Frequency-domain mesh-free finite-difference operator for visco-acoustic wave equation

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In the present study, we apply a mesh-free finite difference method (MF-FDM) to frequency-domain visco-acoustic wave propagation. Although the full-waveform inversion (FWI) is a powerful tool to investigate the subsurface quantitatively, FWI requires a huge amount of computer resources. This poses an obstacle to apply FWI to practical studies especially in three-dimensional cases. MF-FDM has a potential to decrease the computational costs for making synthetic data sets in a simple manner. We demonstrate the effectiveness of MF-FDM using numerical experiments. Our results show that the proposed method can improve the numerical efficiency without sacrificing the numerical accuracy.

1. INTRODUCTION

Recently, the full-waveform inversion (FWI) has drawn attention in scientific and engineering fields because of its strong possibility to extract quantitative information of the subsurface¹⁾. One of the main problems of FWI is high computational costs. Especially in the frequency-domain FWI, we have to solve linear equations to make synthetic data sets. This poses an obstacle to apply FWI to real practices in three-dimension. To develop a novel method for reducing the CPU time and required computational memory is a solid solution to this problem.

A number of numerical methods to solve the wave propagation have been developed^{2,3)}. Almost all of techniques are base on the finite-difference method (FDM). On the other hand, mesh-free methods attract attention as alternatives to the conventional finite-differences because of its high flexibility of discretization. Recently a mesh-free finite-difference method (MF-FDM) has been proposed by Takekawa et al. (2015)⁴⁾. The spatial resolution is locally refined to reduce the number of calculation points without sacrificing the numerical accuracy in a simple manner. This method works well to reduce the computational costs especially in models with high velocity contrasts⁵⁾. Although the effectiveness of the method has been demonstrated in the time-domain modeling, its ability in the frequency-domain modeling has not been revealed yet.

In the present study, we apply the mesh-free method⁴⁾ as a simulator in the frequency-domain modeling. To evaluate the effectiveness of the

MF-FDM, we conduct a numerical experiment using a low velocity anomaly model.

2. METHOD

We briefly explain the visco-acoustic wave equation and the MF-FDM.

(1) Visco-acoustic wave equation

We solve the Helmholtz equation with the attenuation term as follows.

$$(v^2 + i\omega\eta) \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} \right) + \omega^w p = 0 \quad (1)$$

where i is the imaginary unit, ω is the angular frequency, p is the pressure, v is the velocity, $\eta = v^2/(Q\omega)$, Q is the quality factor. We discretize the equation using an operator of the MD-FDM as shown below.

(2) Mesh-free FDM

The mesh-free concept is based on the multi-variable Taylor expansion. The multi-variable Taylor expansion of a scalar function $f(\mathbf{r})$ to M -th order at position $\mathbf{r}_i$ is expressed as follows:

$$f(\mathbf{r}_i + \Delta\mathbf{r}) = f(\mathbf{r}_i) + \sum_{m=1}^M \left\{ \frac{1}{m!} (\Delta\mathbf{r} \cdot \nabla)^m f(\mathbf{r}_i) \right\} + O(\|\Delta\mathbf{r}\|^{M+1}) \quad (2)$$

where $\mathbf{r}_i$ and $\mathbf{r}_i + \Delta\mathbf{r}$ are the position vectors of grid i and its neighboring grid j , respectively. In two-dimensional case, differential operators are calculated as follows:

$$(\delta f)_{\mathbf{r}=\mathbf{r}_i} \approx \left\{ \sum_{j=1}^n (\mathbf{P} \otimes \mathbf{P}) \right\}^{-1} \cdot \left\{ \sum_{j=1}^n (\mathbf{P} \Delta f_{ij}) \right\} \quad (3)$$

$$\mathbf{P} = \left(\Delta r_1, \Delta r_2, \frac{1}{2!} \Delta r_1^2, \Delta r_1 \Delta r_2, \frac{1}{2!} \Delta r_2^2, \dots, \frac{1}{M!} \Delta r_2^M \right)^T \quad (4)$$

$$\boldsymbol{\delta} = \left(\frac{\partial}{\partial r_1}, \frac{\partial}{\partial r_2}, \frac{\partial^2}{\partial r_1^2}, \frac{\partial^2}{\partial r_1 \partial r_2}, \frac{\partial^2}{\partial r_2^2}, \dots, \frac{\partial^M}{\partial r_2^M} \right)^T \quad (5)$$

where $f_{ij} = f_j - f_i$ is the difference between scalar values at positions $\mathbf{r}_i$ and $\mathbf{r}_j$. The details of the derivation of the above equations can be found in a reference⁴⁾.

As an absorbing boundary condition (ABC), we apply a hybrid technique for the frequency-domain modeling^{6,7)}. For the hybrid ABC, we apply a buffer zone in the vicinity of model edges. In the buffer zone, coefficients of a linear system are determined

by a weighted average of the one-way and two-way wave equations. Details can be found in a reference⁶⁾.

3. NUMERICAL RESULTS

In this chapter, we quantitatively evaluate the computational costs of the method with and without the local refinement strategy.

(1) Numerical model

Figure 1 shows a numerical model of the present study. The model includes a low velocity anomaly in the center of the model. The anomaly has a P-wave velocity of 1500 m/s, whereas the background velocity is 3000 m/s. Q-value is set to 10. The source and receiver array are located at the left

Figure 1: Numerical model with a low velocity anomaly. Open square and circle represent the source and the receiver locations, respectively.

Figure 2: Monochromatic wavefield with 10.3 Hz.

Figure 3: Shot gather obtained by the receiver array.

and right side of the anomaly, respectively. The source function is the Ricker wavelet with a central frequency of 10 Hz. We use 10 m lattice structure for background, whereas 5 m interval is applied around the low velocity anomaly.

(2) Wavefield and waveforms

Figure 2 shows a monochromatic wavefield with the harmonic frequency of 10.3 Hz. It can be seen that the wavefield is disturbed by the low-velocity anomaly.

Figure 3 shows a shot gather obtained by the receiver array. We can observe that the waveform can be calculated accurately without numerical dispersion especially behind the low velocity anomaly.

4. CONCLUSIONS

We apply the MF-FDM to the visco-acoustic wave propagation in the frequency-domain. We confirm the effectiveness of the method using an inhomogeneous medium with a low velocity anomaly. Our results show that the proposed method has a potential to be an alternative to the conventional FDM due to its computational efficiency.

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