On Self-Similar Traffic Modeling of Water Resources in Arid Regions

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Abstract. This paper reviews earlier work claiming that water-resource inputs closely follow a self-similar model, which is a manifestation of the more general multi-fractal behavior. Such claims have been repeatedly verified for wet terrains, but are still to be tested for arid regions. The paper presents a tutorial introduction to the concepts and phenomena of self similarity and multi-fractality for water-resource inputs. It also discusses the important implications of these phenomena for water consumption and storage. While conservative water consumption is an all-time virtue, it must be stressed and strictly imposed during periods of water shortages or droughts, which are more likely to be extended than to be brief. Periods of heavy rainfalls and floods should be optimally utilized to improve existing water storage levels. A mandatory policy for water conservation is strongly advocated by this paper, which contributes a scientific confirmation for the intuitively observed self-similarity phenomenon in arid regions. The paper presents a pilot and pioneering statistical study of a rainfall data set in a desert arid region. This study clearly indicates that this data set has a multi-fractal behavior, and that a mono-fractal model does not suit it. To the best of our knowledge, this result is the first published decisive confirmation for the self-similarity and multi-fractality phenomenon for rainfall data in arid regions.

Keywords: Water resources, Arid region, Self-similar, Multi-fractal, Water consumption and Storage, Droughts.

1. Introduction

In many natural phenomena, the arrival of jobs, customers, commodities, etc., follows a Poisson distribution, which is basically based on

neglecting the probabilities of instantaneous arrivals in infinitesimally small periods of time. Two notable exceptions in which the Poisson assumptions are not valid are traffic over the Internet and inputs for water resources such as rainfalls and river (or wadi) flows. These exceptions are typically studied by the theory of multi-fractals or mono-fractals, usually referred to collectively as that of self-similar traffic (though self similarity is precisely synonymous only with mono-fractality).

This paper is a preliminary attempt at studying the phenomenon of self similarity, or more generally, multi-fractality in the realm of water-resource inputs (rainfalls, river and wadi flows). Roughly speaking, these inputs are intuitively noted to have wide fluctuations in which both their maxima and minima do not occur in sharp instances of time but are rather extended over extended periods. This results in the historically recorded alternations between severe droughts and heavy floods. One famous incident of this nature is mentioned in the Old Testament and is described eloquently in the Holy Qur'an (Koran) (12: 43-49) The Prophet Yusuf (Joseph) PBUH was called upon to give an interpretation (exegesis) for a dream repeatedly seen by the King (not Pharaoh) of Egypt. Prophet Yusuf PBUH correctly predicted that the Nile will have high flood continuously for seven years followed by a prolonged drought period of seven years due to extremely low river flow. He again correctly predicted that the water crisis will come to a happy end in the fifteenth year. Based on this prediction, Prophet Yusuf set up and successfully implemented an encompassing fifteen-year plan of water-resource management. While Prophet Yusuf was directly guided by Divine inspiration from Almighty Allah SWT, we need to follow some similar prediction-based management, aided by the knowledge that Allah SWT ordered us to pursue. We note that the erratic behavior of the summer flow of the River Nile that occurred at the time of Prophet Yusuf PBUH has been repeated cyclically over thousands of years, and it is simply a reflection or manifestation of an almost identical behavior for rainfall on the plateau of Abyssinia (Ethiopia).

This paper reviews earlier papers giving scientific support to the aforementioned intuition and demonstrated historical precedents. These earlier papers claim that water-resource inputs closely follow a self-similar model, which is a manifestation of the more general multi-fractal behavior. Such claims have been repeatedly verified for wet terrains, but are still to be tested for arid regions. The paper presents a tutorial

introduction to the concepts and phenomena of self similarity and multi-fractality. These phenomena are related to those of long-range dependence and heavy-tailed probability distributions ^[1]. We note that durations of rain events and drought events over a given region provide important information about the water resources of any specific region. Of particular interest is the shape of upper tails of the probability distributions of such durations. Such a shape is highly important in practice (*e.g.*, for the management of storage and consumption of water resources) ^[2].

The paper presents a tutorial introduction to the concepts and phenomena of self similarity and multi-fractality for water-resource inputs. It also discusses the important implications of these phenomena for water consumption and storage. While conservative water consumption is an all-time virtue, it must be stressed and strictly imposed during periods of water shortages or droughts, which are more likely to be extended than to be brief. Periods of heavy rainfalls and floods should be optimally utilized to improve existing water storage levels. A mandatory policy for water conservation is strongly advocated by this paper, which contributes a scientific confirmation for the intuitively observed self-similarity phenomenon in arid regions. The paper presents a pilot and pioneering statistical study of a rainfall data set in a desert arid region. This study clearly indicates that this data set has a multi-fractal behavior, and that a mono-fractal model does not suit it. To the best of our knowledge, this result is the first published decisive confirmation for the self-similarity and multi-fractality phenomenon for rainfall data in arid regions.

The rest of this paper is organized as follows: Section 2 gives a simple mathematical description of the concepts of self-similarity. Section 3 is an overview of fractals and multi-fractals. Section 4 reviews evidence reported in the literature for the multi-fractal nature of rainfall in wet terrains, and similarly for the flow of rivers originating in regions of heavy rain. The self-similar and self-affine paradigms of multi-fractality are also referred to. Section 5 stresses the need for some future research to test the assumption of self similarity and multi-fractality in specific arid regions, and addresses some of the implications of validating such an assumption. Section 6 presents a pilot and pioneering statistical study of a rainfall data set in a desert arid region, and indicates

that this data set has a multi-fractal behavior, and that a mono-fractal model does not suit it. Section 7 concludes the paper.

2. Self-Similarity

Informally, self-similarity means that a data set when plotted looks the same regardless of what time unit we use. For example, if we plot the number of customer arrivals at a facility per time interval T for different values of T , then the self-similarity is reflected on observing similar-patterns in the different plots. Figures 1(a and b) show two different data sets where each is plotted at a different time unit. The one in Fig. 1(b) is generated from a Poisson process where the plot gets smoother at larger time units. The data set shown in Fig. 1(a) represents the number of packet arrivals at different time units where all plots have a common pattern, and hence look self-similar.

Fig. 1(a). Self-similar process.

Fig. 1(b). Poisson process.

2.1 Mathematical and Statistical Properties

Self-similarity manifests itself statistically in several equivalent ways as shown in the subsequent subsections, in which we will define each property along with methods to test the empirical data for the existence of such a property:

2.1.1 Slowly Decaying Variance

For a self-similar process X^T , the variance decreases more slowly than the reciprocal of the time scale (T) where the time scale is defined as the sampling frequency ($1/T$ is the sample period). In other words, X is the aggregated version of X^T over blocks of length T :

$$\text{Var}[X^T] \sim aT^{-\beta}, \quad 0 < \beta < 1. \quad (1)$$

This is in contrast to the variance of a non-self-similar process (e.g., a Poisson process) where the variance decreases like the reciprocal of the time scale:

$$\text{Var}[X^T] \sim aT^{-1}. \quad (2)$$

The Variance-Time plot ^[3] is a well known technique for testing the behavior of the variance with respect to the time scale. In this method, the variance of the sample data is plotted against the time scale on a log-log plot. The resulting plot is a straight line that can have a slope of -1 for most processes (i.e., for processes that satisfy Equation (2)). For self-similar processes, the slope is still negative but less in magnitude than unity (i.e., these processes satisfy Equation (1)). Figure 2 illustrates this concept.

Fig. 2. Variance versus Time plot.

2.1.2 Non-summable Autocorrelation Function (ACF)

For a self-similar process, the autocorrelation function,

$$r(k) = E[(X_t - \mu)(X_{t+k} - \mu)] \quad (3)$$

decays hyperbolically as compared to the exponential decay exhibited by most processes, which indicates the existence of long range dependence (LRD) among the sample data. Accordingly, self-similar processes are called LRD processes while non-self-similar processes are called short range dependence (SRD) processes. Figure 3 shows the ACF functions of both SRD and LRD processes. It can be seen from Figure 3 that LRD processes (self-similar) have non-summable ACF : $\sum_k r(k) = \infty$.

Fig. 3. ACF of LRD and SRD processes.

2.1.3 The Hurst Effect (R/S Statistic)

Any data set $(X_t, t = 1, 2, 3, \dots, n)$ can be divided into non-overlapping blocks of length m ($m = 1, 2, 3, \dots, n$), and accordingly, the $R(m)/S(m)$ statistic is defined as the rescaled range of the accumulated departures from the mean [4]:

$$\frac{R(m)}{S(m)} = \frac{\max(W_1, \dots, W_m) - \min(W_1, \dots, W_m)}{S(m)}, \quad (4)$$

where $S^2(m)$ is the sample variance of a block and

$$W_i = \sum_{j=1}^i X_j - i\bar{X}(m), \quad i = 1, 2, \dots, m \quad (5)$$

are the accumulated departures from the mean up to the sample point X_j and $\bar{X}(m)$ is the mean of block m .

The expected value of the $R(m)/S(m)$ statistic can be modeled as:

$$E[R(m)/S(m)] \sim am^H \quad (6)$$

The Hurst parameter is $H = 0.5$ for a non-self-similar process and $H > 0.5$ for a self-similar process. This discrepancy is generally called the Hurst effect, which describes the burstiness in the data. For a bursty data set ($H > 0.5$), the high (low) values are likely to be followed by high (low) values. The burstiness itself is a measure of correlations among the block of data.

2.1.4 Heavy-Tailed Distributions

The slowly decaying autocorrelation function (ACF) and the existence of long range dependence (LRD) among the sample data are natural properties that stem from the distribution of any self-similar process that is found to have a heavy tail (*i.e.*, the variance is high or infinite). From the practical point of view, a heavy-tailed distribution gives rise to very large values with non-negligible probability so that sampling from such a distribution results in the bulk of values being small with a few samples having very large values ^[1]. One of the well known heavy-tailed distributions is the Pareto distribution ^[5, 6] that has the probability density function (pdf):

$$f(x) = \frac{ba^b}{x^{b+1}} \quad (7)$$

where $a \leq x < \infty$ and b is the shape parameter. The mean and the variance of x are:

$$\mu = \frac{ba}{b-1} \quad (8)$$

$$\sigma^2 = \frac{ba^2}{(b-1)^2(b-2)} \quad (9)$$

The Hurst parameter, H , is given by:

$$H = \frac{3-b}{2}. \quad (10)$$

For $1 \leq b \leq 2$, the Hurst parameter is such that $0.5 \leq H \leq 1$, and self similarity implies long-range dependence, and vice versa ^[1].

3. Fractals and Multi-Fractals

The word “fractal” was coined from the Latin root ‘fractus’ to capture the rough, broken and irregular characteristics of the objects it describes. In fact, this roughness can be present at all scales, which is described as scale-invariant behaviour ^[7]. A generalization from integer dimensions to fractional dimensions has a profound and intuitive interpretation: non-integer dimensions describe irregular sets consisting of parts similar to the whole. A fractal is therefore defined as a rough or fragmented geometric shape that can be subdivided into parts, each of which is (at least approximately) a reduced-size copy of the whole. A fractal set possesses a fine structure, too much irregularity to be described in traditional geometric language, both locally and globally, some form of self-similarity, perhaps approximate or statistical, a “fractal dimension” (somehow defined) which is greater than its topological dimension, and a simple definition, usually recursive. There are many examples of (approximate) fractals in Nature, such as the distribution of galaxies at large scales, certain mountain ranges, fault networks and earthquake locations, rocks, lightning bolts, snowflakes, river networks, coastlines, patterns of climate change, clouds, cauliflower or broccoli, trees, and mammalian blood vessels.

Many fractals arising in Nature have a far more complex scaling relation than simple fractals, usually involving a range of scales that can depend on their location within the set. Such fractals are called multi-fractals ^[8, 9]. A Multi-fractal process is one that allows an extremely rich scaling structure at small scales, far richer than being simply fractal in general ^[1]. For multi-fractal processes, the local regularity of almost every sample path exhibits an extra-ordinary variability over time, since it is itself fractal-like.

Modeling ^[10] is usually the ultimate goal of studying physical phenomena and mathematical concepts such as those of fractals and multi-fractals. Multi-fractal formalism is applied in many modeling applications, such as modeling the velocity fluctuations in turbulent flows ^[11], rainfall modeling, modeling the Ethernet and Internet traffic ^[1, 6, 12, 13], modeling of swarming in innovative behavior and of innovations time series ^[14], modeling of economic and financial entities such as exchange rates, stock and commodity prices ^[14, 15], and modeling of earthquakes ^[16], river basins ^[17], and environmental pollution ^[18].

The main reason for the increasing use of the multi-fractal formalism lies in its capability to achieve, over a wide range of spatial and/or temporal scales, a strong control on the statistical moments of a given distribution of measures^[11]. The multi-fractal framework is well adapted for providing a parsimonious description of the statistics at all orders, and over a wide range of scales for many turbulent intermittency fields^[19]. Multi-fractals can be studied via elegant and general tools, such as the large-deviation theory and other multi-fractal concepts like singularities and their fractal dimensions, but they can also be understood via standard probability techniques^[20].

3.1 Multi-Fractal Analysis

The fluctuation analysis^[21], which can be considered a generalized form of the variance-time plot, is used to analyze a data set to test its fractal nature as a mono-fractal (self-similar) or a multi-fractal in which the q^{th} moment $F_q(m)$ (Equation 11) of the aggregated time series $(x^{(m)}(k))$ of the original time series at aggregation level m (Equation 12) is plotted against m (also sometimes called the scale level) in the log-log scale. Here x is the original time series and $E[x]$ is the expected value of x . The aggregated time series at aggregation level m is simply computed by dividing the original time series into blocks of length m and then summing up each block to create a single data point, but after subtracting the mean. A linear relation is an indication of a mono- or multi- fractal behaviour. The slope of this linear behavior is defined as a function of the parameter q and the Hurst parameter H as: $\beta(q) = q H(q) - 1$. For a mono-fractal time series (a special case of the multi-fractal one), the function $H(q)$ is independent of q , which means all the moments have the same scaling behavior, and accordingly, $\beta(q) = q H - 1$.

$$F_q(m) = E \left[\left(x^{(m)}(k) \right)^q \right] \quad (11)$$

$$x^{(m)}(k) = \sum_{i=(k-1)m+1}^{km} (x(i) - E[x]), k = 1, 2, \dots, N/m \quad (12)$$

4. Evidence of Multi-Fractality of Rainfall in Wet Terrains

As a physical phenomenon, rainfall is characterized by: (a) irregularity and intermittency between dry and wet states across a wide

range of scales of observations, both spatial and temporal (b) wild variability of rainfall intensity, which can be classified as null, light, mild, moderate, heavy, or violent, abrupt, and cataclysmic ^[2]. These rain characteristics have led to the questioning whether rainfall is self similar ^[22], and then to the recognition of rain rate fields in wet regions as multi-fractal structures ^[23-32]. The phenomenon of self similarity or multi-fractality is now well established for regions of heavy rainfall such as Western Europe or South-Eastern Asia.

Recently, more rigorous mathematical techniques have been used to verify the multi-fractality of rainfall in heavy-rain regions ^[2, 19, 20]. Multi-fractal formalism is also applicable to certain river and stream flow variations, which are mainly the results of climate changes, and precipitation variations ^[33].

5. The Question of Multi-Fractality in Arid Lands

In this paper, we are more interested in arid and semi-arid regions such as the Arabian Peninsula ^[34, 35] rather than wet terrains. Verification of the claims of self-similarity of rainfall seems to have been achieved exclusively for wet regions. For example, the data used by Pavlopoulos, *et al.* ^[2] represent the climate of a tropical oceanic region during the monsoon (*mawsim* or seasons) period, which is very rich in rainfall. The tail heaviness of both wet and dry epochs might be challenged in other types of climate ^[2]. Claims that rainfall follows a self-similar or a multi-fractal model are still to be tested for arid regions.

In a very recent encyclopedic study of water resources in the mainly arid or semi-arid Arab region, Shahin ^[36] devoted his Chapter 4 on *Analysis of Precipitation data* (pp. 135-169) to statistical analysis of rainfall data. He stated that the two functions that have often proved to fit the rainfall data well are the log-normal and the Pearson Type III. However, he did not refer explicitly to self similarity of the rainfall data. Anyhow, the time span of the data given in ^[36] is not long enough to reveal self similarity if it exists.

Shahin ^[36] devoted his Chapter 7, on *Wadis and Wadi Flow* (pp. 279-332) to a discussion that gives some implicit intuitive support for the assumption of self-similarity of rainfall in a typical arid region such as the Arabian Peninsula. Such a region is intersected and dissected by a

large number of transitory, intermittent and ephemeral streams (wadis) that run for only a short time. Usually, a wadi is a single stream, but it can also have a number of tributaries feeding a main stream. A wadi might be totally dry for a prolonged period, and once rain falls, it abruptly runs exhibiting a devastating flood for only a short period after which it becomes shallow and then dry again. The mere existence of wadis, and the nature of their water flow intuitively support the notion of self-similarity for rainfall in arid regions. This notion is consistent with a unique irregular form of agriculture that has been practiced for millennia in arid regions. This form of agriculture is based on spate (flood) irrigation where use is made of occasional heavy floods of very short duration carried by wadis.

Desalination accounts for about 20% of the currently available water resources in an arid region such as Saudi Arabia ^[36]. Many researchers object to fossil-fuel-based desalination for many reasons, including its prohibitively high cost. But once the self similar nature of rainfalls in arid lands is established, the inevitability of using fossil-fuel-based desalination will be clearly seen and strongly defended. Moreover, if sound prediction emanating from a self-similar model is achieved, the optimal amount of desalinated water can be decided.

There are almost two hundred dams in Saudi Arabia. There is a heated controversy among experts about the advantages and disadvantages of these dams ^[37]. Again a mathematical model based on the self similar paradigm can decide the optimal number of dams and their optimal capacity so as to make the most of the water available without inflicting severe side effects.

6. Application of Multi-Fractal Analysis to Arid Land Rainfall Data

As stated earlier, the main contribution of this paper is to test the hypothesis of self similarity or multi-fractality of rainfalls in arid regions. Ideally, we wanted to test rainfall data for the Arab World in general, or for the Arabian Peninsula in particular. So far, we have not been able to access such data in a suitable form and ample quantity. Therefore, we had to conduct our current preliminary study for a data set of an arid region in Western Australia. This data set was collected by the Bureau of Meteorology in Australia (BMA) ^[38]. This Bureau established many offices around Australia that are equipped with monitoring stations to

collect rainfall, temperatures, and solar exposure data. The data set that we use herein represents a sample daily rainfall in mm of a region in western Australia (the city of Giles), which is a desert region. This data set has been collected over a relatively long period of 55 years. Table 1 represents a statistical summary for this data set. From the daily data set, we generated a monthly data set where each data point represents the amount of rain in a month period. In addition to the mean (μ_1) and the median, which are both in mm, Table 1 presents the higher-order moments μ_2 , μ_3 and μ_4 expressed in terms of dimensionless coefficients, namely the coefficient of variation ($\mu_2^{1/2}/\mu_1$), the coefficient of skewness ($\mu_3/\mu_2^{3/2}$), and the coefficient of excess or kurtosis (μ_4/μ_2^2-3). Figures 4 and 5 offer graphical displays of the daily and monthly rainfalls versus time, respectively.

Figures 6 and 7 show the log-log plot of q^{th} moment of the aggregated time series, $F_q(m)$, with respect to the aggregation level m for several q values of the daily and the monthly data sets, respectively. It is clear that for all values of q , the logarithms of the moments scale linearly with $\log(m)$, which is an indication of a multi-fractal behavior. For each plot and using linear fitting, we computed the slope $\beta(q)$ and the corresponding H value. Table 2 summarizes our results. The H values depend on q , which indicates that a mono-fractal model is not suitable for such data sets and that indeed a multi-fractal model is needed. In other words, multifractal data sets have long-range correlation and $F_q(m)$ increases as a power law ($F_q(m) \sim m^{\beta(q)}$) [21].

Table 1. Summary statistics of the GILES Meteorological office station for the daily rainfall during the period from year 1956 to year 2010.

coefficient of kurtosis	coefficient of skewness	coefficient of variation	Median	Mean	Month
4.90	2.02	1.41	20.2	29.2	January
8.67	2.71	1.77	21	45.8	February
11.82	3.08	1.90	14.2	36.7	March
10.57	3.03	1.97	5.8	16	April
4.90	2.10	1.66	6.2	19.8	May
6.78	2.35	1.66	7.6	17.5	June
4.45	2.02	1.68	2.7	11.7	July
7.92	2.59	1.86	1.6	10.6	August
5.46	2.17	1.69	2.3	11.5	September
7.36	2.54	1.83	5.2	14.6	October
5.03	2.07	1.52	12.7	27.6	November
11.45	2.90	1.56	29	43.8	December

Fig. 4. A graphical display of the daily rainfall versus time (in day units) for Giles, Australia.

Fig. 5. A graphical display of the monthly rainfall versus time (in month units) for Giles, Australia.

Fig. 6. Moment plots for the daily data set.

The slope $\beta(q)$ of the linear relation between $\log(F_q(m))$ and $\log(m)$ is a function of the moment order of the aggregated time series q and the Hurst parameter H (defined in section 2.1.3). The plot with q equal to 2 represents the variance time plot presented in section 2.1.1, where the variance decreases linearly with the scale. In other words, the variance decreases linearly with the number of samples.

Fig. 7. Moment plots for the monthly data set.

Table 2. The H values versus q for both daily and monthly data sets.

H	q	
0.61	2	Daily data set
0.48	3	
0.44	4	
0.65	2	Monthly data set
0.52	3	
0.48	4	

For further confirmation of the self-similarity phenomenon, we plot in Fig. 8 the autocorrelation function of the aggregated time series of the daily rainfall versus lag (k) along with the autocorrelation function of a Poisson time series. As expected, Fig. 8 shows that the autocorrelation function of the daily rainfall drops slower than the autocorrelation of the Poisson time series.

As far as we know, this is the first published decisive confirmation for the self-similarity and multi-fractality phenomenon for rainfall data in arid regions. Further investigations are still needed to definitely confirm this phenomenon for other arid regions (in particular, the Arab arid regions). With such a confirmation available, this phenomenon should be taken into consideration while seeking solutions for water problems in these regions ^[39].

Fig. 8. The autocorrelation function of the daily rainfall compared to the autocorrelation function of a poisson time series.

7. Conclusions

This paper presented a brief tutorial introduction to the concepts and phenomena of self similarity and multi-fractality, and their related concepts of long-range dependence and heavy-tailed probability distributions. The paper reviewed earlier work giving scientific support to the claims that water-resource inputs closely follow a self-similar model, or more generally a multi-fractal one. The paper cited many publications which verified the validity of such claims for heavy-rain regions, and stressed the need and importance of similar research dealing with arid regions. Such research can serve as a basis for the optimal management of storage and consumption of water resources. The paper presented what we believe to be the first published decisive confirmation for the self-similarity and multi-fractality phenomenon for rainfall data in arid regions. Further investigations are still needed to definitely confirm this phenomenon for other arid regions. With such a confirmation available, this phenomenon should be taken into consideration while seeking solutions for water problems in these regions.

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حول نمذجة موارد المياه في المناطق الجافة باستخدام السريان ذاتي التشابه

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المستخلص. تراجع ورقة البحث هذه أبحاثاً سابقة تدّعي أن مدخلات موارد المياه تتبع بدقة نموذجاً ذاتي التشابه يعبر عن سلوك أكثر عمومية وهو سلوك عديد الشراذم. وقد تكرر الاستيثاق من هذه الادعاءات في حالة الأقاليم المطيرة، ولكنها لازالت محل الاختبار في حالة المناطق الجافة. تقدم ورقة البحث هذه مدخلا تعليميا لمفاهيم ظواهر التشابه الذاتي والتعدد الشرذمي في حالة مدخلات موارد المياه. كما تناقش الورقة النتائج الهامة لهذه الظواهر بالنسبة لاستهلاك وتخزين المياه. وإذا كان ترشيد استهلاك المياه هو فضيلة في كل وقت وحين، فإنه يتعين تأكيده وفرضه بصرامة خلال الفترات التي تنقص فيها المياه إلى حد القحط والجفاف وهي فترات تكون عادة أميل إلى الطول منها إلى القصر. أما فترات المطر الغزير والفيضانات فينبغي أن تستغل الاستغلال الأمثل في رفع مستويات تخزين المياه. إن هذه الورقة تدعم بقوة سياسة الترشيح الإجباري لاستهلاك المياه من خلال برهانها العلمي على ظاهرة التشابه الذاتي لهطول الأمطار في المناطق الجافة، وهي ظاهرة كانت تستند على مجرد الملاحظة والحدس فيما سبق. تسهم هذه الورقة دراسة إحصائية رائدة لبيانات هطول الأمطار في إحدى المناطق الصحراوية الجافة.

وتوضح هذه الدراسة بجلاء أن هذه البيانات تسلك مسلكاً عديد الشراذم، وأنه يتعذر نمذجتها بنموذج أحادي الشرنمة. وعلى مبلغ علمنا، فإن هذه النتيجة تعد أول تأكيد حاسم لظاهرة ذاتية التشابه وعديدة الشراذم لبيانات هطول الأمطار في المناطق الصحراوية الجافة.

الكلمات الدالة: موارد المياه، المناطق الجافة، ذاتي التشابه، عديد الشراذم، استهلاك وتخزين المياه، الجفاف.