS research is the manufacturer Feedback Instrument Ltd. The company built a mathematical model for TRMS based on Newtonian mechanics, but that mathematical model did not consider the length of the rotating pin and ignores the inertia moment of some small components in the system. To develop and test some algorithms for linear control (PID, LQR, and LQG), nonlinear control (sliding mode, adaptive control, fuzzy, neuron, feedback linearization, etc.), many scholars have researched, built, and adjusted the mathematical model of TRMS under specific conditions. Due to the use of some assumptions when building the model, the structure of the model would be incomplete. Therefore, to develop a control algorithm for TRMS with better output response quality, it is necessary to have an accurate and suitable model with few assumptions.

In this paper, we propose a new approach, whereby we decomposed the system of TRMS into subsystems, then we built a mathematical model of TRMS based on a homogeneous coordinate transformation and the Euler-Lagrange equation. The decomposition of TRMS into components helped the modeling process not to miss a single mechanical detail of the system, thereby build a full dynamics model for TRMS with control torque input, creating a basis for the development and testing of advanced direct torque control algorithms.

The paper was organized as follows: After the introduction, we conducted some literature review in the second section. The third section summarized the physical structure of TRMS. The dynamic model of the TRMS mechanical system was presented in the fourth section and the last section ended with some conclusions.

II. LITERATURE REVIEW

To design suitable controllers for the object, a mathematical model of the object must be obtained. There were two types of modeling conducted for TRMS: exact modeling and approximate modeling. To accurately model the nonlinear kinematics of TRMS, researchers used precise modeling based on Euler - Newton or Euler - Lagrange method. Both of these methods gave accurate object description results and the difference between the two models was very small. In [3] and [4], the authors conducted the modeling according to both Euler - Newton, and Euler - Lagrange methods. The modeling based on the Euler - Lagrange method gave slightly more accurate results than Newton - Euler when compared to the real TRMS model. Research [5] built a white box model for TRMS based on the 2nd Newton law and quantified the model using the recognition tool in MATLAB to produce a gray-box model for TRMS with a voltage control input – a gray-box regression model similar to the study [6] and [7]. When choosing a regression model, to some extent, the studies [5], [6], and [7] lost the completeness of the mathematical model.

Research [8] built a complete dynamic model based on the Euler - Lagrange equation and normalize the model by PD control action. However, because the control signal of the TRMS was voltage, the system had to go through voltage-torque conversion and this stage was not been considered in the modeling process.

In this paper, we used the modeling results based on the Euler - Lagrange method presented in [3] and [4] to build a mathematical model based on the Lagrange equation with the form of control moment inputs. We divided TRMS into 3 parts: (i) The first part included the beam and the other mechanism attached to it (main rotor and tail rotor, main fender, tail fender); (ii) The second part consisted of a counterweight and counterweight bar; and (iii) The last part was the pivot pin.

III. PHYSICAL STRUCTURE OF TRMS

The physical structure of the TRMS system was shown in Figure 1.

Figure 1. The Physical Structure of TRMS [9]

TRMS has the main rotor to change the pitch angle ε and a tail rotor to change the yaw angle β . These two propellers attach to the beam and similar to the helicopter, they are perpendicular to each other. Each rotor is independently driven by a permanently excited DC motor, which can change the direction of rotation and speed. Each motor is attached to a tachometer to measure the angular rate of the propeller. The counterweight bar is attached to the beam at the position of the bearing.

The counterweight position on the counterweight bar can be changed to serve the experiment to change the kinematic parameters of the object. The entire structure is attached to a tower and can rotate around the tower in the horizontal plane. In addition, the system has optical encoders to measure the elevation angle ε and azimuth angle β .

IV. DYNAMIC MODEL OF TRMS MECHANICAL SYSTEM

A. Theoretical basis

The Euler-Lagrange equation of a mechanical system has the following expression:

$$\underline{\tau} = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\underline{q}}} \right)^T - \left(\frac{\partial L}{\partial \underline{q}} \right)^T \quad (1)$$

$$L = K - V \quad (2)$$

$$\begin{cases} K = \frac{1}{2} \int v^2(P) dm(P) \\ V = g \int r_z(P) dm(P) \end{cases} \quad (3)$$

Where:

$\underline{q} = [q_1, q_2, \dots, q_n]^T$ is a joint variable vector;

$\underline{\tau} = [\tau_1, \tau_2, \dots, \tau_n]^T$ is an input torque vector;

L is the Lagrange function;

K is the kinetic the mechanical system;

V is the potential energy of the mechanical system;

P is a mass point in the system.

Suppose, in space, from the coordinate system attached to the body $Bxyz$, it was moved to the standard coordinate system $OXYZ$ by 2 rotations and one translation in the order: Rotation $R_z(\beta)$ around the OZ axis at an angle β , the rotation $R_x(\varepsilon)$ around the OX axis at an angle ε and the translation $T_{xyz}(x_0, y_0, z_0)$ along the axes of $OXYZ$ with coordinates (x_0, y_0, z_0) then in a homogeneous coordinate system we had the transformation T as (4):

$$T = R_z(\beta) R_x(\varepsilon) T_{xyz}(x_0, y_0, z_0) \quad (4)$$

Where:

Rotation $R_z(\beta)$ around the OZ axis:

$$R_z(\beta) = \begin{bmatrix} \cos \beta & \sin \beta & 0 & 0 \\ -\sin \beta & \cos \beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5a)$$

Rotation $R_x(\varepsilon)$ around the OX axis:

$$R_x(\varepsilon) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \varepsilon & \sin \varepsilon & 0 \\ 0 & -\sin \varepsilon & \cos \varepsilon & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5b)$$

$$T_{xyz}(x_0, y_0, z_0) = \begin{bmatrix} 1 & 0 & 0 & x_0 \\ 0 & 1 & 0 & y_0 \\ 0 & 0 & 1 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5c)$$

B. Mathematical Model of TRMS

In Figure 2, we define the orthogonal coordinate systems as follows: The standard coordinate system $OXYZ$ has the center O coincides with the yaw motion of the whole system, the XOY plane is the horizontal plane, the OX axis is specified at 0° of azimuth angle. The $Bxyz$ coordinate system attached to the TRMS has the Bx axis in the direction of the extension of the pivot rod, the By axis along the beam towards the right, and the Bz axis upward in the direction of the extension of the counterweight bar. Thus, if $OXYZ$ is rotated around the OZ axis by an angle β , then OX coincides with Bx ; next rotates around the OX axis by an angle ε , then $OXYZ$ will be parallel to $Bxyz$ and a distance from this system $OB = h$. If $OXYZ$ is translated by a distance h in the OX direction, the $OXYZ$ and $Bxyz$ coordinate systems will coincide.

Define Orthogonal Coordinate Systems

Orthogonal coordinate systems associated with TRMS were performed in Fig. 2.

Figure 2. Orthogonal Coordinate Systems Associated with TRMS [4]

From Fig. 2, with the coordinate transformations (4) and (5a), (5b), (5c), knowing the angles β , ε and then we could calculate the coordinates of any point P on the TRMS according to (6).

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = T \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad (6)$$

$$T = \begin{bmatrix} \cos \beta & \sin \beta \cos \varepsilon & \sin \beta \sin \varepsilon & h \cos \beta \\ -\sin \beta & \cos \beta \cos \varepsilon & \cos \beta \sin \varepsilon & -h \sin \beta \\ 0 & -\sin \varepsilon & \cos \varepsilon & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In this paper, we chose the approach of building a mathematical model of the TRMS system based on the Euler-Lagrange equation on the view of separating the TRMS system into 3 subsystems:

The first one included the beam and the other mechanism attached to it (main rotor and tail rotor, main fender, tail fender); the second one included the counterweight bar and counterweight; the third one was the pivot pin.

Position and Velocity of Subsystems in OXYZ

To calculate the kinetic energy according to formula (2), we need to calculate the velocity of any mass point (every particle) on the TRMS.

a) Position and velocity in the 1st subsystem:

In Figure 2, P_1 was a point on the horizontal bar located in the segment $BP_1 = R_1$ from the center of B whose coordinates in the $Bxyz$ system were $[0, R_1, 0, 1]^T$. For convenience, we denoted the coordinates of P_1 in the $OXYZ$ system as $[r_x(P_1), r_y(P_1), r_z(P_1)]$ and its velocity was respectively $[v_x(P_1), v_y(P_1), v_z(P_1)]$. From (6) we got:

$$\begin{bmatrix} r_x(P_1) \\ r_y(P_1) \\ r_z(P_1) \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \beta & \sin \beta \cos \varepsilon & \sin \beta \sin \varepsilon & h \cos \beta \\ -\sin \beta & \cos \beta \cos \varepsilon & \cos \beta \sin \varepsilon & -h \sin \beta \\ 0 & -\sin \varepsilon & \cos \varepsilon & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ R_1 \\ 0 \\ 1 \end{bmatrix} \quad (7)$$

$$= \begin{bmatrix} R_1 \sin \beta \cos \varepsilon + h \cos \beta \\ R_1 \cos \beta \cos \varepsilon - h \sin \beta \\ -R_1 \sin \varepsilon \\ 1 \end{bmatrix}$$

Take the position derivative $[r_x(P_1), r_y(P_1), r_z(P_1)]$ to have velocity $[v_x(P_1), v_y(P_1), v_z(P_1)]$:

$$\begin{cases} v_x(P_1) = R_1 \dot{\beta} \cos \beta \cos \varepsilon - R_1 \dot{\varepsilon} \sin \beta \sin \varepsilon - h \dot{\beta} \sin \beta \\ v_y(P_1) = -R_1 \dot{\beta} \sin \beta \cos \varepsilon - R_1 \dot{\varepsilon} \cos \beta \sin \varepsilon - h \dot{\beta} \cos \beta \\ v_z(P_1) = -R_1 \dot{\varepsilon} \cos \varepsilon \end{cases} \quad (8)$$

To calculate K , we had to get v^2 :

$$v^2(P_1) = v_x^2(P_1) + v_y^2(P_1) + v_z^2(P_1) \quad (9)$$

After reducing we had equation (10):

$$v^2(P_1) = R_1^2 (\beta^2 \cos^2 \varepsilon + \varepsilon^2) + h^2 \beta^2 + 2R_1 h \dot{\beta} \dot{\varepsilon} \sin \varepsilon \quad (10)$$

b) Position and velocity in the 2nd subsystem:

In Figure 2, the point P_2 was on the counterweight bar at a distance R_2 from the center of B with coordinates in the $Bxyz$ system of $[0, 0, -R_2, 1]^T$. The same calculation as (7), we got (11):

$$\begin{cases} r_x(P_2) = -R_2 \sin \beta \sin \varepsilon + h \cos \beta \\ r_y(P_2) = -R_2 \cos \beta \sin \varepsilon - h \sin \beta \\ r_z(P_2) = -R_2 \cos \varepsilon \end{cases} \quad (11)$$

From (11), take the position derivative to get velocity, we got (12):

$$v^2(P_2) = R_2^2 (\beta^2 \sin^2 \varepsilon + \varepsilon^2) + h^2 \beta^2 - 2R_2 h \dot{\beta} \dot{\varepsilon} \cos \varepsilon \quad (12)$$

c) Position and velocity in the 3rd subsystem:

In Figure 2, the point P_3 was on the pivot pin at a distance R_3 from the center of O , whose coordinates in the $Bxyz$ system were $[(R_3 - h), 0, 0, 1]^T$. Calculating according to (7), we got (13) and (14):

$$\begin{cases} r_x(P_3) = R_3 \cos \beta \\ r_y(P_3) = R_3 \sin \beta \\ r_z(P_3) = 0 \end{cases} \quad (13)$$

$$v^2(P_3) = R_3^2 \beta^2 \quad (14)$$

Kinetic and Potential Energy of Each Component

Applying formulas (2) and (3) with the 3 components of TRMS as follows:

a) For members on the beam: the 1st subsystem:

From (10) we have:

$$\begin{cases} K_1 = \frac{1}{2} (\dot{\beta}^2 \cos^2 \varepsilon + \dot{\varepsilon}^2) J_1 \\ \quad + \frac{1}{2} h^2 \dot{\varepsilon}^2 m_{T_1} + h \dot{\beta} \dot{\varepsilon} m_{T_1} l_{T_1} \sin \varepsilon \\ V_1 = g m_{T_1} l_{T_1} \sin \varepsilon \end{cases} \quad (15)$$

Where:

m_{T_1} - the total mass on the beam;

l_{T_1} - the center of gravity of the beam;

J_1 - the inertia moment of the beam with the center of rotation B .

$$m_{T_1} = \int dm(R_1) \quad (15a)$$

$$= m_t + m_{tr} + m_{ts} + m_m + m_{mr} + m_{ms}$$

$$l_{T_1} = \frac{\int R_1 dm(R_1)}{\int dm(R_1)}$$

$$= \frac{(m_m / 2 + m_{mr} + m_{ms}) l_m}{m_{T_1}} \quad (15b)$$

$$+ \frac{-(m_t / 2 + m_{tr} + m_{ts}) l_t}{m_{T_1}}$$

$$J_1 = \int R_1^2 dm(R_1)$$

$$= \left(\frac{m_t}{3} + m_{tr} + m_{ts} \right) l_t^2$$

$$+ \left(\frac{m_m}{3} + m_{mr} + m_{ms} \right) l_m^2 \quad (15c)$$

$$+ \frac{1}{2} m_{ms} r_{ms}^2 + m_{ts} r_{ts}^2$$

Where:

$m_t, m_{tr}, m_{ts}, m_m, m_{mr}, m_{ms}$ - the mass of the beam on the tail rotor side, tail rotor, tail fender; the mass of the beam on the main rotor side, main rotor, and main fender, respectively;

$l_m, l_{tr}, r_{ms}, r_{ts}$ - the length and radius of the main beam, the tail beam, the main fender, and the tail fender, respectively.

b) For the counterweight: the 2nd subsystem:

From (12) we got:

$$\begin{cases} K_2 = \frac{1}{2} (\dot{\beta}^2 \sin^2 \varepsilon + \dot{\varepsilon}^2) J_2 \\ \quad + \frac{1}{2} h^2 \dot{\beta}^2 m_{T_2} - h \dot{\beta} \dot{\varepsilon} m_{T_2} l_{T_2} \cos \varepsilon \\ V_2 = -g m_{T_2} l_{T_2} \cos \varepsilon \end{cases} \quad (16)$$

Where:

m_{T_2} - the total mass on the counterweight;

l_{T_2} - the center of gravity of the counterweight bar including the counterweight;

J_2 - the inertia moment of the counterweight bar to the center of rotation B.

$$m_{T_2} = \int dm(R_2) \quad (16a)$$

$$= m_b + m_{cb}$$

$$l_{T_2} = \frac{m_b l_b / 2 + m_{cb} l_{cb}}{m_{T_2}} \quad (16b)$$

$$J_2 = \int R_2^2 dm(R_2)$$

$$= \frac{1}{3} m_b l_b^2 + m_{cb} l_{cb}^2 + \frac{1}{4} m_{cb} r_{cb}^2 + \frac{1}{12} m_{cb} L_{cb}^2 \quad (16c)$$

With $l_b, l_{cb}, L_{cb}, m_b, m_{cb}$ were the counterweight bar length, counterweight to pivot distance, counterweight length, counterweight bar mass, and counterweight, respectively.

c) For the pivot pin: the 3rd subsystem:

$$\begin{cases} K_3 = \frac{1}{2} \dot{\beta}^2 J_3 \\ V_3 = 0 \end{cases}$$

$$J_3 = \int R_3^2 dm(R_3) \quad (17)$$

$$= \frac{m_h}{3} h^2$$

Energy Balance with Euler-Lagrange Equation

From equation (1), take partial derivatives by azimuth and elevation angles as (18):

$$\begin{cases} \tau_h = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\beta}} \right)^T - \left(\frac{\partial L}{\partial \beta} \right)^T \\ \tau_v = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\varepsilon}} \right)^T - \left(\frac{\partial L}{\partial \varepsilon} \right)^T \end{cases} \quad (18)$$

Where:

- τ_h : The sum of the torque acting in the yaw motion (azimuth channel);
- τ_v : The sum of the moments acting in the pitch motion (elevation channel);
- L: The Lagrange energy function with 3 components taken according to formulas (15), (16a), (16b), (16c), and (17).

Ignoring the intermediate steps, we had the following results:

$$\begin{aligned}\tau_h = & \left[J_1 \cos^2 \varepsilon + J_2 \sin^2 \varepsilon + h^2 (m_{T_1} + m_{T_2}) + J_3 \right] \ddot{\beta} \\ & + h(m_{T_1} l_{T_1} \sin \varepsilon - m_{T_2} l_{T_2} \cos \varepsilon) \ddot{\varepsilon} \\ & + h(m_{T_1} l_{T_1} \cos \varepsilon + m_{T_2} l_{T_2} \sin \varepsilon) \dot{\varepsilon}^2 \\ & + 2(J_2 - J_1) \dot{\beta} \dot{\varepsilon} \sin \varepsilon \cos \varepsilon\end{aligned}\quad (19)$$

$$\begin{aligned}\tau_v = & (J_1 + J_2) \ddot{\varepsilon} \\ & + h(m_{T_1} l_{T_1} \sin \varepsilon - m_{T_2} l_{T_2} \cos \varepsilon) \ddot{\beta} \\ & + g(m_{T_1} l_{T_1} \cos \varepsilon + m_{T_2} l_{T_2} \sin \varepsilon) \\ & + \dot{\beta}^2 (J_1 - J_2) \sin \varepsilon \cos \varepsilon\end{aligned}\quad (20)$$

Combining (19) and (20) and writing in vector form, we got the TRMS dynamics equation according to (21):

$$\begin{aligned}\begin{bmatrix} \tau_h \\ \tau_v \end{bmatrix} = & \begin{bmatrix} J_1 \cos^2 \varepsilon + J_2 \sin^2 \varepsilon + h^2 (m_{T_1} + m_{T_2}) + J_3 & h(m_{T_1} l_{T_1} \sin \varepsilon - m_{T_2} l_{T_2} \cos \varepsilon) \\ h(m_{T_1} l_{T_1} \sin \varepsilon - m_{T_2} l_{T_2} \cos \varepsilon) & J_1 + J_2 \end{bmatrix} \begin{bmatrix} \ddot{\beta} \\ \ddot{\varepsilon} \end{bmatrix} \\ & + \begin{bmatrix} h(m_{T_1} l_{T_1} \cos \varepsilon + m_{T_2} l_{T_2} \sin \varepsilon) \dot{\varepsilon}^2 + 2(J_2 - J_1) \dot{\beta} \dot{\varepsilon} \sin \varepsilon \cos \varepsilon \\ \dot{\beta}^2 (J_1 - J_2) \sin \varepsilon \cos \varepsilon + g(m_{T_1} l_{T_1} \cos \varepsilon + m_{T_2} l_{T_2} \sin \varepsilon) \end{bmatrix}\end{aligned}\quad (21)$$

The moment components applied to the TRMS in yaw motion include:

- The tail rotor thrust moment $M_h = l_t F_h (\omega_h) \cos \varepsilon$ (where ω_h - the angular velocity of the tail rotor and F_h - the thrust of the tail fan wing);
- The frictional moment M_{msh} ;
- The drag torque produced by the M_{cab} signal cable
- The inter-channel moment $k_m \dot{\omega}_v \cos \varepsilon$ (k_m - the inter-channel factor and ω_v - the angular velocity of the main rotor).

The total torque acting on the TRMS in yaw motion:

$$\tau_h = M_h - M_{msh} - M_{cab} + k_m \dot{\omega}_v \cos \varepsilon \quad (22)$$

Similarly, the total torque acting on the TRMS during pitch motion:

$$\tau_v = M_v - M_{msv} + k_t \dot{\omega}_h + M_{gyro} \quad (23)$$

Where:

$M_v = l_m F_v (\omega_v)$ - the torque of the main rotor thrust (F_v - the main rotor thrust);

M_{msv} - the frictional moment in pitch motion;

$k_t \dot{\omega}_h$ - the inter-channel moment (k_t - the coefficient of inter-channel);

$M_{gyro} = k_g F_v \omega_h \cos \varepsilon$ - the component was caused by the gyroscope effect when the TRMS rotates in the horizontal plane with angular velocity $\omega_h = \dot{\beta}$.

From the mathematical model (21) along with all the parameters in (22), (23), we found that TRMS had the following basic characteristics:

- A nonlinear, uncertain system: a strong inter-channel due to the influence of the main and tail rotor movements; parameter uncertainty due to changes in the inertia moment inertia and coefficient of friction on joints; parameters could only be determined through identification.
- Being affected by noise, including system disturbance, measurement noise, noise caused by environmental conditions such as wind speed, temperature, humidity, etc.

Because of the above characteristics of the TRMS system, we need to have specialized control methods to be able to ensure the desired control quality. The block diagram of TRMS was shown in Figure 3.

Figure 3. The block diagram of TRMS

In Figure 3, the main actuators are permanently excited DC motors, in which the usual control method is to change the field current or the armature voltage. With each different control algorithm approach, the control signal u can be the angular velocity of the fans or the current and voltage supplied to the DC motors that rotate the fans.

V. CONCLUSION

The article proposed a new approach in building a dynamic model of the TRMS system based on the homogeneous coordinate transformation and the Euler-Lagrange equation on the view of separating the TRMS system into 3 subsystems. The model building process did not omit any mechanical details and a mechanical effect, so the result was a complete model with torque control input, serving the purpose of testing modern control algorithms.

In the next studies, the authors will evaluate the effectiveness of the model built with some algorithms of direct torque control with some sustainable adaptive controllers.

Conflict of Interest

The authors declare no conflict of interest.

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