Predicting the losses and the efficiency of a 1-3 piezo-composite transducer through a partial homogeneization method

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The topic of this work is the theoretical study of a disc-shaped acoustic transducer made with a 1-3 piezoelectric composite material. This material consists of PZT ceramic rods embedded in a polymer matrix. A modeling of this ideal transversally periodic structure is proposed. It is based on a finite element approach derived from homogenization techniques mainly used for composite material studies. The analysis focuses on a representative unit cell with specific boundary conditions on the lateral surfaces taking accurately into account the periodicity of the structure. The first step proposed is the development of a three-dimensional Fortran code with complex variables, especially adapted for this problem. Using the principle of correspondence of Lee-Mandel, this technique allows the prediction of the damping properties of the transducer from the complex modulus of the constituents. Both the versatility of the method and the rigorous character of the model are pointed out through various boundary conditions and mixed loadings. An interesting result is that, despite the lossy polymer matrix, a 1-3 composite can advantageously replace a much heavier massive transducer, in terms of efficiency and loss factor.

Keywords: piezo-composite, transducer, finite elements, correspondence principle, losses.

Predicción de las pérdidas y la eficacia de un transductor piezocomposite 1-3 mediante un método de homogeneización parcial

El objetivo de este trabajo es el estudio teórico de un transductor acústico con forma de disco construido con un composite piezoeléctrico 1-3. Este material consiste en barras cerámicas de PZT embebidas en una matriz polimérica. Se propone un modelo de su estructura periódica transversal ideal basándose en una aproximación mediante elementos finitos derivada de técnicas de homogeneización usadas principalmente para estudios de materiales compuestos. El análisis se enfoca a una celdilla unidad representativa con unas condiciones de contorno específicas sobre las superficies laterales, teniendo en cuenta adecuadamente la periodicidad de la estructura. El primer paso propuesto es el desarrollo de un código Fortran tridimensional con variables complejas, adaptado especialmente a este problema. Usando el principio de correspondencia de Lee-Mandel, esta técnica permite la predicción de las propiedades amortiguadas del transductor a partir de los módulos complejos de los constituyentes. Tanto la versatilidad del método como el carácter riguroso del modelo se ponen de manifiesto a través de varias condiciones de contorno y mezcla de cargas. Un resultado interesante es que, a pesar de la matriz polimérica, un composite 1-3 puede reemplazar ventajosamente a un transductor masivo mucho más pesado en términos de eficacia y factor de pérdidas.

Palabras clave: Piezo-composite, transductor, elementos finitos, principio de correspondencia, pérdidas dieléctricas

1. INTRODUCTION

The electromechanical transducer considered in this study is disc-shaped, a few centimeters in diameter and 5 to 10 mm thick. It is made with a piezoelectric composite material presenting a 1-3 connectivity pattern (1). This means that it contains several hundreds of rods of PZT (Figure 1) periodically distributed and embedded into a polymer resin. Both faces of the disk are electroded with conductive ink for electrical connection to the rods. The excitation can be an acoustic pressure or an imposed voltage depending on the operating mode. A purpose of the present study is the ultrasonic transducer efficiency prevision. This is of great importance especially for power transducers for underwater or medical applications, or other industrial applications such as ultrasonic cleaning or sonochemistry.

In this paper the modeling of the representative

Figure 1 : Schematic representation of a 1-3 piezocomposite plate (after W.A. Smith)

elementary cell by 3D finite elements is proposed. This model considers the cell as a structure, and adequate two dimensional periodicity conditions are implemented in the plane orthogonal to the rods also taking into account the experimental macro boundary conditions of the whole specimen.

From the so-called Principle of correspondence of Lee-Mandel (2,3), the viscoelastic problem is solved in C^n , using complex characteristics of polymer matrix and PZT rods. A specific three-dimensional finite element code in C^n has been developed for this purpose. The problem is solved here by partial homogenization (4). This means with periodicity conditions applied only in the planes perpendicular to the z-axis (see Figure 2). To achieve this, the formulation proposed in (5) has been modified from the case of pure piezoelectricity.

2. POSITION OF THE PROBLEM AND ITS FEM IMPLEMENTATION

2.1. Physical considerations

The basis of the analysis is the REC (Representative Elementary Cell) volume Ω shown in Figure 2 and consisting of a PZT rod surrounded by an epoxy matrix layer. It is placed in an orthogonal (Oxyz) coordinates system. Axis Oz, also considered as direction 3, is the poling direction of the piezoelectric material ; it is the PZT rod axis. The periodicity of the composite material is then defined in the (Oxy) plane. $\partial\Omega_L$ and $\partial\Omega_T$ are respectively the lateral (i.e. the four vertical planes) and the transversal (i.e. the two planes perpendicular to Oz) parts of the boundary. Both ends forming $\partial\Omega_T$ are also electric equipotential surfaces where the voltage can be applied. They can also be submitted to a defined pressure, or to an uniform vertical displacement.

The variables of the problems are then defined as follows:

$$\begin{cases} u_i = \tilde{u}_i(x, y, z) + E_{ij}(z)x_j ; \quad \varphi = \tilde{\varphi}(x, y, z) + G_j(z)x_j ; \quad i, j = x, y, z \\ \text{with } \tilde{u} \in V_0 ; \quad \tilde{\varphi} \in \Psi_0 \text{ and :} \\ V_0 = \{v \in C^3, v \text{ periodic on } \partial\Omega_L\} , \\ \Psi_0 = \{\varphi \in C, \varphi \text{ periodic on } \partial\Omega_L\} , \\ \sigma_{ij}n_j \text{ and } D_jn_j \text{ antiperiodic on } \partial\Omega_L . \end{cases} \quad (1)$$

In these equations u is the complex (mechanical) displacement vector, σ is the stress tensor, and φ is the electric complex potential whose gradient is opposed to the electric field. The E_{ij} are essentially free variables representing the macroscopic strain superimposed to the periodic local strain $\tilde{\epsilon}$. In the same way, the G_i are the components of a macroscopic potential gradient, which is superimposed to the local periodic gradient of potential $\tilde{\varphi}$. These variables allow to take into account a global strain and a global gradient of the electric potential corresponding to partially (horizontally) homogenized composite material. The last two assumptions complete the periodicity conditions to apply to the REC.

The considered state equations for the piezoelectric material are:

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl} + e_{mij}\varphi_{,m} ; \quad D_i = e_{ikl}\epsilon_{kl} - K_{ij}\varphi_{,j} \quad (2)$$

where D is the electric displacement (or induction), C is the elastic stiffness tensor, e is the piezoelectric field stress tensor and K is the dielectric permittivity tensor. All the material

Figure 2 : The horizontally periodic (REC) unit cell

parameters are complex, except e from lack of experimental data. The imaginary parts of the C and D complex modulus are respectively representative of the mechanic and dielectric losses.

Gauss theorem and static equilibrium equations have to be respected and can be written as:

$$\sigma_{ij,j} = 0 \text{ and } D_{j,j} = 0 \quad (3)$$

Boundary and loading conditions complete the system of equations to solve.

2.2. Variational formulation and finite element discretisation

The following real test functions are considered:

$$\tilde{v} \in V_0 ; \quad \tilde{\psi} \in \Psi_0 \quad (4)$$

Under the considered conditions and according to the divergence theorem, equations (3) become:

$$\begin{aligned} \int_{\Omega} \epsilon_{ij}(\tilde{v}) \sigma_{ij}(u, \varphi) dV &= 0 \quad \forall \tilde{v} ; \\ \int_{\Omega} \tilde{\psi}_{,j} D_j(u, \varphi) dV &= 0 \quad \forall \tilde{\psi} ; \end{aligned} \quad (5)$$

Let's now consider the REC discretization in 3D-finite elements. As usual the same class of interpolation functions will be used for the complex FEM-solution fields and the real test fields. From the equations (5), the following linear system is obtained:

$$\begin{bmatrix} \int B^T C B & \int B^T e^T B_{el} \\ \int B_{el}^T e B & - \int B_{el}^T K B_{el} \end{bmatrix} \begin{Bmatrix} \{u_n\} \\ \{\varphi_n\} \end{Bmatrix} = \begin{Bmatrix} \{F_n\} \\ \{Q_n\} \end{Bmatrix} \quad (5)$$

In these equations $\{u_n\}$ and $\{\varphi_n\}$ represent the nodal u and φ : subscript n stands for the nodal unknowns. Matrix B sets the relations between $\epsilon(u)$ and $\{u_n\}$ and matrix B_{el} sets the relations between the gradient of φ and $\{\varphi_n\}$.

The integrals, whose differential elements have been omitted for simplicity, are sums on the volume of the mesh and allow the classical assembly of elementary matrices. C is

the complex stiffness matrix (complex modulus in visco-elasticity), e is the piezoelectric coupling matrix and K is the complex dielectric permittivity matrix.

This system has three displacement type degrees of freedom (dof) and one electric potential dof at each node. The equipotential boundary conditions for the two horizontal end surfaces ($\partial\Omega_T$) are established at the assembly of the problem by condensation of the electric dof of these surfaces with two additional variables Φ_{inf} et Φ_{sup} . These last variables are then affected with the needed values.

The periodicity conditions are involved as below, figure 3, on each horizontal plane ABCD.

From the periodicity of $\tilde{u}$, the following relations

$$\begin{aligned} u_i(N) - u_i(M) &= u_i(D) - u_i(B) ; u_i(J) - u_i(I) = u_i(B) - u_i(A) ; i = x, y, z \\ \varphi(N) - \varphi(M) &= \varphi(D) - \varphi(B) ; \varphi(J) - \varphi(I) = \varphi(B) - \varphi(A) \end{aligned} \quad (7)$$

are to be imposed for all opposite points (I, J) an (M, N) in each nodal plane of the FEM mesh. They are taken into account by condensation of the corresponding lines and rows, as usual.

To generate a solution for the whole transducer by horizontal periodicity, relations (7) allow to post-calculate the macroscopic quantities E_{ij} (except E_{zz} , E_{yz} , E_{xz} , not defined) and G_x and G_y then to define from (1) the periodic $\tilde{u}_i$ and $\tilde{\varphi}$ variables.

The final complex linear system is solved using a classical Gauss type method. The code is written in Fortran for Q8 type parallelepipedic finite element, so with multilinear variation of the displacements and the potential within the element. The development and running of the program have been executed on an Apple Power-PC G3-400 computer.

2.3. Numerical testing method

Parallelepipedic elements, with rectangular sections, are especially adapted to the meshing of the representative volume element. Figure 4 gives a typical mesh of a section perpendicular to the z-axis. With five nodal horizontal planes the mesh consists in 768 elements and gives a linear complex system of 4502 rows and columns.

Various cases are tested, always with periodicity conditions,

- $u_{normal} = 0$ on lateral boundary $\Omega\partial_L$, or free, depending on the application. The first case simulates the conditions largely in use in the literature, in spite of the fact they are too restrictive. However, they allow to retrieve the results of some analytical uniaxial theories and experimental data for transducers operated in the thickness mode. Free velocity on lateral boundary $\Omega\partial_L$ is more representative of the actual conditions on the lateral surface of the transducer operating in the quasi-static mode.

- Loadings are imposed either by vertical displacements on $\Omega\partial_T$, or by pressure p on $\Omega\partial_T$. In both cases φ can be either free or imposed.

From the solution, the following computations are conducted:

- the mean stresses in the polymer matrix, and in the ceramic rod are determined,
- the distribution of the complex energy W defined by

$$\int \frac{1}{2} (\sigma_{ij} \epsilon_{ji}^* + E_i D_i^*) d\Omega$$

Figure 3 : Localization of periodicity conditions

Figure 4 : Typical meshing of horizontal planes

where upper-script * means conjugate value, is calculated.

- the global loss factor $\tan\delta_w$ is evaluated to give a rigorous measure of the efficiency of the composite transducer, versus massive PZT versions.

3. APPLICATIONS

3.1. Influence of boundary conditions on the loss factor

In terms of energy and dissipation, boundary conditions have a significant influence: Table I gives the loss factor of P188/epoxy and P189/epoxy piezo-composites for various volume rates of PZT and different loadings and boundary conditions. For the computations, the material properties are

Table 1. MECHANICAL LOSS FACTOR (%) FOR 1-3 P188/ARALDITE D AND P189/ARALDITE D COMPOSITES UNDER VARIOUS LATERAL BOUNDARY AND LOADING CONDITIONS: A - ZERO NORMAL DISPLACEMENT ON $\partial\Omega_L$ AND IMPOSED NORMAL DISPLACEMENT ON $\partial\Omega_T$; B - FREE LATERAL EDGES AND IMPOSED NORMAL DISPLACEMENT ON $\partial\Omega_T$; C - FREE LATERAL EDGES AND IMPOSED UNIFORM PRESSURE ON $\partial\Omega_T$.

PZT vol. %	PZT-P188			PZT-P189		
	A	B	C	A	B	C
0		3.9			3.9	
20	2.45	1.97	2.42	1.4	0.8	1.77
50	1.99	1.47	1.75	0.78	0.3	0.8
80	1.95	1.31	1.33	0.7	0	0.18
100		1.25			0.08	0

Figure 5 : Maximum axial stress as a function of the PZT fraction for a PZT P189/Epoxy composite with a 1m/s front radiating face vibration speed. The stress of the radiating transducer (air backed, one radiating face in water) is modeled with the KLM monodimensional model.

Figure 6 : Loss angle ($\tan(\delta_m)$) as a function of the PZT volume fraction for PZT P188/Epoxy composite (dots) and PZT P189/Epoxy composites (triangles) obtained with the proposed modeling approach and with clamped lateral boundary conditions. The lines correspond to a simple parallel model with complex material data (7).

those of PZT P188 and PZT P189 (commercial PZT made by Saint-Gobain Quartz) and Araldite D (the commonly used epoxy resin from Ciba Chemical Specialties). Note that for pure PZT, only free boundary conditions losses are given because it has been evaluated on longitudinal 33 mode as specified in (6). Indeed, in the piezo-composite the PZT rods are more or less working in this specific mode.

3.2. Ultrasonic transducer analysis

The determination of the losses of a piezo-composite transducer and their distribution between active and passive phases is a key point for defining the adequate PZT volume fraction and to better define the accurate PZT and polymeric material for a given application (7).

Two phenomena appear to be in competition in a composite. On the one hand, the acoustic impedance mismatch with water is lower for low PZT volume fraction composites, this leads to lower stresses in the transducer for a given velocity of the front (radiating) face or for a given radiated acoustic power. This is illustrated in Figure 5 showing the maximum axial stresses as a function of the PZT contents obtained with the KLM model (7,8). Consequently this also results in lower losses in the transducer. On the other hand, while reducing the acoustic impedance of the transducer, the (lossy) polymer volume fraction increases, increasing the mechanical loss angle of the composite material. Figure 6 shows the mechanical loss angle of the composite versus the PZT volume fraction as it is predicted by the proposed approach. Two data sets are plotted for a soft and a hard PZT respectively P188 and P189. For each data set the results of the model with clamped lateral boundaries and no electric field is compared to the prediction of a simple parallel model with complex material data (7).

Both results are in good agreement. It is interesting to note that the loss angle is not a simple linear function of the PZT

Figure 7: Efficiency as a function of the PZT volume fraction for PZT P188/Epoxy and PZT P189/Epoxy composites. The total acoustic output power is held constant. Obtained with a mono- dimensional KLM model of an air baked transducer radiating in water.

volume fraction, but is rather constant down to 20% PZT while it finally increases up to the polymer loss angle. From these two opposite tendencies, it can be shown using a mono-dimensional piezoelectric transducer model with losses (KLM or other) that the result is favourable to the lowest PZT fractions. Figure 7 shows the global efficiency (output acoustic power to electric input power ratio) of P188/Epoxy and P189/Epoxy composite transducers. It has been obtained with a constant acoustic output power with various PZT volume fractions. This globally shows that even for a piezo-composite made essentially of a polymer resin, the piezoelectric ceramic losses are still an important parameter and that PZT fraction ranging between 10 and 40% are preferred. This agrees with previously reported experiments showing best results for a 50% P189/Epoxy composite (7).

Concerning the losses distribution, Figure 8 shows that nearly 50% of the total losses are due to the PZT in the case of the PZT P188 while only 10% of the losses are due to the ceramic in a P189/Epoxy composite.

3.3. Longitudinal mode positioning device

Another important application in the field of piezoelectric materials is the linear actuation or positioning. In this case the frequency is usually low and the piezomaterial is working in the quasi-static mode. In this configuration, the piezoelectric rod is driven with a high electric field which mainly leads to dielectric losses. The proposed model is used with free displacement boundary conditions and imposed electric potentials. The aim here is to consider the losses and the advantage of using a composite instead of a bulk ceramic. Figure 9 shows the axial displacement and the total losses for one strain cycle for a P188/Epoxy composite. The modeled piezo-composite is driven with a 100V/mm electric field. The extremity is mechanically free and only very small variations are observed with a 0.1 MPa superimposed pressure along the axis. As it is known for a 1-3 piezo-composite, the displacement is not a linear function of the PZT volume fraction. Instead, the displacement is nearly invariant down to a 30% PZT fraction and then it decreases rapidly down to zero. At the same time, in this case mainly influenced by the dielectric losses of the materials, the total losses are a linear function of the PZT volume fraction.

Another way to consider this result is to plot the axial displacement versus the PZT volume fraction at constant loss. In this case, as the losses are proportional to the square of the electric field, the driving electric field is a decreasing function of the PZT volume fraction. Then, the displacement obtained in figure 9 has to be corrected according to the electric field value, taking into account that the displacement is proportional to the electric field. Figure 10 shows the corrected axial displacement versus the PZT volume fraction for a constant 47 $\mu\text{J}/\text{cycle}$ loss (loss value corresponding a 50% PZT volume fraction composite driven with a 100V/mm electric field). This shows that if the losses are to be taken into account, in the case of a periodic motion, piezo-composite with a low PZT content could be advantageously used instead of bulk ceramic.

4. CONCLUSIONS

The originality of this work consists of a specific 3D-modeling of a piezoelectric periodic composite with losses and taking into account boundary conditions that are truly representative of the whole transducer.

With null normal displacement the results agree well with the usual parallel model approach with complex data. The losses are not simply proportional to the lossy polymer volume fraction; they depend strongly on the PZT type (soft or hard) and their variations are also strongly connected to the and solicitation boundary conditions. For a 50% P189-epoxy composite with blocked normal displacements, losses are shown to be twice as important as in the free edges configuration ones. With free lateral edges for the P188-epoxy composite, the losses present very few variations between 50 and 100% PZT; this allows the use of composite to decrease the global transducer mass without affecting its losses.

Figure 8: Comparison of the losses between the two composite components for PZT P188/Epoxy and PZT P189/Epoxy composites.

Figure 9: Axial displacement and total losses as a function of the PZT volume fraction for a PZT P188/Epoxy composite in quasi-static conditions. Modeling with free boundary condition and imposed electric field (100V/mm).

Figure 10: Axial displacement as a function of the PZT volume fraction for a PZT P188/Epoxy composite at constant losses. The electric field and consequently the displacement is corrected from Figure 9 to keep a 47 μJ loss/cycle.

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