Two Novel Characterizations of the DE Flip Flop

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Abstract. Modern digital circuits, especially those based on large-scale integration devices employ DE flip flops, which are an extension of the D type with the capacity to store an input value only upon request or enabling. The DE flip flop could possibly be described algebraically by its characteristic equation or tabularly by its next-state table (used for analysis purposes) and its excitation table (used for synthesis purposes). This paper explores two novel characterizations of DE flip flops. First, equational and implicational descriptions are presented, and the Modern Syllogistic Method is utilized to produce complete statements of all propositions that are true for a general DE flip flop. Next, methods of Boolean-equation solving are employed to find all possible ways to express the excitations in terms of the present state and next state. The concept of Boolean quotient plays a crucial role in exposing the pertinent concepts and implementing the various desired derivations. This paper is expected to be of an immediate pedagogical benefit, and to facilitate the analysis and synthesis of contemporary sequential digital circuits.

Keywords: Flip flop, Characterization, Modern Syllogistic Method, Boolean-equation solving, Boolean quotient, Excitation.

1. Introduction

An essential building block of a sequential digital circuit is an elementary memory cell called a flip flop, a one-bit register, a bistable or a bistable multivibrator^[1]. A flip-flop is a circuit that has two stable states and can be used to store a single bit (binary digit) of data; with one of the two states of the flip flop representing a "one" and the other representing a "zero". A flip flop is usually named in terms of its inputs or excitations. In the following, we use the symbols y_i and Y_i , to denote the present and next states of flip flop number i in a given circuit. We also use upper-case subscripted letters to designate the excitations of a flip flop.

There are many well-known types of flip flops, including those with a single input (such as the D flip flop and the T flip flop), those with two inputs (such as the SR flip flop, the JK flip flop, and the DE flip flop), and those with three or more inputs. Out of these, we single out the JK flip-flop as a very versatile device that was commonly used to construct circuits with discrete components. The JK flip flop is discussed in many publications on digital design (See, e.g., ^[1-8]) and its various characterizations are detailed in ^[1]. However, it has been mostly replaced by a variation of the D flip flop, which is conveniently called the DE flip flop (or occasionally, the E-type flip flop)^[1,9-12]. This variation is simply a D flip flop that is equipped with an enable input E .

This paper is a tutorial exposition about flip flops in general, and about *DE* flip flops, in particular. After surveying conventional characterizations of *DE* flip flops (in comparison with those of *JK* flip flops), we employ novel mathematical methods of logic deduction and Boolean-equation solving for further characterization of the *DE* flip flop.

The organization of the remainder of this paper is as follows. Section 2 offers an introductory characterization of the *DE* flip flop in comparison with the *JK* flip flop. Section 3 uses the Modern Syllogistic Method (MSM) of logic deduction to ferret out all that can be said about the *DE* flip flop in the most compact form. Section 4 applies methods of 'big' Boolean-equation solving to find all possible solutions of the excitations of the *DE* flip flop in terms of its present and next states. Section 5 concludes the paper. To make the paper self-contained, it is supplemented with an appendix on the "Boolean Quotient", a crucial concept for some of the current derivations.

2. The DE Flip Flop Versus the JK Flip Flop

We recall that the *DE* flip flop is a variation of the *D* flip flop, which is simply a *D* flip flop that is equipped with an enable input E ^[1,9-12]. Despite the contemporary widespread use of the *DE* flip flop, it is definitely less familiar (to most readers) than the *JK* flip flop. We devote this section to a brief introduction of the *DE* flip flop, in comparison to the *JK* flip flop. Figure 1 presents the characteristic maps of both types of flip flops, in both conventional-map form and variable-entered form. Figure 2 displays the transition maps for both types of flip flops. The transition variable δy_i is a four-valued variable defined to be $\delta y_i = 0$ when $y_i = Y_i = 0$, $\delta y_i = 1$ when $y_i = Y_i = 1$, $\delta y_i = \Delta$ when $y_i = 0$ and $Y_i = 1$, and finally $\delta y_i = \nabla$ when $y_i = 1$ and $Y_i = 0$ ^[9]. Figure 3 presents

the excitation maps for both the *JK* and the *DE* flip flops. Of course, the excitation map of the *DE* flip flop (as shown in Fig. 3(b)) is virtually unknown in the literature, but it will be verified and formally derived in the forthcoming sections.

We now digress a little bit to explain why the *JK* flip flop is superior from the theoretical point of view, and why it was dominantly used in the era of discrete components. A quick glimpse at Fig. 1-3 demonstrates clearly that the *JK* flip flop has a more balanced behavior and a more versatile capability than the *DE* flip flop. In fact, the *JK* flip flop enjoys the following distinctive advantages:

- By contrast to the *SR* flip-flop, which has an indeterminate outcome when $S_i = R_i = 1$, (or must, otherwise, be operated under the constraint $S_i R_i = 0$), the *JK* flip-flop has the merit of guaranteed determined outcome under unconstrained operation.

- The *JK* flip-flop is intimately related to other famous flip flops. It is exactly an *SR* flip flop with an additional external feedback provided *via* the relations $S_i = J_i \bar{y}_i$ and $R_i = K_i y_i$. These relations ensure the satisfaction of the aforementioned constraint ($S_i R_i = 0$) and allow the designer, if he/she wishes, to dispense with external feedback from the outputs of a particular flip flop to its own inputs. The *JK* flip-flop also combines the *delay* function of a *D* flip-flop and the *toggle* function of a *T* flip-flop. The *JK* flip-flop reduces to a *D* flip-flop by making its excitations complementary ($J_i = D_i$, $K_i = \bar{D}_i$), and reduces to a *T* flip-flop by making its excitations equal ($J_i = K_i = T_i$).

- The *JK* flip-flop possesses the minimum number of excitations that can control its next-state variable to assume any of the four possible values that might be taken by

a Boolean function of a single variable representing the present-state value, namely: the present state itself, its complement, logic 0, and logic 1. In fact the algebraic characteristic equation (next-state equation) of the *JK* flip-flop

$$Y_i = J_i \bar{y}_i \vee \bar{K}_i y_i, \quad (1)$$

might be rewritten in the minterm-expansion form

$$Y_i = (y_i) \bar{J}_i \bar{K}_i \vee (0) \bar{J}_i K_i \vee (\bar{y}_i) J_i K_i \vee (1) J_i \bar{K}_i. \quad (2)$$

Equations (1) and (2) can be readily deduced from Fig. 1.

- The excitations of a *JK* flip-flop can be made independent of its present state by setting them as the Boolean quotients (See Appendix A) ^[1]

$$J_i = Y_i / \bar{y}_i, \quad (3a)$$

$$K_i = \bar{Y}_i / y_i. \quad (3b)$$

As stated earlier, a modern digital network, especially one that is based on large-scale integration devices (such as a Field-Programmable Gate Array (*FPGA*)) employs a *DE* flip flop which is an extension of the *D* type ^[1, 9-12]. This flip flop is similar to a standard *D* flip flop except that the *D* input is only enabled when the input *E* (for “Enable”) is equal to logic 1. When the input *E* is equal to logic 0, the flip flop remains in its current state. Hence, the *DE* flip flop has the capacity to store an input value only upon request or enabling. This behavior differs from that of the *D* flip flop, which stores a new value (unconditionally) at each active edge of the clock. The characteristic equation for the *DE* flip flop might be written as

$$Y_i = y_i \bar{E}_i \vee D_i E_i. \quad (4)$$

Detailed information about the *DE* flip flop is available in ^[11], where it is given the designation *E-PET* with “*PET*” standing for

“Positive-Edge-Triggered”. Equation (4) might be rewritten as a minterm expansion in the form

$$Y_i = (y_i) \bar{D}_i \bar{E}_i \vee (0) \bar{D}_i E_i \vee (1) D_i E_i \vee (y_i) D_i \bar{E}_i \quad (5)$$

Again, Equations (4) and (5) can be deduced from Fig. 1. Note that Y_i for a *DE* flip flop can assume any of the *three* values 0, 1, and y_i , while for a *JK* flip flop, it can assume any of the *four* values 0, 1, y_i , and $\bar{y}_i$. Though the *DE* flip flop seems, theoretically, less versatile than the *JK* flip flop, it is by no means inferior to it within the modern circuits in which it is used such as an *FPGA*, in which a look-up table (*LUT*) is used rather than conventional logic gates ^[9-12].

3. Equational and Implicational Descriptions

In this section, we use the Modern Syllogistic Method (*MSM*) ^[13-23] to ferret out from the characteristic Equation (4) (viewed as a premise) all that can be concluded from it, with the resulting conclusions cast in the simplest or most compact form. A similar study was conducted in ^[1] for the *JK* flip flop.

First, we reformulate (4) as an equation whose R.H.S. is 0, *i.e.*,

$$f = f(Y_i, y_i; D_i, E_i) = 0, \quad (6)$$

where

$$\begin{aligned} f &= Y_i \oplus (y_i \bar{E}_i \vee D_i E_i) \\ &= Y_i (\overline{y_i \bar{E}_i \vee D_i E_i}) \vee \bar{Y}_i (y_i \bar{E}_i \vee D_i E_i) \\ &= Y_i (\bar{y}_i \bar{E}_i \vee \bar{D}_i E_i) \vee \bar{Y}_i (y_i \bar{E}_i \vee D_i E_i). \end{aligned} \quad (7)$$

Then we replace f in (7) by its complete-sum form using the Blake-Tison Method ^[13-27]. We note that (7) involves four variables, which are all biform. There is no consensus *w.r.t.* any of the three variables Y_i , y_i , and D_i . However, there are two consensuses $Y_i \bar{y}_i \bar{D}_i$ and $\bar{Y}_i y_i D_i$, which are obtained *w.r.t.* E_i . The resulting formula is absorptive (*i.e.*, it has no term that can absorb another) ^[13], and hence it represents the

complete sum $CS(f)$ of f , and by virtue of (6), we obtain

$$CS(f) = Y_i(\bar{y}_i\bar{E}_i \vee \bar{D}_iE_i \vee \bar{y}_i\bar{D}_i) \vee \bar{Y}_i(y_i\bar{E}_i \vee D_iE_i \vee y_iD_i) = 0. \quad (8)$$

The expression in (8) is a disjunction of six terms equated to zero. This is exactly equivalent to each of the terms in (8) being individually equated to zero. These six equations (See Table 1) constitute all the propositions that can be stated about the DE flip flop. However, we might use the equivalence^[13]

$$\{A \rightarrow B\} \equiv \{A\bar{B} = 0\}, \quad (9)$$

to convert each of the six equational statements in Table 1 to any of eight equivalent implicational forms, as shown in Table 1.

Complete information about the DE flip flop is possible by using one of the nine equivalent statements given in each of the six main (major or double) rows in Table 1. Though Table 1 provides a wealth of facts about the DE flip flops, many of these facts are redundant as they are deducible from other facts. In fact, only four independent statements suffice (albeit with some inconvenience occasionally) to characterize the DE flip flop. By the "independence" requirement we rule out the following cases:

a) Selection of the first three successive equations $Y_i\bar{y}_i\bar{E}_i = 0$, $Y_i\bar{D}_iE_i = 0$, and $Y_i\bar{y}_i\bar{D}_i = 0$ or the next three consecutive equations $\bar{Y}_iy_i\bar{E}_i = 0$, $\bar{Y}_iD_iE_i = 0$, and $\bar{Y}_iy_iD_i = 0$. In each case, the third equation is simply the consensus of the former two, and is deducible from them.

b) Selection of two statements that belong to the same major double row, since the nine statements in the same major double row are simply equivalent.

c) Selection of three statements that belong to the first three major rows or to the last three major rows.

The simplest characterization is naturally the characterization *via* the equational forms in major rows 1, 2, 4, and 5, (highlighted in blue in Table 1). These equations are deducible from the original characteristic function (7) equated to zero, and they are neutral about utility to analysis or synthesis. By contrast, there are six redundant analysis-oriented implicational characterizations (highlighted in yellow), in which the antecedents depend on the excitations and the consequents depend on the present and next states. There are also six redundant synthesis-oriented implicational characterizations (highlighted in green), in which the antecedents depend on the present and next states and the consequents depend on the excitations. Let us consider the four analysis-oriented implicational statements selected from major rows 1, 2, 4, and 5, namely

$$\bar{E}_i \rightarrow \bar{Y}_i \vee y_i, \quad (10a)$$

$$\bar{D}_iE_i \rightarrow \bar{Y}_i, \quad (10b)$$

$$\bar{E}_i \rightarrow Y_i \vee \bar{y}_i, \quad (10c)$$

$$D_iE_i \rightarrow Y_i. \quad (10d)$$

The implicational statements (10) have the excitation E_i and (possibly) the excitation D_i in the antecedents of the implications, and have the next state Y_i and (possibly) the present state y_i in the consequents. Using techniques of Boolean reasoning, we can view Equations (10) as a precise translation of the characteristic map of the DE flip flop (Figs. 1(b) and 1(d)). The implication $\bar{D}_iE_i \rightarrow \bar{Y}_i$ in (10b) is equivalent to the second column in Fig. 1(b) or Fig. 1(d), which can be read as

$$\{D_i = 0, E_i = 1\} \rightarrow \{Y_i = 0\}. \quad (11)$$

Note that this implication keeps silent about y_i , which is its way of saying that y_i is a don't care and could be either 0 or 1, when $D_i = 0$ and $E_i = 1$. Likewise, the implication $D_i E_i \rightarrow Y_i$ in (10d) is equivalent to the third column in Fig. 1(b) or Fig. 1(d), which can be read as

$$\{D_i = 1, E_i = 1\} \rightarrow \{Y_i = 1\}. \quad (12)$$

Finally, the two implications in (10a) and (10d) can be combined as

$$\begin{aligned} \{E_i = 0\} \rightarrow \{(\bar{Y}_i \vee y_i)(Y_i \vee \bar{y}_i) = 1\} \\ = \{(y_i \equiv Y_i) = 1\}, \end{aligned} \quad (13)$$

which is equivalent to the first and fourth columns combined in Fig. 1(b) or Fig. 1(d).

We now consider the four synthesis-oriented implicational statements selected from major rows 1, 2, 4, and 5, namely

$$Y_i \bar{y}_i \rightarrow E_i, \quad (14a)$$

$$Y_i \rightarrow D_i \vee \bar{E}_i, \quad (14b)$$

$$\bar{Y}_i y_i \rightarrow E_i, \quad (14c)$$

$$\bar{Y}_i \rightarrow \bar{D}_i \vee \bar{E}_i. \quad (14d)$$

The implications in (14) are the converses of those in (10) and hence they have antecedents involving the next state Y_i and (possibly) the present state y_i and consequents involving the excitation E_i and (possibly) the excitation D_i . The implications in (14) are precisely equivalent to the excitation map of the DE flip flop (Fig. 3(b)). The conditions (14b) and (14d) might be expanded as

$$Y_i \rightarrow \bar{D}_i \bar{E}_i \vee D_i \bar{E}_i \vee D_i E_i, \quad (14b1)$$

$$\bar{Y}_i \rightarrow \bar{D}_i \bar{E}_i \vee \bar{D}_i E_i \vee D_i \bar{E}_i. \quad (14d1)$$

These mean that if $Y_i = 1$ then (D_i, E_i) belongs to $S_1 = \{(0,0), (1,0), (1,1)\} = \{(d,0), (1,1)\}$. The condition (14a) means that if further to $Y_i = 1$, we have $y_i = 0$ then $E_i = 1$, and hence S_1 reduces to $\{(1,1)\}$, i.e., $D_i = 1$. If $Y_i = 0$ then (D_i, E_i) belongs to $S_2 = \{(0,0), (0,1), (1,0)\} = \{(0,d), (d,0)\}$.

The condition (14c) means that if further to $Y_i = 0$, we have $y_i = 1$ then $E_i = 1$, and hence S_2 reduces to $\{(0,1)\}$, i.e., $D_i = 0$. These arguments suffice (albeit with difficulty) to verify all entries in the two excitation maps of Fig. 3(b). It would have been more convenient if we complete the picture by augmenting Equations (14) by the two synthesis-oriented implicational statements in major rows 3 and 6, namely

$$Y_i \bar{y}_i \rightarrow D_i, \quad (14e)$$

$$\bar{Y}_i y_i \rightarrow \bar{D}_i. \quad (14f)$$

4. Boolean-Equation Solving for Excitations

In this section, we employ methods of Boolean-equation solving^[27-35] to obtain all possible solutions for the excitations of a DE flip flop in terms of its present state and next state. We consider a single flip flop i and seek solutions of its excitations D_i and E_i in terms of its present state y_i and next state Y_i . The characteristic equation of this flip flop (4) is our starting point. To solve for D_i and E_i in terms of Y_i and y_i , we first convert (4) to the form of a single equation of a function equated to 1, i.e., to the complement of (6), namely

$$g(Y_i, y_i; D_i, E_i) = 1, \quad (15)$$

where g is the complement of f in (7), and is given by

$$\begin{aligned} g &= Y_i \odot (y_i \bar{E}_i \vee D_i E_i) \\ &= Y_i (y_i \bar{E}_i \vee D_i E_i) \vee \bar{Y}_i (\bar{y}_i \bar{E}_i \vee \bar{D}_i E_i) \\ &= (Y_i y_i \vee \bar{Y}_i \bar{y}_i) \bar{D}_i \bar{E}_i \vee (Y_i \bar{y}_i \vee \bar{Y}_i y_i) D_i E_i \\ &\quad \vee (Y_i y_i \vee \bar{Y}_i \bar{y}_i) D_i \bar{E}_i \\ &\quad \vee (Y_i) D_i E_i. \end{aligned} \quad (16)$$

The function g in (16) can be viewed as $g(D_i, E_i)$ where $g: B^2 \rightarrow B$, and $B = B_{16} = FB(Y_i, y_i)$ is the free Boolean algebra with 2 generators Y_i and y_i , $2^2 = 4$ atoms given by $\bar{Y}_i \bar{y}_i, \bar{Y}_i y_i, Y_i \bar{y}_i$ and $Y_i y_i$, and $2^4 = 16$ elements. These elements can be identified as all the switching functions of the two variables Y_i and y_i . Figure 4(a) is the natural (also

called variable-entered) map for $g(D_i, E_i)$. Figure 4(b) is a replica of Fig. 4(a) with its map entries being expanded as disjunctions of minterms of Y_i and y_i (i.e., as atoms of $FB(Y_i, y_i)$). The four atoms $\bar{Y}_i\bar{y}_i$, $\bar{Y}_iy_i$, $Y_i\bar{y}_i$, and Y_iy_i make their appearances in the cells of the map of Fig. 4(b) 3, 1, 1, and 3 times, respectively. Since none of the four atoms is absent in Fig. 4(b), the consistency condition for Equation (15) is satisfied trivially as an identity $\{0 = 0\}$. This means that Equation (15) is unconditionally consistent. It has a number of particular solutions equal to $3 * 1 * 1 * 3 = 9$ [1, 30, 32, 33].

Figure 4(b) can be used to construct the auxiliary function $G(D_i, E_i, \mathbf{p})$ in Fig. 5. Each of the appearances of the four atoms $\bar{Y}_i\bar{y}_i$, $\bar{Y}_iy_i$, $Y_i\bar{y}_i$ and Y_iy_i in Fig. 4(b), is appended in Fig. 5 by a distinguishing binary tag selected from the orthonormal sets $\{\bar{p}_1\bar{p}_2, \bar{p}_1p_2, p_1\}, \{1\}, \{1\}, \{\bar{p}_3\bar{p}_4, \bar{p}_3p_4, p_3\}$, respectively [32, 33]. For example, Fig. 5 shows the atom $\bar{Y}_i\bar{y}_i$ appended (ANDed with) $\bar{p}_1\bar{p}_2$ in the cell $\bar{D}_i\bar{E}_i$, appended with $\bar{p}_1p_2$ in the cell $D_i\bar{E}_i$, and appended with p_1 in the cell $\bar{D}_iE_i$. Finally the solution for D_i and E_i is written as [32, 33]

$$\begin{aligned} D_i &= \text{Disjunction of entries of the domain } D_i \text{ in Fig. 5} \\ &= \bar{Y}_i\bar{y}_i\bar{p}_1p_2 \vee Y_iy_i\bar{p}_3p_4 \vee Y_i\bar{y}_i(1) \\ &\quad \vee Y_iy_ip_3 \\ &= \bar{y}_i\bar{p}_1p_2 \vee Y_i\bar{y}_i \vee Y_iy_i(p_3 \vee p_4). \end{aligned} \quad (17a)$$

$$\begin{aligned} E_i &= \text{Disjunction of entries of the domain } E_i \text{ in Fig. 5} \\ &= \bar{Y}_i\bar{y}_ip_1 \vee \bar{Y}_iy_i(1) \vee Y_i\bar{y}_i(1) \vee Y_iy_ip_3 \\ &= \bar{y}_ip_1 \vee \bar{Y}_iy_i \vee Y_i\bar{y}_i \vee y_ip_3. \end{aligned} \quad (17b)$$

Equations (17) are a parametric solution for D_i and E_i , where each of the four independent parameters p_1, p_2, p_3 and p_4 belongs to $B_2 = \{0, 1\}$, and hence (17) can be used to deduce the nine particular solutions of (15). These nine particular solutions for D_i and E_i in terms of Y_i and y_i are shown in algebraic and map forms in Figs. 6 and 7, respectively.

We might elect to replace the two parameters p_3 and p_4 by the two parameters p_1 and p_2 (provided we let these parameters p_1 and p_2 belong to the underlying Boolean algebra ($B_{16} = FB(Y_i, y_i)$)). In this case, the solution becomes

$$D_i = \bar{y}_i\bar{p}_1p_2 \vee Y_i\bar{y}_i \vee Y_iy_i(p_1 \vee p_2), \quad (18a)$$

$$E_i = p_1 \vee \bar{Y}_iy_i \vee Y_i\bar{y}_i. \quad (18b)$$

Equations (18) are another parametric solution for (15) that uses a minimum number of parameters (two) belonging to $B_{16} = FB(Y_i, y_i)$. The solutions (17) and (18) are equivalent, since they produce the same set of nine particular solutions shown in Figs. 7 and 8.

An alternative way to solve for D_i and E_i is to use the concept of atomic decomposition [35]. Figures 8(a) and 8(b) present the atomic decompositions of the variables D_i and E_i , namely

$$D_i = (D_{i0})\bar{Y}_i\bar{y}_i \vee (D_{i1})\bar{Y}_iy_i \vee (D_{i2})Y_i\bar{y}_i \vee (D_{i3})Y_iy_i, \quad (19a)$$

$$E_i = (E_{i0})\bar{Y}_i\bar{y}_i \vee (E_{i1})\bar{Y}_iy_i \vee (E_{i2})Y_i\bar{y}_i \vee (E_{i3})Y_iy_i, \quad (19b)$$

where the four atomic components of $D(D_{i0}, D_{i1}, D_{i2}, D_{i3})$ and those of $E(E_{i0}, E_{i1}, E_{i2}, E_{i3})$ are arbitrary binary values. Substituting these decompositions into (16) we obtain the atomic decomposition of $g(D_i, E_i)$ as

$$\begin{aligned} g(D_i, E_i) &= (\bar{E}_i \vee \bar{D}_iE_i) \bar{Y}_i\bar{y}_i \vee (\bar{D}_iE_i) \bar{Y}_iy_i \vee (D_iE_i) Y_i\bar{y}_i \\ &\quad \vee (\bar{E}_i \vee D_iE_i) Y_iy_i \\ &= ((\bar{E}_{i0} \vee \bar{D}_{i0}E_{i0}) \bar{Y}_i\bar{y}_i \vee (\bar{E}_{i1} \vee \bar{D}_{i1}E_{i1}) \bar{Y}_iy_i \\ &\quad \vee (\bar{E}_{i2} \vee \bar{D}_{i2}E_{i2}) Y_i\bar{y}_i \\ &\quad \vee (\bar{E}_{i3} \vee \bar{D}_{i3}E_{i3}) Y_iy_i) \bar{Y}_i\bar{y}_i \\ &\quad \vee ((\bar{D}_{i0}E_{i0}) \bar{Y}_i\bar{y}_i \vee (\bar{D}_{i1}E_{i1}) \bar{Y}_iy_i \\ &\quad \vee (\bar{D}_{i2}E_{i2}) Y_i\bar{y}_i \vee (\bar{D}_{i3}E_{i3}) Y_iy_i) \bar{Y}_iy_i \\ &\quad \vee ((D_{i0}E_{i0}) Y_i\bar{y}_i \vee (D_{i1}E_{i1}) Y_iy_i) Y_i\bar{y}_i \\ &\quad \vee ((D_{i2}E_{i2}) Y_i\bar{y}_i \vee (D_{i3}E_{i3}) Y_iy_i) Y_iy_i \\ &\quad \vee ((\bar{E}_{i0} \vee D_{i0}E_{i0}) \bar{Y}_i\bar{y}_i \\ &\quad \vee (\bar{E}_{i1} \vee D_{i1}E_{i1}) \bar{Y}_iy_i \\ &\quad \vee (\bar{E}_{i2} \vee D_{i2}E_{i2}) Y_i\bar{y}_i \\ &\quad \vee (\bar{E}_{i3} \vee D_{i3}E_{i3}) Y_iy_i) Y_iy_i \\ &= (\bar{E}_{i0} \vee \bar{D}_{i0}E_{i0}) \bar{Y}_i\bar{y}_i \vee (\bar{D}_{i1}E_{i1}) \bar{Y}_iy_i \vee (D_{i2}E_{i2}) Y_i\bar{y}_i \\ &\quad \vee (\bar{E}_{i3} \vee D_{i3}E_{i3}) Y_iy_i \end{aligned} \quad (20)$$

Figure 8(c) illustrates the atomic decomposition of $g(D_i, E_i)$ as given by (20). The equation to be solved forces each individual entry in the map of Fig. 8(c) to be 1. Hence, we obtain

$$\bar{E}_{i0} \vee \bar{D}_{i0} E_{i0} = 1, \quad (21a)$$

$$\bar{D}_{i1} E_{i1} = 1, \quad (21b)$$

$$D_{i2} E_{i2} = 1, \quad (21c)$$

$$\bar{E}_{i3} \vee D_{i3} E_{i3} = 1. \quad (21d)$$

The solutions of Equations (21) are precisely those reported in Fig. 7. For convenience, we follow Brown^[36] in expressing the parametric solutions (18) for D_i and E_i in terms of Boolean quotients (See Appendix A), namely

$$D_i = (D_i/\bar{y}_i)\bar{y}_i \vee (D_i/y_i)y_i, \quad (22a)$$

$$E_i = (E_i/\bar{y}_i)\bar{y}_i \vee (E_i/y_i)y_i. \quad (22b)$$

Here, the Boolean quotients are

$$D_i/\bar{y}_i = \bar{p}_1 p_2 \vee Y_i, \quad (23a)$$

$$D_i/y_i = Y_i(p_1 \vee p_2), \quad (23b)$$

$$E_i/\bar{y}_i = p_1 \vee Y_i, \quad (23c)$$

$$E_i/y_i = p_1 \vee \bar{Y}_i. \quad (23d)$$

5. Conclusions

This paper is a tutorial exposition about two widely used two-input types of flip flops, namely, *DE* and *JK* flip flops, with a stress on the *DE* type. The paper contributes two novel characterizations of *DE* flip flops, using the methods of logic deduction and Boolean-equation solving. The immediate benefit to be gained from this paper is that it might help facilitate the analysis and synthesis of sequential digital circuits. Future work in support and continuation of the present analysis might include the provision of some simulation and implementation results.

Table 1. All possible statements that can be made about *DE* flip flops (arranged in six major double rows). Four irredundant equational characterizations are highlighted in blue. Six redundant analysis-oriented implicational characterizations are highlighted in yellow. Six redundant synthesis-oriented implicational characterizations are highlighted in green.

Equational form	Implicational form			
$Y_i \bar{y}_i \bar{E}_i = 0$	$Y_i \bar{y}_i \bar{E}_i \rightarrow 0$	$Y_i \bar{y}_i \rightarrow E_i$	$Y_i \bar{E}_i \rightarrow y_i$	$\bar{y}_i \bar{E}_i \rightarrow \bar{Y}_i$
	$Y_i \rightarrow y_i \vee E_i$	$\bar{y}_i \rightarrow \bar{Y}_i \vee E_i$	$\bar{E}_i \rightarrow \bar{Y}_i \vee y_i$	$1 \rightarrow \bar{Y}_i \vee y_i \vee E_i$
$Y_i \bar{D}_i E_i = 0$	$Y_i \bar{D}_i E_i \rightarrow 0$	$Y_i \bar{D}_i \rightarrow \bar{E}_i$	$Y_i E_i \rightarrow D_i$	$\bar{D}_i E_i \rightarrow \bar{Y}_i$
	$Y_i \rightarrow D_i \vee \bar{E}_i$	$\bar{D}_i \rightarrow \bar{Y}_i \vee \bar{E}_i$	$E_i \rightarrow \bar{Y}_i \vee D_i$	$1 \rightarrow \bar{Y}_i \vee D_i \vee \bar{E}_i$
$Y_i \bar{y}_i \bar{D}_i = 0$	$Y_i \bar{y}_i \bar{D}_i \rightarrow 0$	$Y_i \bar{y}_i \rightarrow D_i$	$Y_i \bar{D}_i \rightarrow y_i$	$\bar{y}_i \bar{D}_i \rightarrow \bar{Y}_i$
	$Y_i \rightarrow y_i \vee D_i$	$\bar{y}_i \rightarrow \bar{Y}_i \vee D_i$	$\bar{D}_i \rightarrow \bar{Y}_i \vee y_i$	$1 \rightarrow \bar{Y}_i \vee y_i \vee D_i$
$\bar{Y}_i y_i \bar{E}_i = 0$	$\bar{Y}_i y_i \bar{E}_i \rightarrow 0$	$\bar{Y}_i y_i \rightarrow E_i$	$\bar{Y}_i \bar{E}_i \rightarrow \bar{y}_i$	$y_i \bar{E}_i \rightarrow Y_i$
	$\bar{Y}_i \rightarrow \bar{y}_i \vee E_i$	$y_i \rightarrow Y_i \vee E_i$	$\bar{E}_i \rightarrow Y_i \vee \bar{y}_i$	$1 \rightarrow Y_i \vee \bar{y}_i \vee E_i$
$\bar{Y}_i D_i E_i = 0$	$\bar{Y}_i D_i E_i \rightarrow 0$	$\bar{Y}_i D_i \rightarrow \bar{E}_i$	$\bar{Y}_i E_i \rightarrow \bar{D}_i$	$D_i E_i \rightarrow Y_i$
	$\bar{Y}_i \rightarrow \bar{D}_i \vee \bar{E}_i$	$D_i \rightarrow Y_i \vee \bar{E}_i$	$E_i \rightarrow Y_i \vee \bar{D}_i$	$1 \rightarrow Y_i \vee \bar{D}_i \vee \bar{E}_i$
$\bar{Y}_i y_i D_i = 0$	$\bar{Y}_i y_i D_i \rightarrow 0$	$\bar{Y}_i y_i \rightarrow \bar{D}_i$	$\bar{Y}_i D_i \rightarrow \bar{y}_i$	$y_i D_i \rightarrow Y_i$
	$\bar{Y}_i \rightarrow \bar{y}_i \vee \bar{D}_i$	$y_i \rightarrow Y_i \vee \bar{D}_i$	$D_i \rightarrow Y_i \vee \bar{y}_i$	$1 \rightarrow Y_i \vee \bar{y}_i \vee \bar{D}_i$

(a) The JK flip flop (conventional map)(b) The DE flip flop (conventional map) (c) The JK flip flop (with present state as entered variable)(d) The DE flip flop (with present state as entered variable)**Fig. 1. Characteristic maps for the JK and DE flip flops.** **Fig. 2. Transition maps for the JK and DE flip flops.**

(a) The JK flip flop

(b) The DE flip flop

Fig. 3. Excitation maps for the JK and DE flip flops.

	D_i	
E_i	$Y_i y_i \vee \bar{Y}_i \bar{y}_i$	$Y_i y_i \vee \bar{Y}_i y_i$
	$\bar{Y}_i$	Y_i

(a) $g(D_i, E_i)$

	D_i	
E_i	$\bar{Y}_i \bar{y}_i \vee Y_i y_i$	$\bar{Y}_i \bar{y}_i \vee Y_i y_i$
	$\bar{Y}_i \bar{y}_i \vee \bar{Y}_i y_i$	$Y_i \bar{y}_i \vee Y_i y_i$

(b) $g(D_i, E_i)$ Fig. 4. Natural map for the function $g(D_i, E_i)$ as obtained from (16) and with minterm-expanded entries.

	D_i	
E_i	$\bar{Y}_i \bar{y}_i \bar{p}_1 \bar{p}_2 \vee Y_i y_i \bar{p}_3 \bar{p}_4$	$\bar{Y}_i \bar{y}_i \bar{p}_1 p_2 \vee Y_i y_i \bar{p}_3 p_4$
	$\bar{Y}_i \bar{y}_i p_1 \vee \bar{Y}_i y_i (1)$	$Y_i \bar{y}_i (1) \vee Y_i y_i p_3$

 $G(D_i, E_i, \mathbf{p})$ Fig. 5. Natural map for the auxiliary function $G(D_i, E_i, \mathbf{p})$.

			p_1	
		$\begin{bmatrix} \bar{Y}_i y_i \vee Y_i \bar{y}_i \\ Y_i \bar{y}_i \end{bmatrix}$	$\begin{bmatrix} \bar{Y}_i y_i \vee Y_i \bar{y}_i \\ \bar{y}_i \end{bmatrix}$	$\begin{bmatrix} \bar{Y}_i \vee \bar{y}_i \\ Y_i \bar{y}_i \end{bmatrix}$
		$\begin{bmatrix} \bar{Y}_i y_i \vee Y_i \bar{y}_i \\ Y_i \end{bmatrix}$	$\begin{bmatrix} \bar{Y}_i y_i \vee Y_i \bar{y}_i \\ Y_i \vee \bar{y}_i \end{bmatrix}$	$\begin{bmatrix} \bar{Y}_i \vee \bar{y}_i \\ Y_i \end{bmatrix}$
p_3	$\begin{bmatrix} Y_i \vee y_i \\ Y_i \end{bmatrix}$	$\begin{bmatrix} Y_i \vee y_i \\ Y_i \vee \bar{y}_i \end{bmatrix}$	$\begin{bmatrix} 1 \\ Y_i \end{bmatrix}$	p_4
			p_2	
$\begin{bmatrix} E_i \\ D_i \end{bmatrix}$				

Fig. 6. The nine particular solutions for E_i and D_i expressed algebraically in terms of Y_i and y_i . One of these solutions ($E_i = 1, D_i = Y_i$) asserts that upon enabling, the DE flip flop behaves as a standard D flip flop.

		Y_i	
		0	1
		0	1
		1	0
y_i	1	1	1
		D_i	

		Y_i	
		1	1
		0	1
		0	0
y_i	0	0	1
		E_i	

Fig. 7. The nine particular solutions for D_i and E_i expressed in terms of maps of map variables Y_i and y_i . This figure is an expanded version of the excitation map of the DE flip flop in Fig. 3(b).

Fig. 8. Atomic decompositions of the variables D_i and E_i as well as of the function $g(D_i, E_i)$ equated to 1.

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Appendix A

Boolean Quotients and the Boole-Shannon Expansion

The concept of a Boolean quotient is a switching-algebraic concept that can be conveniently used to facilitate Boolean manipulations. Given a two-valued Boolean function f and a term t , the Boolean quotient of f with respect to t , denoted by (f/t) or $(f|t)$, is defined to be the function formed from f by explicitly imposing the condition $\{t = 1\}$ (See Brown^[13], Rushdi and Rushdi^[37], or Rushdi^[38]), *i.e.*,

$$f/t = [f]_{t=1}, \quad (\text{A.1})$$

The Boolean quotient is also known as a ratio^[3], a subfunction^[39, 40] or a restriction^[41]. An important feature of Boolean quotients is that the conjunction of a term with a function is equal to the conjunction of the term with the Boolean quotient of the function with respect to the term, *viz.*,

$$t \wedge f = t \wedge (f/t). \quad (\text{A.2})$$

If the term t is implied by the function f (*i.e.*, $f \leq t$, $f \rightarrow t$, $f = t \wedge f$), then (A.2) reduces to

$$f = t \wedge (f/t). \quad (\text{A.3})$$

The concept of the Boolean quotient has a striking similarity to that of conditional probability^[37, 42, 43], but perhaps the most important utilization of the Boolean quotient is its use in the Boole-Shannon Expansion, which constitutes the most fundamental theorem of Boolean algebra (See Brown^[13], Rushdi and Rushdi^[37], or Rushdi and Ghaleb^[44])

$$f(\mathbf{X}) = (\bar{X}_i \wedge (f(\mathbf{X})/\bar{X}_i)) \vee (X_i \wedge (f(\mathbf{X})/X_i)), \quad (\text{A.4})$$

For example, the next state Y_i of flip flop number i can be expressed in terms of the present state y_i of the same flip flop as

$$Y_i = (Y_i/\bar{y}_i)\bar{y}_i \vee (Y_i/y_i)y_i \quad (\text{A.5})$$

The two Boolean quotients $(Y_i/\bar{y}_i)$ and (Y_i/y_i) in (A.5) are independent of the state of flip flop i . They are functions of other variables of the circuit, including inputs to the excitation logic and (possibly) the present states of other flip flops. For the *DE* flip flop, they are given by $D_i\bar{E}_i$ and $\bar{E}_i \vee D_i$, respectively.

توصيفان مبتكران لبدال تمكين التأخير (م خ)

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المستخلص. تستعمل الدوائر الرقمية الحديثة، وخاصة تلك المبنية علي النبائط ذات الكاملة كبيرة النطاق بدالات تُعرف باسم بدالات تمكين التأخير (م خ) تُمثل امتدادا لبدالات التأخير، تتمتع بالمقدرة علي تخزين قيمة مدخلة بحيث يكون هذا التخزين بالطلب أو التمكين. يتم وصف هذا البديل جبريا بواسطة معادلته المميزة، أو جدوليا بواسطة جدول الحالة التالية (المستعمل لأغراض التحليل) أو جدول الاستثارة (المستعمل لأغراض التركيب والاصطناع). تستكشف ورقة البحث هذه توصيفين مبتكرين لبدال تمكين التأخير (م خ). بداية، يتم وصف البديل بتوصيفات ضامنية أو معادلية، وتُستعمل الطريقة الاستدلالية الحديثة لإنتاج تعبيرات كاملة عن كل الأخبار التي تصح بالنسبة لبدال تمكين التأخير. يلي ذلك توظيف طرائق حل المعادلات البولانية لإيجاد جميع الأساليب الممكنة للتعبير عن مدخلي الاستثارة (التغذية) بدلالة الحالة الراهنة والحالة التالية. وهنا يلعب مفهوم "خارج القسمة البولاني" دورا هاما في توضيح المفاهيم المعنية وفي إنجاز الاشتقاقات المطلوبة المختلفة. ويُرجي لهذه الورقة أن تكون ذات نفع تعليمي مباشر وأن تيسر تحليل وتركيب الدوائر الرقمية المتتابعة.

كلمات مفتاحية: البديل، التوصيف، الطريقة الاستدلالية الحديثة، حل المعادلات البولانية، خارج القسمة البولاني، الاستثارة (التغذية).