Waveform Disturbance by Transversely Isotropic Medium with a Tilted Axis of Symmetry around a Borehole

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Sonic logging is an essential tool to understand the properties of reservoirs in detail. In particular, the more accurate method for understanding the anisotropy in the vicinity of well is demanded to develop the unconventional resources such as shale oil and gas. We use 3D Hamiltonian Particle Method (HPM) to simulate a seismic wave propagation including a borehole and transmit from dipole source to make use of the shear wave. We apply the cross-dipole orientation as the system of source and receivers. We set the Transversely Isotropic medium with a Horizontal axis of symmetry (HTI) as an anisotropic layer around a borehole. It is known that the shear wave splits into two polarized waves, i.e., as shear wave splitting. This is the distinctive feature to detect the azimuthal anisotropy. In addition, we conduct numerical simulation for Transversely Isotropic medium with a Tilted axis of symmetry (TTI). We use the conventional Slowness Time Coherence (STC) to detect the slowness of formation for both HTI and TTI. Our results clearly show that obvious difference appears as received waveform. We conclude that the Full Waveform Inversion (FWI) could apply to estimate anisotropic properties around a borehole with high accuracy and resolution.

1. INTRODUCTION

Anisotropy under the ground usually exists caused by in-situ stress and the cluster of cracks. The anisotropic properties caused by stress correspond to the properties of wave propagation. It is important to understand anisotropic property around a borehole for effective production of oil and gas, efficient control of hydraulic fracturing, etc. Because it is difficult to measure the stress under the ground directly, sonic logging is useful way to understand the anisotropic properties. In the conventional research, Vertical Seismic Profiling (VSP) has been used to estimate the anisotropic properties around a borehole. However the 3-dimensional anisotropic properties around a borehole may not be well obtained using VSP because seismic wave covers only a low frequency band that degrades the spatial resolution. So this uncertainty causes some problems in practical fields for which some improvements have been waited for.

After introducing dipole transmitter, information about shear waves has been obtained (Kitsunezaki, 1980)¹⁾. When shear wave passes through the transversely isotropic medium with a horizontal axis of symmetry (HTI), it splits into two polarized waves: fast shear and slow shear. To detect the azimuthal anisotropy, Alford Rotation has been applied in the previous studies (Alford, 1986)²⁾.

Since these studies extract the information about anisotropy from travel time, utilization of waveforms has not been conducted yet. Furthermore, little study has been done to investigate the effect of transversely isotropic medium with a tilted axis of symmetry (TTI) case on the received waveforms.

In the present study, we investigate the effect of anisotropic layer, especially for TTI medium, across a borehole on the waveforms. We show the results of received waveforms and travel time, and point out the importance of introduction of waveforms to the analysis of anisotropic properties.

2. METHOD

(1) Hamiltonian Particle Method

To express seismic wave propagation, we use the Hamiltonian Particle Method with staggered particles (HPM) (Takekawa et al., 2014)³⁾. In this method, main particles are assigned for phase velocity and density, and staggered particles are assigned for stress and strain. It is simple to apply the free surface and flexible structure in the numerical model. HPM has the advantages in computational efficiency over the conventional lattice based methods. It discretizes the continuum by finite number of particles. And then equation of motion is discretized by introducing influence domain.

(2) Governing equation

Governing equation for linear elastic material is given by

$$\rho \frac{\partial v_\alpha}{\partial t} = \frac{\partial}{\partial x_\beta} [\lambda \varepsilon_{\gamma\gamma} \delta_{\alpha\beta} + 2\mu \varepsilon_{\alpha\beta}] \quad (1)$$

where ρ is the density, v_α is the particle velocity, $\varepsilon_{\gamma\gamma}$ is volume strain, $\delta_{\alpha\beta}$ is Kronecker delta, and λ and μ is Lamé's constants.

Although the number of the independent elements of the stiffness matrix is 36 formally, these become 21 because of the symmetric property of matrix. As anisotropic layer in geophysical exploration, HTI is preferable. HTI has the 5 independent elements written as below.

$$\begin{pmatrix} C_{11} & C_{13} & C_{13} & & \\ & C_{33} & C_{33} - 2C_{44} & & 0 \\ & & C_{33} & & \\ & \text{Sym.} & & C_{44} & \\ & & & & C_{66} & C_{66} \end{pmatrix} \quad (2)$$

The spatial arrangement of medium in relation to the coordinate system of TTI is defined by the inclination of HTI. We derive the general relationship between the matrix C of HTI and C' of TTI expressed as

$$C' = RCR^T \quad (3)$$

where R is rotation matrix while the R^T is the transpose of the matrix R .

(3) Numerical Model

Our numerical model includes an anisotropic layer in a homogeneous medium (Figure 1). The model consists of $200 \times 200 \times 100$ particles that equals to $2\text{m} \times 2\text{m} \times 1\text{m}$. The diameter of a borehole is 20cm. The time interval is 3.0×10^{-6} seconds. We set 8 receivers inside of the borehole. We apply absorbing boundary around the model which occupies 50 particles (Cerjan et. al, 1985)⁴⁾. The physical characters of model are shown in Table 1.

Figure 1: Numerical model

Table 1: Physical characters of model

Mud	
P wave velocity [m/s]	1600
S wave velocity [m/s]	0.00
Density [g/cm ³]	1.60
Formation	
P wave velocity [m/s]	
Isotropy	2000
Anisotropy	1690 (15.5%)
S wave velocity [m/s]	
Isotropy	1154
Anisotropy	975 (15.5%)
Density [g/cm ³]	2.30
Elastic modulus	
C_{11} [GPa]	9.20
C_{13} [GPa]	3.27
C_{33} [GPa]	7.67
C_{44} [GPa]	2.56
C_{66} [GPa]	3.07

3. RESULTS

(1) Shear wave splitting

Distinctive feature can be observed when shear wave passes through the HTI medium which is known as shear wave splitting. When shear wave reaches the HTI medium, it splits into two polarized shear waves: fast shear and slow shear. When the initial direction of vibration as to shear wave correspond with the direction of the isotropic plane, it doesn't show splitting (Figure 2). On the other hand, when the initial direction of vibration as to shear wave differs from the direction of the isotropic plane, it shows ellipse motion because fast

shear and slow shear arrives different time (Figure 3). To detect the azimuthal anisotropy, Alford rotation has been applied in the previous studies (Alford, 1986). The wave excited from dipole source is recorded both the same and orthogonal components. After applying Alford rotation using all 4 waveforms, we could estimate the azimuth from the energy and degree of anisotropy from the phase difference (Figure 4).

Figure 2: Orbit motion when angle between shear wave and isotropic plane is 0 degree.

Figure 3: Orbit motion when angle between shear wave and isotropic plane is 30 degree.

Figure 4: Waveform after Alford Rotation. Blue and red line is recorded by the parallel system of source and receiver which correspond with the azimuth of anisotropy. Green line is recorded by the diagonal system of source and receiver which is minimized.

(2) TTI medium

For TTI medium, we tilt the axis of symmetry from the horizontal to vertical (Figure 5). When the axis of symmetry is tilted 30 degree from the horizontal, we can see big difference about phase and amplitude. On the other hand, when the axis of symmetry is tilted 60 degree from the horizontal, the difference becomes small. However there is explicit difference between fast shear and slow shear (Figure 6).

Figure 5: The process of axis of symmetry becomes tilt

Figure 6: Waveform when the axis of symmetry is tilted 60 degree from HTI. Blue line shows fast shear and Red line shows slow shear.

(3) Velocity analysis

In the conventional study, Slowness Time Coherence (STC) (Kimball, 1984)⁵⁾ has been used to analyze the formation velocity around a borehole. The semblance is calculated by

$$\rho^2(s, \tau) = \frac{\int_{t=0}^{T_w} (\sum_{m=1}^M r_m[t + s(z_m - z_1) + \tau])^2 dt}{M \sum_{m=1}^M \int_{t=0}^{T_w} (r_m[t + s(z_m - z_1) + \tau])^2 dt} \quad (4)$$

where s is the formation slowness, τ is time, T_w is time window, m is the number of receiver, M is the total number of receivers, r_m is the recorded waveform at receiver m , z_m is the position of m 's receiver.

We apply the STC for the data after applying the Alford rotation (Figure 7, 8). Then, we evaluate how inclination of axis of symmetry affects to the velocity analysis (Figure 9, 10). It shows if we misunderstand the HTI medium as TTI medium, we cannot estimate the phase velocity correctly.

Figure 7: STC analysis for slow shear

Figure 8: STC analysis for fast shear

Figure 9: The error of fast shear velocity caused by HTI becomes tilted

Figure 10: The error of slow shear velocity caused by HTI becomes tilted

4. CONCLUSION

In the present study, we investigate the influence of the properties as axis of symmetry is tilted. If the anisotropic layer is HTI medium, fast shear and slow shear can be detected explicitly by applying Alford rotation. On the other hand, the symmetric axis of the anisotropic layer becomes tilted from HTI medium, the phase difference between fast shear and slow shear becomes smaller. This result indicates that if anisotropic layer tilts, we cannot estimate anisotropic parameters correctly just by using the Alford rotation. On the other hand, we can observe an evident difference between fast and slow shear waveforms. Since this difference stems from the tilted angle of the anisotropic layer, inversion technique utilizing full-waveform could estimate anisotropic properties around a borehole with high accuracy and resolution.

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