Vibration Analysis of the Tool Bearing Spindle in Case of Superfinishing Technological Processes. Insertion of External Strains and Determining the Balance Conditions

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Abstract— The paper presents the main spindle model at superfinishing operations taking into account the external strains determined by the resulting forces and moments in the grinding process. An important role is played by the geometric configuration of the main spindle. The initial conditions of the movement are presented for each geometric section. There are also emphasized the inertial forces and moments determined by the tool position. There are also established the balance conditions, situation in which the spindle is deformed but without vibrating. There are computed the expressions for the cutting forces, bending moments as well as the continuity equations regarding spindle deformation in the considered sections.

Keywords— Balance conditions, eccentric, external strains, main spindle, superfinishing.

I. INTRODUCTION

Establishing the general movement equations that describe the evolution of the vibration movement of the tool bearing spindle in case of superfinishing operations emphasizes the propagation of the oscillating movement along the coordinate axes. Their establishment was done with respect to a proper reference system attached to the main spindle.

For determining the external strains it is necessary to know the specific work conditions of the spindle in case of superfinishing.

In this context, the geometric configuration of the spindle plays a major role [1]. It is required to transpose the movement equations with respect to a fixed reference system. Thus can be evaluated the optimal movement conditions [2,3].

II. INSERTION OF THE EXTERNAL STRAINS

The analyzed tool bearing spindle model is given in fig. 1:

Figure 1. Ideal model of the spindle

Figure 2. Action of bearing upon spindle

Corresponding to the upper bearing A, the reactions of the bearing upon the eccentric spindle are given in fig. 2.

The forces, moments and impulses for the two bearings are given by:

$$\begin{aligned}
 \{p\}^{(1)} &= \begin{Bmatrix} F_{11} \\ F_{21} \\ F_{31} \end{Bmatrix} \delta(x_1 - 30) \\
 \{m\}^{(1)} &= \begin{Bmatrix} 0 \\ M_{21} \\ M_{31} \end{Bmatrix} \delta(x_1 - 30) \\
 \{p\}^{(2)} &= \begin{Bmatrix} F_{12} \\ F_{22} \\ F_{32} \end{Bmatrix} \delta(x_1 - 170) \\
 \{m\}^{(2)} &= \begin{Bmatrix} 0 \\ M_{22} \\ M_{32} \end{Bmatrix} \delta(x_1 - 170) \quad (1)
 \end{aligned}$$

where [4]: $\delta(x_1 - l_i) = \begin{cases} 1, x_1 = l_i \\ 0, x_1 \neq l_i \end{cases}$

The bearings produce a damping whose expression is:

$$\{p\}^{(a)} = - \begin{Bmatrix} C_1 \dot{u}_1 \\ C_2 \dot{u}_2 \\ C_3 \dot{u}_3 \end{Bmatrix} [\delta(x_1 - 30) + \delta(x_1 - 170)] \quad (2)$$

In stabilized regime, the movement equations become:

$$\begin{aligned}
 \rho A \ddot{u}_1 - E A u_{1,11} &= F_{11} \delta(x_1 - 30) + F_{12} \delta(x_1 - 170) - \\
 &- c_1 \dot{u}_1 [\delta(x_1 - 30) + \delta(x_1 - 170)] \\
 \rho A \ddot{u}_3 - \rho I \ddot{u}_{3,11} + 2 \rho A (\Omega + \omega') \dot{u}_2 - \rho A (\Omega + \omega')^2 u_3 + \\
 &+ E u_{3,1111} = M_{21} \delta(x_1 - 30) + M_{22} \delta(x_1 - 170) + \\
 &+ F_{31} \delta(x_1 - 30) + F_{32} \delta(x_1 - 170) - c_3 \dot{u}_3 [\delta(x_1 - 30) + \\
 &+ \delta(x_1 - 170)] \\
 - \rho A \ddot{u}_2 + \rho I \ddot{u}_{2,11} + 2 \rho A (\Omega + \omega') \dot{u}_3 + \rho A (\Omega + \omega')^2 u_2 - \\
 &- E u_{2,1111} = M_{31} \delta(x_1 - 30) + M_{32} \delta(x_1 - 170) - \\
 &- F_{21} \delta(x_1 - 30) - F_{22} \delta(x_1 - 170) - c_2 \dot{u}_2 [\delta(x_1 - 30) + \\
 &+ \delta(x_1 - 170)] \quad (3)
 \end{aligned}$$

Also, the movement of the abrasive tools will produce:

$$\vec{F}_{in} = -m_T \cdot \vec{a}_c = m_T (\ddot{u}_1 \vec{i}_1 + \ddot{u}_2 \vec{i}_2 + \ddot{u}_3 \vec{i}_3) \quad (4)$$

$$\begin{aligned}
 \vec{M}_{in} &= [J_d \ddot{u}_{3,1} - (J_0 - J_d)(\Omega + \omega') \dot{u}_{2,1}] \vec{i}_2 - \\
 &- [J_d \ddot{u}_{2,1} - (J_0 - J_d)(\Omega + \omega') \dot{u}_{3,1}] \vec{i}_3 \quad (5)
 \end{aligned}$$

The use of abrasive tool introduces specific elements. In this case, the limit conditions will be:

$$E A u_{1,1} = 0$$

$$E u_{2,111} - \rho I \ddot{u}_{2,1} - E A u_{1,1} u_{2,1} = 0$$

Figure 3. The position of the fixed reference system with respect to the mobile reference system

$$E u_{3,111} - \rho I \ddot{u}_{3,1} - E A u_{1,1} u_{3,1} = 0$$

$$E u_{2,11} = 0$$

$$E u_{3,11} = 0$$

$$E A u_{1,1} + m_p \ddot{u}_1 = R_1$$

$$E u_{2,111} - \rho I \ddot{u}_{2,1} - E A u_{1,1} u_{2,1} - m_p \ddot{u}_2 = -R_2$$

$$E u_{3,111} - \rho I \ddot{u}_{3,1} - E A u_{1,1} u_{3,1} - m_p \ddot{u}_3 = -R_3$$

$$E u_{3,11} + J_d \ddot{u}_{3,1} - (J_0 - J_d)(\Omega + \omega') \dot{u}_{2,1} = -M_{R2}$$

$$E u_{2,11} + J_d \ddot{u}_{2,1} + (J_0 - J_d)(\Omega + \omega') \dot{u}_{3,1} = M_{R3} \quad (6)$$

in which:

m_T – mass of abrasive tool

J_0 – inertial moment of the grinding wheel with respect to the symmetry axis;

J_d – inertial moment of the grinding wheel with respect to a diametral axis;

R_2, R_3, M_{R2}, M_{R3} – components of the external force and moment produced by the abrasive process.

Observation: Since the deformations of the main spindle with respect to the proper reference system of the spindle cannot be measured due to the rotation movement, they are projected on the axes of a fixed reference system.

The relations for transforming the fixed system coordinates are:

$$[Q] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\Omega + \omega') & -\sin(\Omega + \omega') \\ 0 & \sin(\Omega + \omega') & \cos(\Omega + \omega') \end{bmatrix} \quad (7)$$

This represents the reference system change matrix.

If we denote by $\vec{v} = v_1 \vec{i}_{1f} + v_2 \vec{i}_{2f} + v_3 \vec{i}_{3f}$, the point displacement on the spindle axis can be written as:

$$\begin{Bmatrix} v_1 \\ v_2 \\ v_3 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\Omega + \omega') & -\sin(\Omega + \omega') \\ 0 & \sin(\Omega + \omega') & \cos(\Omega + \omega') \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \quad (8)$$

and, conversely,

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\Omega + \omega') & \sin(\Omega + \omega') \\ 0 & -\sin(\Omega + \omega') & \cos(\Omega + \omega') \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \end{Bmatrix} \quad (9)$$

In more detail:

$$\begin{aligned} u_1 &= v_1 & u_{1,1} &= v_{1,1} \\ \dot{u}_1 &= \dot{v}_1 & u_{1,11} &= v_{1,11} \\ \ddot{u}_1 &= \ddot{v}_1 \end{aligned} \quad (10)$$

$$\begin{aligned} u_2 &= \cos(\Omega + \omega')tv_2 + \sin(\Omega + \omega')tv_3 \\ \dot{u}_2 &= \cos(\Omega + \omega')t(\dot{v}_2 + (\Omega + \omega')v_3) + \\ &+ \sin(\Omega + \omega')t(\dot{v}_3 - (\Omega + \omega')v_2) \end{aligned}$$

$$\begin{aligned} \ddot{u}_2 &= \cos(\Omega + \omega')t(\ddot{v}_2 + 2(\Omega + \omega')\dot{v}_3 - (\Omega + \omega')^2 v_2) + \\ &+ \sin(\Omega + \omega')t(\ddot{v}_3 + 2(\Omega + \omega')\dot{v}_2 - (\Omega + \omega')^2 v_3) \end{aligned}$$

$$u_{2,1} = \cos(\Omega + \omega')tv_{2,1} + \sin(\Omega + \omega')tv_{3,1}$$

$$u_{2,11} = \cos(\Omega + \omega')tv_{2,11} + \sin(\Omega + \omega')tv_{3,11}$$

$$\begin{aligned} u_{2,1111} &= \cos(\Omega + \omega')tv_{2,1111} + \sin(\Omega + \omega')tv_{3,1111} \\ \ddot{u}_{2,11} &= \cos(\Omega + \omega')t(\ddot{v}_{2,11} + 2(\Omega + \omega')\dot{v}_{3,11} - (\Omega + \omega')^2 v_{2,11}) + \\ &+ \sin(\Omega + \omega')t(\ddot{v}_{3,11} - 2(\Omega + \omega')\dot{v}_{2,11} - (\Omega + \omega')^2 v_{3,11}) \end{aligned} \quad (11)$$

$$\begin{aligned} u_3 &= -\sin(\Omega + \omega')tv_2 + \cos(\Omega + \omega')tv_3 \\ u_{3,1111} &= -\sin(\Omega + \omega')tv_{2,1111} + \cos(\Omega + \omega')tv_{3,1111} \\ \dot{u}_3 &= \cos(\Omega + \omega')t(\dot{v}_3 - (\Omega + \omega')v_2) - \\ &- \sin(\Omega + \omega')t(\dot{v}_2 - (\Omega + \omega')v_3) \end{aligned}$$

$$\begin{aligned} \ddot{u}_3 &= \cos(\Omega + \omega')t(\ddot{v}_3 - 2(\Omega + \omega')\dot{v}_2 - (\Omega + \omega')^2 v_3) - \\ &- \sin(\Omega + \omega')t(\ddot{v}_2 + 2(\Omega + \omega')\dot{v}_3 - (\Omega + \omega')^2 v_2) \\ \ddot{u}_{3,11} &= \cos(\Omega + \omega')t(\ddot{v}_{3,11} - 2(\Omega + \omega')\dot{v}_{2,11} - (\Omega + \omega')^2 v_{3,11}) - \\ &- \sin(\Omega + \omega')t(\ddot{v}_{2,11} + 2(\Omega + \omega')\dot{v}_{3,11} - (\Omega + \omega')^2 v_{2,11}) \end{aligned} \quad (12)$$

The forces and torques in the reference system are given by:

$$\begin{aligned} F_{11} &= F_{11}^f \\ F_{21} &= \cos(\Omega + \omega')tF_{21}^f + \sin(\Omega + \omega')tF_{31}^f \\ F_{31} &= -\sin(\Omega + \omega')tF_{21}^f + \cos(\Omega + \omega')tF_{31}^f \\ M_{21} &= \cos(\Omega + \omega')tM_{21}^f + \sin(\Omega + \omega')tM_{31}^f \\ M_{31} &= -\sin(\Omega + \omega')tM_{21}^f + \cos(\Omega + \omega')tM_{31}^f \end{aligned} \quad (13)$$

The movement equations become:

$$\begin{aligned} \rho A \ddot{v}_1 - EA v_{1,11} &= F_{11}^f \delta(x_1 - 30) + F_{12}^f \delta(x_1 - 170) - \\ &- c_{11}^f \dot{v}_1 [\delta(x_1 - 30) + \delta(x_1 - 170)] \\ \rho A \ddot{v}_3 - \rho I \ddot{v}_{3,11} + 2\rho I(\Omega + \omega')\dot{v}_{2,11} + \rho I(\Omega + \omega')^2 v_{3,11} + \\ &+ EI v_{3,1111} &= M_{21}^f \delta(x_1 - 30) + M_{22}^f \delta(x_1 - 170) + \\ &+ F_{31}^f \delta(x_1 - 30) + F_{32}^f \delta(x_1 - 170) - c_{33}^f \dot{v}_3 [\delta(x_1 - 30) + \\ &+ \delta(x_1 - 170)] \\ \rho A \ddot{v}_2 + \rho I \ddot{v}_{2,11} + 2\rho I(\Omega + \omega')\dot{v}_{3,11} - \rho I(\Omega + \omega')^2 v_{2,11} - \\ &- EI v_{2,1111} &= M_{31}^f \delta(x_1 - 30) + M_{32}^f \delta(x_1 - 170) - \\ &- F_{21}^f \delta(x_1 - 30) - F_{22}^f \delta(x_1 - 170) + c_{22}^f \dot{v}_2 [\delta(x_1 - 30) + \\ &+ \delta(x_1 - 170)] \end{aligned} \quad (14)$$

The limit conditions will be:

$$\begin{aligned}
 EAv_{1,1} &= 0 \\
 EIV_{2,111} - \rho I \ddot{v}_{2,1} - 2\rho I(\Omega + \omega') \dot{v}_{3,1} + \rho I(\Omega + \omega')^2 v_{2,1} - \\
 - EAv_{1,1} v_{2,1} &= 0 \\
 EIV_{3,111} - \rho I \ddot{v}_{3,1} + 2\rho I(\Omega + \omega') \dot{v}_{2,1} + \rho I(\Omega + \omega')^2 v_{3,1} - \\
 - EAv_{1,1} v_{3,1} &= 0 \\
 EIV_{2,11} &= 0 \\
 EIV_{3,11} &= 0 \\
 EAv_{1,1} + m_p \ddot{v}_1 &= R_1^f \\
 EIV_{2,111} - \rho I \ddot{v}_{2,1} + 2\rho I(\Omega + \omega') \dot{v}_{3,1} + \rho I(\Omega + \omega')^2 v_{2,1} - \\
 EAv_{1,1} v_{2,1} - m_p (\ddot{v}_2 + 2(\Omega + \omega') \dot{v}_3 - (\Omega + \omega')^2 v_2) &= \\
 = -R_2^f \\
 EIV_{3,111} - \rho I \ddot{v}_{3,1} + 2\rho I(\Omega + \omega') \dot{v}_{2,1} + \rho I(\Omega + \omega')^2 v_{3,1} - \\
 EAv_{1,1} v_{3,1} - m_p (\ddot{v}_3 + 2(\Omega + \omega') \dot{v}_2 - (\Omega + \omega')^2 v_3) &= \\
 = -R_3^f \\
 EIV_{3,11} + J_d (\ddot{v}_{3,1} + 2(\Omega + \omega') \dot{v}_{2,1} - (\Omega + \omega')^2 v_{3,1}) + \\
 + (J_0 - J_d)(\Omega + \omega') \cdot (\dot{v}_{2,1} - v_{3,1}) &= -M_{R2}^f \\
 EIV_{2,11} + J_d (\ddot{v}_{2,1} + 2(\Omega + \omega') \dot{v}_{3,1} - (\Omega + \omega')^2 v_{3,1}) + \\
 + (J_0 - J_d)(\Omega + \omega') (\dot{v}_{3,1} - (\Omega + \omega') v_{2,1}) &= M_{R3}^f
 \end{aligned} \tag{15}$$

In order to establish the balance configuration, we consider that on the ensemble spindle-bearings-tool actions a system of external actions that produces a deformation of the spindle, without causing it to vibrate [5].

At balance, the initial conditions become:

$$\begin{aligned}
 EAv_{1,11} &= 0 \\
 \rho I(\Omega + \omega')^2 v_{2,11} + EIV_{2,1111} &= 0 \\
 \rho I(\Omega + \omega')^2 v_{3,11} + EIV_{3,1111} &= 0
 \end{aligned} \tag{16}$$

and:

$$\begin{aligned}
 x_1 &= 0 \\
 EAv_{1,1}^{(0)}(0) &= 0 \\
 EIV_{2,111}^{(0)}(0) - \rho I(\Omega + \omega')^2 v_{2,1}^{(0)}(0) - EAv_{1,1}^{(0)}(0) v_{2,1}^{(0)}(0) &= 0 \\
 EIV_{3,111}^{(0)}(0) - \rho I(\Omega + \omega')^2 v_{3,1}^{(0)}(0) - EAv_{1,1}^{(0)}(0) v_{3,1}^{(0)}(0) &= 0
 \end{aligned}$$

$$\begin{aligned}
 EIV_{2,11}^{(0)}(0) &= 0 \\
 EIV_{3,11}^{(0)}(0) &= 0 \\
 x_1 &= 30 : x_1 = 170 \\
 [EAv_{1,1}]_{x_1=30^\circ;170^\circ} &= F_{ij}^{f*} \\
 [EIV_{2,111} + \rho I(\Omega + \omega')^2 v_{2,1} - EAv_{1,1} v_{2,1}]_{x_1=30^\circ;170^\circ} &= \\
 = -F_{1+1,j}^{f*} \\
 [EIV_{3,111} + \rho I(\Omega + \omega')^2 v_{3,1} - EAv_{1,1} v_{3,1}]_{x_1=30^\circ;170^\circ} &= \\
 = -F_{1+2,j}^{f*} \\
 [EAv_{3,11}]_{x_1=30^\circ;170^\circ} &= -M_{i+1,j}^{f*} \\
 [EAv_{2,11}]_{x_1=30^\circ;170^\circ} &= M_{i+2,j}^{f*} \\
 v_1^{(0,1)}(x_1) &= v_1^{(1,2)}(x_1) \\
 v_2^{(0,1)}(x_1) &= v_2^{(1,2)}(x_1) \\
 v_3^{(0,1)}(x_1) &= v_3^{(1,2)}(x_1) \\
 v_{2,1}^{(0,1)}(x_1) &= v_{2,1}^{(1,2)}(x_1) \\
 v_{3,1}^{(0,1)}(x_1) &= v_{3,1}^{(1,2)}(x_1)
 \end{aligned} \tag{17}$$

$$\begin{aligned}
 x_1 &= 250 \\
 EAv_{1,1}^{(2)}(x_1) &= R_1^{f*} \\
 EIV_{2,111}^{(2)}(x_1) + \rho I(\Omega + \omega')^2 v_{2,1}^{(2)}(x_1) - \\
 - EAv_{1,1}^{(2)}(x_1) v_{2,1}^{(2)}(x_1) + m_p \Omega^2 v_2^{(2)}(x_1) &= -R_2^{f*} \\
 EIV_{3,111}^{(2)}(x_1) + \rho I(\Omega + \omega')^2 v_{3,1}^{(2)}(x_1) - \\
 - EAv_{1,1}^{(2)}(x_1) v_{3,1}^{(2)}(x_1) + m_p \Omega^2 v_3^{(2)}(x_1) &= -R_3^{f*} \\
 EIV_{3,11}^{(2)}(x_1) + J_0(\Omega + \omega')^2 v_{3,1}^{(2)}(x_1) &= -MR_2^{f*} \\
 EIV_{2,11}^{(2)}(x_1) - J_0(\Omega + \omega')^2 v_{2,1}^{(2)}(x_1) &= MR_3^{f*}
 \end{aligned} \tag{18}$$

$$\begin{aligned}
 EIV_{2,11}^{(2)}(x_1) - J_0(\Omega + \omega')^2 v_{2,1}^{(2)}(x_1) &= MR_3^{f*}
 \end{aligned} \tag{19}$$

III. CONCLUSIONS

- Insertion of external strains concerns the movement initial conditions and the geometric configuration of the main spindle.
- At balance, the initial conditions regard the axial cutting force and the bending moments determined for the main characteristic points.
- Establishing the final form of the movement equations and the balance conditions creates the premises for the stability analysis of the system composed of the main spindle and abrasive tool.

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