Analysis of the different techniques for calculation of bearing capacity of flexural reinforced concrete deep beams

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Abstract. The paper deals with the singularities of the design of deep beams without shear reinforcement according to different design codes, namely, STR 2.05.05:2005, EC2, EC2-SMM and ACI-318. The comparative analysis of calculation models, experimental verification, accuracy of models and the ranking of models according to the Modified Merit Points Classification (MDP) method are presented. The results shown that despite the systematic error of the STR 2.05.05:2005 method is closest to 1 if compared to the other methods, it is classified as dangerous one according to the MDP ranking.

Keywords: reinforced concrete, deep beam, carrying capacity, analysis, design methods, accuracy.

Introduction

Deep reinforced concrete beams could be used for the construction of ground beams, foundation caps, column supports, bridge transverse beams and other important structural members. However, the design methodologies (codes) in the different countries vary considerably. It shows that the factors like concrete strength, longitudinal reinforcement ratio, shear span and effective height ratio are estimated in a different way. Besides, the main assumptions, calculation schemes and calculation models are also different. These factors lead to different calculation results, including load bearing capacity, material economy and different overall design reliability. In addition, there are two design codes in Lithuania with three essentially different calculation models (STR 2.05.05:2005, EC2 and EC2-SMM), which result in three different load bearing capacities when designing the same structure in the same country. Thus, it is necessary to analyze the above-mentioned design models by comparing them with experimental data, determining their accuracy and the main factors influencing carrying capacity of the deep beams. In a same context, it is also interesting and valuable to include the American design codes (ACI-318) in this analysis.

Calculation models of deep beams according to different design codes

According to Lithuanian design codes, deep beams could be calculated in three different ways: (1) according to STR 2.05.05:2005, where calculation model is based on the failure of compression zone in projection of shear section; (2) according to EC2 empirical model, which determines shear stress acting in cross-sectional area; and (3) according to EC2-SMM (strut-tie) model, which estimates the failure of the strut. All these methods are fundamentally different and subsequently leads to different results of carrying capacity. There are some similarities of the methods: all of them evaluate the strength of the concrete, the amount of longitudinal reinforcement, the shear span to effective height ratio (a/d). However, all these factors are evaluated by different empirical expressions, for example, according to STR 2.05.05:2005 concrete shear stresses can be expressed as $0.3f_c^{2/3}$, according to EC2 – $(100 \cdot \rho_1 \cdot f_c)^{1/3}$, according to EC2-SMM – $0.36f_c \left[1 - \frac{f_c}{250} \right]$. American design codes estimate concrete strength as $0.156\sqrt{f_c}$. It must be pointed out, that one of the most important factors is the ratio of shear span to effective height (or acting moment to shear force ratio M/V). Unfortunately, in the design codes STR2.05.05: 2005 this factor is not evaluated at all. Meanwhile, EC2, EC2-SMM and ACI-318 evaluate this factor, but in a different way. The a/d ratio in EC2 model is assessed in calculation directly. Instead, the the beam height effect expressed through the coefficient k . EC2-SMM model for

such purpose uses the strut angle θ , and the ACI-318 model uses directly the M/V ratio. The key formulas of all mentioned design models are given in Appendix 1, Table 1. The typical calculation scheme is presented in Figure 1.

Figure 1. Typical calculation scheme of the deep beam

Verification by experimental data

Calculation models given in different design codes were verified by the numerical calculations and comparison to the experimental data of deep beam tests carried out by various authors. The experimental data of 212 beams without shear reinforcement were collected in available literature sources (Appendix 1, Table 2). The shear span to effective height ratio of the analyzed beams vary from 0.67 to 2 and the concrete compressive strength vary from 12 MPa to 87 MPa. The amount of longitudinal reinforcement in the tested deep beams vary between 0.57 to 4.76%, although the maximum reinforcement ratio values allowed in the design codes (when exceeding) were used in numerical calculations. Other parameters of experimental beams were also different, but they considered as less significant. Some deep beam specimens do not match the sizes used in construction, such as a beam width only 51 mm, concrete strength greater than 80 MPa or longitudinal reinforcement ratio that do not meet the general detailing requirements ($0.5\% < \rho_l < 2\%$). All of the test beams were tested by hydraulic testing machines. The load was applied through the steel plates and the load application velocity ranged from 0.02 to 0.05 kN/s. The main results of the numerical calculations are shown graphically in the Figures 2 to 5.

Figures 2–5 presents the influence of different factors to the V_{test} / V_{calc} value, namely, the width of deep beam b , shear span to effective height ratio a/d , concrete compressive strength f_c and longitudinal reinforcement ratio ρ_l . The linear regression analysis is performed based on experimental data presented in Table 1. The figures show the regression equations of the calculation models and the coefficients of determination R^2 .

Figure 2. V_{test}/V_{calc} ratio versus the width of deep beam b

Figure 2 shows the influence of the width of the beam b by comparing the the experimental and theoretical results of the deep beams without shear reinforcement. The presented regression equations show, that in this case the maximum value of the free member is 2.07 for the model provided by ACI-318. It means that the experimental load bearing capacity of an average deep beam is 2 times higher than the theoretical loading bearing capacity determined by this model. The minimum error is in the model STR 2.05.05:2005 and equals to 0.5%. A positive direction of regression lines indicates that by increasing the beam width b , the V_{test} / V_{calc} ratios also increase. The given determination coefficients R^2 show that the relationship between all models is very weak. The conclusion can be done that the ratio of V_{test} / V_{calc} is linearly independent from the width of the beam b .

Figure 3 shows the influence of the a/d ratio by comparing the the experimental and theoretical results of the deep beams without shear reinforcement. The regression equations presented in Figure 3 show that the maximum value of the free member is 3.42 for the model given in ACI-318, which means that the average deep beam experimental load capacity is 3.42 times higher than the theoretical load bearing capacity determined by this calculation model. The minimum error is again found in the STR 2.05.05:2005 model and equals to 25%. The negative angle of regression lines indicates that by increasing the a/d ratio, the V_{test} / V_{calc} ratios decrease. The obtained determination coefficients R^2 shows that the relationship between the ACI-318 code model is weak, and the other models are even weaker. Similary to previous example, the ratio V_{test} / V_{calc} is not linearly dependent on the a/d ratio.

Figure 3. V_{test}/V_{calc} ratio versus shear span to effective height ratio a/d

Figure 4 shows the influence of the compressive strength of the concrete f_c by comparing the the experimental and theoretical results of the deep beams without shear reinforcement. The presented regression equations shows that the maximum value of the free member is 2.2 for the EC2-SSM calculation model, which means that the average deep beam experimental load bearing capacity is 2.2 times higher than the theoretical load bearing capacity obtained by this model. The minimum error is 13% for the STR 2.05.05:2005 model. The determination coefficients R^2 shows that the relationship of all models is very weak. The ratio of V_{test} / V_{calc} is not linearly dependent on the compressive strength of the concrete f_c .

Figure 5 shows the influence of the longitudinal reinforcement ratio ρ_l by comparing the the experimental and theoretical results of the deep beams without shear reinforcement. It should me noted that the effect of longitudinal reinforcement ratio is not evaluated at all in the STR 2.05.05:2005 and EC2-SSM calculation models. The regression equations show that the maximum value of the free member is 1.99 for the model given in ACI-318 code, which means that the experimental load bearing capacity of an average deep beam is 99% higher than the theoretical one. The determination coefficients R^2 show that the relation between V_{test} / V_{calc} and ρ_l is very weak. Also, the ratio V_{test} / V_{calc} is not linearly dependent on the longitudinal reinforcement ratio ρ_l .

Figure 4. V_{test}/V_{calc} ratio versus concrete compressive strength f_c

Figure 5. V_{test}/V_{calc} ratio versus longitudinal reinforcement ratio ρ_l

The accuracy of calculation models

The analysis of systematic and random errors of the models under consideration is carried out by comparing the experimental and numerical results of 212 specimens (tested deep beams). The main results of this analysis are presented in Appendix 1, Table 3.

The highest systematic error is resulted by using the ACI-318 design code model. The most accurate results are obtained by using the calculation model given in STR 2.05.05:2005 design code. The systematic error is equal to 1.02. The summarized V_{test}/V_{calc} graphical interpretation with a 67% confidence interval is presented in Figure 6.

The calculation models given in different design codes could be also evaluated by other criteria, such as Modified Demerit Points Classification (DPC). The methodology based on the following evaluation system: when the systematic error is greater than 2 – 2 penalty points are awarded for the calculation model; when the systemic error ranges between 1.15 and 2 – penalty is 1 point; when systemic error varies between 0.85 and 1.15 – 0 penalty points are given; when systemic error ranges between 0.50 and 0.85 – 5 penalty points are given and finally, 10 penalty points are awarded when systemic error is less than 0.5. The calculation models are then ranked according to the sum of the awarded penalty points. The results presented in Appendix 1, Table 4, show that according to this evaluation method all analyzed design codes could be ranked in the following order: the most accurate method is EC2, second best is ACI-318, the third is EC2-SSM and the least accurate is STR 2.05.05:2005.

Figure 6. V_{test}/V_{calc} ratio with a 67% confidence interval according to different design codes

Conclusions

The regression analysis of experimental and numerical results of deep beams with varying the width of the beam b , shear span to effective height ratio a/d , concrete compressive strength f_c and longitudinal reinforcement ratio ρ_l has shown that the V_{test} / V_{calc} ratio is not linearly dependent on all these factors.

The comparison of experimental and numerical load bearing capacity values for deep beams has shown, that the systematic error of all design codes exceeds value of 1.0. The most accurate code based on systematic error is STR 2.05.05:2005. According to the systematic and random errors the codes could be ranked in the following order: STR 2.05.05:2005 (1.02; 0.26), EC2 (1.56; 0.46), EC2-SSM (1.83; 0.62) and ACI-318 (2.01; 0.70).

The calculation models were also ranked according to Qualification of Models of the Demerit Points Classification (DPC) method (Neto, Barros, & Melo, 2013). Unfortunately, this evaluation method gives the opposite ranking results. The codes are ranked in the following order: EC2, ACI-318, EC2-SSM and STR 2.05.05:2005.

According to DPC method, the calculation model given in STR 2.05.05:2005 collects the equal number of penalty points in three categories: Dangerous, Appropriate safety and Conservative. It is the main reason which caused the highest number of penalty point among all evaluated models. It must be taken into account by structural engineers while designing the deep beams and by using probabilistic methods, which assess systematic and random errors of the models.

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Appendix 1

Table 1. Calculation models provided in the analyzed design codes

Design code	Key formulas of calculation model
EC2	$V_{c1} = \left[C_c \cdot k \left(100 \cdot \rho_1 \cdot f_c \right)^{\frac{1}{3}} \right] b \cdot d \geq v_{c,min} \cdot b \cdot d$ $v_{c,min} = 0.035 \cdot k^{3/2} \cdot f_c^{1/2}$ $k = 1 + \sqrt{\frac{200}{d}} \leq 2.0$
EC2-SMM	$V_c = \left(l_b \cdot \sin^2(\theta_s) + c \cdot \sin(2 \cdot \theta_s) \right) \cdot f_{csb} \cdot b$ $f_{csb} = 0.6 \cdot f_c \cdot v$ $\theta_s = \arctan(\theta_s) = \frac{h - c - c_2}{a}$ $v = 0.6 \left[1 - \frac{f_c}{250} \right]$

End of Table 1

Design code	Key formulas of calculation model
STR2.05.05:2005	$V_c = \frac{\varphi_{c4} \cdot (1 + \varphi_n) \cdot f_{ctd} \cdot b \cdot d^2}{c}$ $V_{c,\min} = \varphi_{c3} \cdot (1 + \varphi_n) \cdot f_{ctd} \cdot b \cdot d$ $V_{c,\max} = 2.5 \cdot f_{ctd} \cdot b \cdot d$ $c_{\max} = 2.5 \cdot d$
ACI-318	$V_c = \left(3.5 - 2.5 \frac{M_u}{V_u} \right) \left[0.156 \cdot \sqrt{f_c} + 17.96 \cdot \rho_l \left(\frac{V_u \cdot d}{M_u} \right) \right] b \cdot d$ $1.0 \leq \left(3.5 - 2.5 \frac{M_u}{V_u} \right) \leq 2.5$ $V_c \leq 0.5 \sqrt{f_c} \cdot b_w \cdot d$

d is the effective beam height;
 a is the shear span;
 f_c is the compressive concrete strength;
 ρ_l is the amount of longitudinal reinforcement;
 b is the width of beam;
 l_b is the length of support plate;
 f_y is the yield strength of the longitudinal reinforcement.

Table 2. Experimental beams tested by various authors

Author(s)	b (mm)	a/d	f_c (MPa)	ρ_l	Number of specimens
Tan, Kong, Teng, and Guan (1995) Tan, Cheng, and Cheong (2005) Tan and Cheng (2006)	110–140	0.84–1.7	32.7–77.6	2.6	8
Zhang and Tan (2007)	80–230	1.1	28.3–32.4	1.15–1.28	8
Moody, Viest, Elstner, and Hognestad (1954, 1955)	178	0.76–2	17.8–41.5	0.95–4.76	51
Rogowsky, MacGregor, and Ong (1986)	127–200	1.05–2	23.1–79.8	1.12–3.34	5
Ismail (2016)	100–150	0.91–1.67	35.8–85.2	1.38–3.83	6
Morrow and Viest (1957)	305	0.95–1.96	12.7–46.8	0.57–3.8	22
Chang and Kesler (1958)	137	1.85	14.9	2.37	1
Watstein and Mathey (1958)	203	1.51	22.4–26.4	0.75–3.05	22
Rodriguez, Bianchini, Viest, and Kesler (1959)	152–156	1.36–1.38	17.9–26.1	1.59	6
de Cassio and Seiss (1960)	152	1.52–2	25.8–28.3	1.35–3.33	2
Leonhardt and Walther (1964)	190	1–2	22.3–38.1	1.32–2.47	10
de Pavia and Siess (1965)	51–102	0.67–1.34	24.6–37	0.83–1.67	5
Kani (1967)	149–157	0.99–2	17.4–35.2	0.77–2.69	20
Manuel (1974)	102	0.9–1.06	32.3–38.1	1.88	2
Walraven and Lehwalter (1994)	250	1–1.5	15–26.4	1.12–1.9	19
Yang, Chung, Lee, and Eun (2003)	160	1.06–1.13	31.4–78.5	0.97–1.05	7
Smith and Vantsoitis (1982)	100–102	0.77–1.31	13.1–21.7	1.93–3	5
Oh and Shin (2001)	130	0.85–1.25	23.7–49.1	1.56	3
Adebar (2000)	250	1.75–2	19.5–20.3	0.84–1.75	3
Pendyala and Mendis (2000)	80	2	63–87	2.02	2
Shin, Lee, Moon, and Ghosh, (1999)	110	1.13–1.69	77.6	3.77	2
Xie, Ahmad, Yu, Hino, and Chung (1994)	127	1–2	41.4–47	2.07	2
Niwa, Maekawa, and Okamura (1981)	100	0.88	13.1	3	1
Total					212

Table 3. The accuracy of calculation models

Calculation model	EC2	EC2 SSM	STR	ACI-318
Systematic error	1.56	1.83	1.02	2.01
Random error	0.46	0.62	0.26	0.7

Table 4. Classification of models by DPC method

V_{test} / V_{calc}	Classification	Penalty points			
		EC2	EC2-SSM	STR2.05.05:2005	ACI-318
<0.5	Extremely dangerous	0	0	0	0
0.5–0.85	Dangerous	14	16	68	1
0.85–1.15	Appropriate safety	34	27	78	18
1.15–2	Conservative	122	66	66	96
≥ 2	Very conservative	42	103	0	97
Sum		276	352	406	295
Range		1	3	4	2