Non linear effects in piezoelectric materials

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The static and dynamic non-linear behaviours of a soft and a hard zirconate titanate composition are investigated in this paper as a function of electrical and mechanical fields. The calculated Rayleigh coefficients show that they are similar for the permittivity ϵ_{33}^T and the piezoelectric constant d_{33} and nul for the voltage constant g_{33} and the compliance at zero D (D = dielectric displacement). A non-linear electromechanical equivalent circuit is built up with components proportional to D . Finally an extended model to non-Rayleigh type behaviours is proposed.

Keywords : Piezoelectric ceramics, non-linearities, high fields, equivalent circuit.

Efectos no lineales en materiales piezoeléctricos

Los comportamientos no lineales estáticos y dinámicos de composiciones blandas y duras de titanato circonato de plomo se investigan en este trabajo en función de campos eléctricos y mecánicos. Los coeficientes de Rayleigh calculados son similares para la permitividad ϵ_{33}^T y la constantes piezoeléctrica d_{33} y nulos para la constante g_{33} y la complianza a D cero (D =desplazamiento dieléctrico). Se construye un circuito electromecánico no lineal equivalente con componentes proporcionales a D . Finalmente se propone un modelo extendido a comportamientos de tipo no-Rayleigh.

Palabras clave: Cerámicas piezoeléctricas, no linealidades, campos altos, circuito equivalente.

1. INTRODUCTION

Piezoelectric ceramic materials convert electrical energy into mechanical energy and vice versa. They are the key components of high precision sensors actuators or ultrasonic probes which play an important role in the maintenance of many critical industrial processes. Piezoelectric functional materials made of bulk ceramics or multilayers, are also widely used in a lot of applications to miniaturised systems.

The three available tools to help industrial to develop piezoelectric devices are :

- Analytical approach
- Finite element analysis
- Electromechanical equivalent circuits.

All of them are built on the linear equations of the piezoelectricity based on thermo-dynamical considerations.

These equations rely the stress T to the electric displacement D or the electric field E to the relative strain S with piezoelectric coefficients (d_{33} , d_{31} , d_{15}). In the same way linear relations are assumed between D and E (permittivities ϵ_{33}^T , ϵ_{33}^S ...) or between S and T (compliances s_{33}^E , s_{33}^D ...).

Practically at high power none of these coefficients are constant but they present a non-linear-behaviour that is to say they are dependent on electric field, stress and frequency. Such non linearities are also linked with increased losses and hysteresis phenomena.

Our purpose is to establish phenomenological behaviour laws of the electromechanical coefficients which could be used in models for designing piezoelectric devices for power dynamic applications.

In a first part experimental results will be given on non-linear behaviours as a function of static and dynamic electric field and stress. In a second part a Rayleigh type approach, which is often sufficient when the level of solicitation is rela-

tively limited, is considered. In a third part a more elaborated model will be developed in order to take into account quadratic non-linearities and/or saturation effects.

2. EXPERIMENTAL PROCEDURES AND RESULTS

2.1. Experimental procedures

Measurements are achieved on a soft composition (PZT 5A) and a hard type one (PZT 4D) on disks (10 mm in diameter, 1 mm in thickness) or rods ($t = 15$ mm, diameter 6.35 mm). Dielectric losses and permittivity are measured with the help of a Schering bridge at 1 kHz. Measurements of the converse piezoelectric coefficients as function of the electric field are made at 1 kHz with the help of a capacitive sensor device described elsewhere (1).

Measurements of the direct piezoelectric coefficient were made at 1.5 Hz as function of the uniaxial compressive stress using the experimental set up in Figure 1. The static prestress

Figure 1. Mechanical cycles set up

Figure 2. Relative permittivity as a function of the electric field (no offset, 1 kHz, disk) ● ■ experimental ---- fitting with equation [1]

Figure 3. Converse piezoelectric coefficient as a function of the electric field (no offset, 1 kHz, disk) ● + experimental ---- fitting with equation [1]

Figure 4. Dielectric loss tangent versus the electric field (no offset, 1 kHz, disk) ● ■ experimental ---- fitting with equation [3]

(bias stress) can be adjusted by a screw and the amplitude of the dynamic stress is adjusted by the cam eccentricity (up to around 30 Mpa). Measurement of the short circuit charges is performed on two piezoelectric rods connected head to foot.

As the compressive stress causes domain rearrangement the samples are submitted to high bias stress for at least one hour before measurement.

Mechanical losses and frequency shift at the series resonance are measured with a network analyser a power amplifier and current probes (2)

b) PZT 4D

Figure 5. Converse piezoelectric coefficient d_{33c} versus the DC field ($f = 1$ kHz, $E_{AC} = 30$ V/mm)

a) PZT 5A

b) PZT 4D

Figure 6. Relative permittivity ϵ_{33r} versus the DC bias ($f = 1$ kHz, $E_{AC} = 3$ V/mm)

2.2. Results

Figures 2 and 3 show that ϵ_{33}^T and d_{33} follow a quasi Rayleigh law as a function of the electric field with some deviations at high field ($> 400 \cdot 10^3$ Vm⁻¹). The variations of the dielectric loss tangent ($\tan \delta$) are not so linear (Figure 4).

Figures 5 and 6 give the variations of d_{33c} and ϵ_{33}^T at constant AC drive and in function of the DC bias. The corresponding static Rayleigh coefficients are smaller than the dynamic one's and are dependent of the bias sign.

Figure 7. Direct piezoelectric coefficient d_{33} as a function of dynamic stress (1.5 Hz, offset $T_{st} = T_{dyn} + T_{min}$, T_{dy} = peak value, $T_{min} = 2.8$ Mpa)

Figure 8. Direct piezoelectric coefficient d_{33} as a function of static stress ($f = 1.5$ Hz, $T_{dy} \ll T_{st}$)

Figures 7 and 8 represent respectively the dependence of d_{33} on the dynamic stress at constant bias stress and of static stress at low dynamic stress.

The decrease of d_{33} as a function of the static uniaxial stress is in agreement with results obtained by G. Robert (3) but in contradiction with results obtained by D. Guyomar (4) and G. Zhang (5). Such discrepancies could be due to different experimental procedure or to ageing and deageing mechanisms.

Figure 9 shows a linear variation of the displacement u as function of the current I absorbed by the sample at zero stress.

Therefore the voltage constant $g_{33} = \frac{S}{D} = \frac{A\omega u}{t I}$ is constant.

The variations of the admittance at the longitudinal resonance mode at various electric fields (6) show clearly that the parallel resonance (frequency), is independent of the level of solicitation (Figure 10).

It can be concluded that the compliance s_{33}^D at constant dielectric displacement is constant.

A) PZT 5A

b) PZT 4D

Figure 9. Displacement u as a function of the absorbed current (disk, 1 kHz)

Figure 10. Shift of the series resonance frequency and stability of the parallel resonance as a function of the applied voltage

3. RAYLEIGH APPROACH OF THE NON-LINEARITY

Although intensively studied since the discovery of PZT the physical mechanisms at the origin of the non-linearities are not really cleared up. At present it is generally admitted that, for a medium level of solicitations, non-linearities are the result of extrinsic contributions such as domain wall motion or to a lesser extent interphase boundary motion and displacement of defects points. In the case of electrical non-linearities all domain walls are concerned whereas in the case of piezoelectrical non-linearities only "non - 180°" domain walls are involved.

Extrinsic contribution can be reversible for small solicitations or irreversible for higher solicitations with depinning of the domain walls or reversal of a dipolar defect and consequently emergence of the hysteresis (Figure 11).

Empirical Rayleigh-type models have been introduced in piezoelectricity by Damjanovic et al (8) and used by other (1) (9). They appropriately describe the hysteresis of the piezoelectric response and are in agreement with experimental dependences of the electromechanical coefficients as a function of electric field and stress.

Our purpose is to develop here a normalized Rayleigh model applied to the dielectrical ($\epsilon_{33}^T, \epsilon_{33}^S$), piezoelectrical (d_{33}, g_{33}) and mechanical coefficients (s_{33}^E, s_{33}^D) as well as to the associated losses. The Rayleigh coefficients have to be differentiated whether the solicitation is static (bias) or dynamic : indeed at high static solicitations some slow reorganization of the domain walls (relaxation processes) is occurring for a rearrangement of the internal stresses and the static coefficients are affected independently of the dynamic one's. Moreover these static coefficients will depend on the sign of the solicitation (positive or negative electric bias).

3.1. Static Rayleigh coefficients

It is assumed that the electrical bias E_{DC} and the static compressive stress T_{dy} don't induce any permanent partial depoling of the material.

The whole static non-linearities are taken into account through the three following Rayleigh-type laws (coupled terms $E_{DC} T_{dy}$ are neglected)

$$\epsilon_{33}^T = \epsilon_{330}^T \left(1 + \alpha_{AC}^T E_{DC} + \gamma_{dy}^T T_{dy} \right)$$

$$d_{33}^{st} = d_{330} \left(1 + \gamma_{DC} E_{DC} + \delta_{st} T_{dy} \right)$$

$$s_{33}^E = s_{330}^E \left(1 + \beta_{DC}^E E_{DC} + \rho_{st}^E T_{dy} \right)$$

Other Rayleigh coefficients (see annex) can be easily derived from these six Rayleigh coefficients using the following relationships :

$$g_{33} = \frac{d_{33}}{\epsilon_{33}^T} \quad k_{33}^2 = \frac{d_{33}^2}{\epsilon_{33}^T s_{33}^E} \quad s_{33}^D = s_{33}^E (1 - k_{33}^2) \quad \text{and} \quad \epsilon_{33}^S = \epsilon_{33}^T (1 - k_{33}^2)$$

The electric bias field E_{DC} being either polarizing or depolarizing the coefficients of E_{DC} differ following the sign of E_{DC} . So the number of independent static Rayleigh coefficients is nine : $\alpha_{DC+}^T, \alpha_{DC-}^T, \dots$ for a full characterization of the static non-linearities.

Figure 11. Reversible and irreversible effects in a free energy diagram with a potential well (after G Robert (3))

3.2. Dynamic Rayleigh coefficients

Experimental results show that the dynamic non-linearities can also be described by Rayleigh laws :

$$\epsilon_{33}^T = \epsilon_{33st}^T \left(1 + \alpha_{AC}^T E_{AC} + \gamma_{dy}^T T_{dy} \right)$$

$$d_{33} = d_{33st} \left(1 + \gamma_{AC} E_{AC} + \delta_{dy} T_{dy} \right) \quad \text{where } E_{AC} \text{ and } T_{dy} \text{ are peak values}$$

$$s_{33}^E = s_{33st}^E \left(1 + \beta_{AC}^E E_{AC} + \rho_{dy}^E T_{dy} \right)$$

The number of independent coefficients is theoretically six but if we consider that the dynamic non-linearities have a common origin they could be reduced at only one.

3.3 Determination of the Rayleigh coefficients

Standardized Rayleigh coefficients calculated from the experimental results are given in Table I. For comparison the Rayleigh coefficients calculated from results given in reference (7) on Ceramic Multilayer Actuators are given in Table II : in every case the similarity of the α_{AC} and γ_{AC} Rayleigh coefficients confirm that the voltage coefficient g_{33} is constant. For hard compositions (PZ26, PZ24, PZT 4D) the variation are not really linear, so the given values of α_{AC+}^T and γ_{AC} correspond to a mean slope.

The main comments on experimental results at low frequency are :

- Only electromechanical coefficients at constant dielectric displacement D are true constants :

$$g_{33}^D = g_{330}^D \quad s_{33}^D = s_{330}^D \quad (\text{stability of the antiresonance frequency at high level}) \quad \text{or} \quad \beta_{AC}^D = 0$$

- Relative variations of the permittivity ϵ_{33}^T and of the piezoelectric coefficient (d_{33}) are identical : $\alpha_{AC+}^T \neq \gamma_{AC}$

- α_{AC+}^T and γ_{AC} are nearly frequency independent (1)

The dielectric loss tangent is increasing at high field.

3.4 Electrical equivalent circuit in Rayleigh approximation

From comments made in 3.3. it can be expected that the non-linearities are mainly dependent on the dielectric displacement D induced by an electrical or mechanical solicitation.

The conventional low frequency equivalent circuit (Figure 12a) must be completed by a resistance r and a reactance X such as:

$$r = \frac{aD}{C_{LF_0} \omega} \quad \text{and} \quad X = \frac{bD}{C_{LF_0} \omega} \quad \text{with} \quad C_{LF_0} = \frac{\epsilon_{33_0}^T A}{t}$$

(A = disk area, t = thickness) or in a standardized form :

$$\rho = \frac{aD}{\epsilon_{33_0}^T \omega} \quad \text{and} \quad \epsilon' = -\frac{\epsilon_{33_0}^T}{bD} \quad (\epsilon' \text{ represents a negative capacitance}).$$

For small non-linearities or at relatively low electrical field ($E \ll 1/4 \epsilon_{33_0}^T b$) and neglecting ρ a Rayleigh law is found for ϵ_{33}^T and d_{33} :

$$\frac{\epsilon_{33}^T}{\epsilon_{33_0}^T} = \frac{d_{33}}{d_{33_0}} \quad \# \quad 1 + b \epsilon_{33_0}^T E \quad (1)$$

$$\alpha_{AC}^T = \gamma_{AC} = b \epsilon_{33_0}^T \quad \text{therefore} \quad g_{33} = \text{constant}$$

$$s_{33}^E = s_{33}^D + d_{33} g_{33}^D \quad \text{leads to :} \quad \beta_{AC}^E = k_{33_0}^2 \gamma_{AC}$$

For mechoelectrical conversion the circuit of Figure 12b becomes as in Figure 13.

In open electrical circuit ϵ' and ρ are inactive as a function of the dynamic stress. So :

$$s_{33}^D = s_{33_0}^D \rightarrow \rho_{dy}^D = 0, \quad g_{33} = g_{33_0} \rightarrow \delta_{dy} = \chi_{dy}^T$$

$$\frac{d_{33}}{d_{33_0}} \# 1 + bD \quad \text{and} \quad \frac{s_{33}^E}{s_{33_0}^E} = 1 + k_{33_0}^2 bD \# 1 + k_{33_0}^2 d_{33_0} bT_{dy}$$

Therefore :

$$\delta_{dy} = \chi_{dy}^T = d_{33_0} b = g_{33_0} \gamma_{AC} \quad (2)$$

$$\rho_{dy}^E = k_{33_0}^2 \delta_{dy} = k_{33_0}^2 g_{33_0} \gamma_{AC}$$

Experimental values of δ are higher than the theoretical one's calculated with equation (2) :

$$\text{PZT 5A : } \delta_{dy \text{ exp}} = 46 \cdot 10^{-9} \text{ m}^2 / \text{N} \quad g_{33_0} \gamma_{AC} = 40 \cdot 10^{-9} \text{ m}^2 / \text{N}$$

$$\text{PZT 4D : } \delta_{dy \text{ exp}} = 26.6 \cdot 10^{-9} \text{ m}^2 / \text{N} \quad g_{33_0} \gamma_{AC} = 8.6 \cdot 10^{-9} \text{ m}^2 / \text{N}$$

γ_{AC} measurements are made around 1 kHz whereas δ_{dy} measurements are made around 1.5 Hz. It seems therefore that frequency dependence effects are not accurately taken into account in the present model.

The value of "a" is obtained by fitting the dielectric loss tangent of Figure 4 with the following equation derived from the model of Figure 12b (10).

$$\tan \delta \# \tan \delta_0 + \epsilon_{33_0}^T (a + b \tan \delta_0) E + a b \epsilon_{33_0}^T E^2 \quad (3)$$

TABLE I. RAYLEIGH COEFFICIENTS FOR CERAMICS PZT 5A AND PZT 4D (RAYLEIGH COEFFICIENTS OF g_{33} AND s_{33}^D ARE NUL)

Type	x_i	$\frac{1}{x_i} \frac{\delta x_i}{\delta E_{DC}} 10^{-7} \frac{\text{m}}{\text{V}}$	$\frac{1}{x_i} \frac{\delta x_i}{\delta E_{AC}} 10^{-7} \frac{\text{m}}{\text{V}}$	$\frac{1}{x_i} \frac{\delta x_i}{\delta T_{st}} 10^{-9} \frac{\text{m}^2}{\text{N}}$	$\frac{1}{x_i} \frac{\delta x_i}{\delta T_{dy}} 10^{-9} \frac{\text{m}^2}{\text{N}}$
PZT 5A	ϵ_{33}^T	$\alpha_{DC+}^T = -1.8$ $\alpha_{DC-}^T = -1.8$	$\alpha_{AC}^T = 14.8$	$\chi_{st} = 0.7$	
	d_{33}	$\gamma_{DC+} = -1$ $\gamma_{DC-} = -0.6$	$\gamma_{AC} = 14.1$	$\delta_{st} = -6.7$	$\delta_{dy} = 46$
PZT 4D	ϵ_{33}^T	$\alpha_{DC+}^T = 0$ $\alpha_{DC-}^T = -2.7$	$\alpha_{AC}^T = 3.4$	$\chi_{st} = 3$	
	d_{33}	$\gamma_{DC+} = 1.2$ $\gamma_{DC-} = -2.3$	$\gamma_{AC} = 3.1$	$\delta_{st} = -2.6$	$\delta_{dy} = 26.6$

TABLE II. RAYLEIGH COEFFICIENTS FOR CERAMIC MULTILAYER ACTUATOR (CALCULATED FROM MEASUREMENTS OF REF (7))

Type	PZ 21	PZ 27	PZ 29	PZ 26	PZ 24
$\alpha_{AC}^T 10^{-7} \frac{\text{m}}{\text{V}}$	16.7	11.2	15.1	2.2	3.9
$\gamma_{AC} 10^{-7} \frac{\text{m}}{\text{V}}$	16.1	11.4	16.2	2.5	3.9

a) Conventional

b) New non linear circuit

$$N = d_{33_0} / s_{33_0}^E \quad J = \omega \cdot D \quad J_m = N \dot{S} = N \omega S$$

Figure 12. Low frequency equivalent circuit

Figure 13. Non-linear mechoelectrical equivalent circuit

Figure 14. Non-linearities of ϵ_{33}^T and d_{33} as a function of the dynamic field

TABLE III. VALUES OF NON-LINEAR COEFFICIENTS FOR VARIOUS SOFT AND HARD COMPOSITIONS

b: RAYLEIGH APPROXIMATION VERSUS ELECTRIC DISPLACEMENT [EQU (1)]

b_1, n : EXTENSION TO NON-RAYLEIGH TYPE BEHAVIOURS [EQU (2)]

A: NON LINEAR LOSS COEFFICIENT WITH RAYLEIGH APPROXIMATION [EQU (3)]

* COEFFICIENTS CALCULATED FROM EXPERIMENTAL MEASUREMENTS GIVEN IN 2.2

** COEFFICIENTS CALCULATED FROM MEASUREMENTS IN REF. [7]

Type		ϵ_{33}^T				d_{33}				Losses
		ϵ_{330}^T r	b m ² /C	b_1 m ² /C	n	d_{330} pC/N	b m ² /C	b_1 m ² /C	n	
Soft	PZT 5A*	1640	102	135	0.67	400	97	135	0.61	25.8
	PZ 27**	1950	65	112	0.52	310	66	104	0.61	
	PZ 29**	2950	58	130	0.48	445	62	153	0.46	
	PZ 21**	3800	50	142	0.42	500	48	78	0.59	
Hard	PZT 4D*	1160	33	30	1	265	30	25	1	4.4
	PZ 26**	1400	16	15	1	210	20	16.5	1	
	PZ 24**	460	96	74	1	120	96	72	1	

4. EXTENDED MODEL TO NON-RAYLEIGH TYPE BEHAVIOURS

The non-linear behaviour of electromechanical coefficients as a function of the applied field shows three distinct areas (Figure 14) :

A very small threshold region (9) that will be neglected in this study (region ①)

A Rayleigh region in which coefficients are linearly varying (region ②)

A quadratic or saturated region at high solicitations (region ③)

Behaviours in regions ② and ③ can be described following a modified behaviour law for ϵ_{33}^T and d_{33}

$$\frac{\epsilon_{33}^T}{\epsilon_{330}^T} = \frac{d_{33}}{d_{330}} = \frac{1 + b_1 D}{1 + b_1 (1 - n) D}$$

with $n = 1$ no saturation effect (ϵ_{33}^T and $d_{33} \rightarrow \infty$ for $D \rightarrow \infty$)

$$n < 1 \text{ saturation effect } \left(\epsilon_{33 \max}^T = \frac{\epsilon_{330}^T}{1 - n}, d_{33 \max} = \frac{d_{330}}{1 - n} \right)$$

At low frequency ($f \ll f_s$) : $D = \epsilon_{33}^T E_{AC}$, therefore :

$$\frac{\epsilon_{33}^T}{\epsilon_{330}^T} = \frac{d_{33}}{d_{330}} = \frac{1 + b_1 \epsilon_{33}^T E_{AC}}{1 + b_1 (1 - n) \epsilon_{33}^T E_{AC}} \quad (4)$$

The series negative permittivity is then : $\epsilon' = -\epsilon_{330}^T \frac{1 + b_1 D}{n b_1 D}$

The variations of the compliance s_{33}^E are : $\frac{s_{33}^E}{s_{330}^E} = \frac{1 + b_1 D}{1 + b_1 D (1 - k_{330}^2) n}$

Table III gives the values of b_1 and n calculated with equation (4) and the values of b and a calculated with equations (1) and (3). For hard type compositions no saturation effects are observed ($n = 1$) whereas they are noticeable in soft type one's ($n < 1$).

5. EXAMPLE OF THE INFLUENCE OF NON-LINEARITIES IN PIEZOMOTOR APPLICATION (11)

Piezomotors are typically an application at a resonance mode of the structure. Two unwanted effects occur as a function of the applied voltage : a shift of the series resonance frequency (as in Figure 10) and a saturation of the vibration velocity of the stator (Figure 15).

Figure 15. Vibration velocity for 3 "hard" materials : the more sensitive material A is the less linear one

Such phenomenon are easily described by the non-linear components r and X (Figure 16). The apparent increase of the compliance ϵ_{33}^T due to the reactance X in series with the motional capacity C_m (s_{33}^E) causes a decrease of the propagation velocity $v=(\rho s_{33}^E)^{-1/2}$ and consequently a decrease of the series resonance frequency.

At high level the increase of the total resistance at the series resonance due to r in series with the motional resistance R_m decreases the vibration velocity of the stator.

In a Rayleigh approximation : $v = (r + R_m) i_m = \left(\frac{aD}{C_{Lr} \omega} + R_m \right) i_m$.

With $D = i_m / \omega A$ and $i_m = 2 N \dot{u}$ (N = electromechanical transformer ratio) it is obtained: $V = k_1 \dot{u} + k_2 \dot{u}^2$ (non linear law in accordance with experimental results of Figure 15).

As previously mentioned(12) the driving of piezomotors at high voltage around the parallel resonance frequency better preserves the linearity (s_{33}^D # constant and $J = \omega D$ small at fp).

The equivalent circuit of Figure 16 can be also used for modelling the non-linear behaviour of piezotransformers.

6. CONCLUSIONS

The design of high power devices requires the correct knowledge of the behaviour laws of piezoelectric materials at high field.

A Rayleigh approach of non-linearities at medium power is proposed. A non-linear model is developed with two com-

Figure 16. Non-linear equivalent circuit of a piezomotor

ponents, in series at the input of the classical Mason equivalent circuit, whose values are proportional to the dielectric displacement D . Many correlations between the AC electric field Rayleigh coefficients have been found, as well as between the dynamic stress coefficients. They are in good agreement with the experimental results. Relationships between electric field coefficients and stress coefficients are not so well validated perhaps because the frequency effects are not correctly taken into account by the model.

An extended model taking into account non-Rayleigh behaviours is proposed.

At last the importance of non-linearities is illustrated on one application.

ANNEX. OTHER RAYLEIGH COEFFICIENTS (STATIC OR DYNAMIC) DERIVED FROM THE RAYLEIGH COEFFICIENTS OF ϵ_{33}^T (α^T, χ^T), d_{33} (γ, δ) AND s_{33}^E (β^E, ρ^E).

x_i	$\frac{1}{x_i} \frac{\delta x_i}{\delta E}$	$\frac{1}{x_i} \frac{\delta x_i}{\delta T}$
g_{33}	$\gamma - \alpha^T$	$\delta - \chi^T$
k_{33}	$\frac{2\gamma - \alpha^T - \beta^E}{2}$	$\frac{2\delta - \chi^T - \rho^E}{2}$
s_{33}^D	$\beta_D = \frac{\beta^E - (2\gamma - \alpha^T) k_{33o}^2}{1 - k_{33o}^2}$	$\rho^D = \frac{\rho^E - (2\delta - \chi^T) k_{33o}^2}{1 - k_{33o}^2}$
ϵ_{33}^S	$\alpha^S = \frac{\alpha^T - (2\gamma - \beta^E) k_{33o}^2}{1 - k_{33o}^2}$	$\chi^S = \frac{\chi^T - (2\delta - \rho^E) k_{33o}^2}{1 - k_{33o}^2}$

For dynamic coefficients we obtain assuming

$$\left\{ \begin{array}{l} \gamma_{AC} = \alpha_{AC}^T \\ \delta_{dy} = \chi_{dy}^T \\ \text{and} \\ \beta_{AC}^E = k_{33o}^2 \gamma_{AC} \\ \rho_{dy}^E = k_{33o}^2 \delta_{dy} \end{array} \right.$$

x_i	$\frac{1}{x_i} \frac{\delta x_i}{\delta E_{AC}}$	$\frac{1}{x_i} \frac{\delta x_i}{\delta T_{dy}}$
g_{33}	0	0
k_{33}	$\gamma_{AC} \cdot \frac{1 - k_{33o}^2}{2}$	$\delta_{dy} \cdot \frac{1 - k_{33o}^2}{2}$
s_{33}^D	$\beta_{AC}^D = 0$	$\rho_{dy}^D = 0$
ε_{33}^S	$\alpha_{AC}^S = \gamma \left(1 - k_{33o}^2 \right)$	$\chi_{dy}^S = \delta_{dy} \left(1 - k_{33o}^2 \right)$

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