STUDY OF SINGLE-SPECIES POPULATION MODELS

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Abstract

In this research paper one dimensional population models developed centuries ago shows that growth and/decay of single homogeneous populations But environmental effects spatial heterogeneity or age-structure deterministic models prevailing single species population models.

Key Words: Pedagogical, heterogeneity, Species, geometrical

Introduction:

The dynamics of single populations are generally described in terms of one-dimensional differential equations. In this chapter, we consider one-dimensional population models that were developed over the past several centuries to describe the growth and/or decay of single homogeneous populations. There is a pedagogical aim here in asserting the simple and fundamental principles at work in most continuous population models, and in this vein, we will begin from very simple foundations.

Following a progression similar to that taken in Edelstein-Keshet's text *Mathematical Models in Biology* (2005), we aim to elucidate the development of various models by augmenting in gradual increments. The addition of each new parameter will be accompanied by an empirical and/or theoretical justification that will, in any case, provide the reader with a gradual (as opposed to what one might call *saltationist*) sense of the model's evolution.

It should be noted that, in most scenarios, the models outlined herein are inaccurate and oversimplified. They do not consider stochasticity (chance events), environmental

effects, spatial heterogeneity, or age-structure. With regard to stochasticity, it is assumed that the deterministic model will produce results that, on average, would be produced by the analogous stochastic model (Maynard Smith, 1974).

The absence of certain realistic features does not negate the importance of these models or the principles they convey. Whereas the illustrative power of the models outlined below is pedagogical in nature; therefore, the explanatory power of the underlying principles, in large part, dominates the striving we might otherwise have for realism or accuracy.

MALTHUSIAN EXPONENTIAL GROWTH MODEL :

Thomas Robert Malthus proposed that a population's growth will proceed exponentially if growth goes unchecked (1798). The Malthus equation is denoted

$$\frac{dN}{dt} = rN \quad (1)$$

where N represents the number of individuals in the population (or more precisely, the *biomass* of the population) and r is a constant representing the intrinsic rate of growth. The growth rate r is also called the *Malthusian parameter* or the *net intrinsic growth rate* (i.e., $r \equiv b-d$, where b and d are intrinsic birth and death rates, respectively).

Units of time t vary depending on the organism of inquiry. For instance, for rapidly multiplying organisms (e.g., bacteria), t may be measured in minutes, whereas for slowly multiplying organisms (e.g., elephants), t may be measured in years.

Analytic Solution :

The solution to Eq. (1) is easily achieved by separation of variables and integrating both sides of the equation, assuming that $N(0) = N(t = 0)$, to yield

$$\int_{N(0)}^{N(T)} \frac{dN}{N} = \int_{t=0}^{t=T} r dt \quad (2)$$

$$\ln | \ln ||N||_{N(0)}^{N(T)} = rt + C|_0^T \quad (3)$$

and evaluating the upper and lower limits yield

$$\ln N(T) - \ln N(0) = (rT + c) - (r * 0 + c)$$

$$\ln(N(T)) - \ln(N(0)) = rT$$

$$\frac{N(T)}{N(0)} = e^{rT} \quad (4)$$

where $N(T)$ and $N(0)$ are both positive. Rearranging the equation, we get the exact solution

$$N(t) = N_0 e^{rt} \quad (5)$$

where the *initial condition* $N(0) = N_0$

Geometrical Analysis :

Malthusian growth described by Eq. (1) can manifest as both exponential growth and exponential decay (Figure 1); for instance, exponential growth occurs for all $r > 0$ (Figure 2); however, reversing the sign of r , the model becomes one in which a population decays exponentially in time as the fraction r of individuals is removed per unit time (Figure 3).

Viewing the phase line (Figure 1), the linear rate of change in population size, or density, is portrayed for both exponential growth and exponential decay. The only equilibrium solution $\frac{dN}{dt} = 0$ occurs when $N^* = 0$.

Qualitatively, we can judge the equilibrium point's stability based on whether trajectories approach $+\infty$, for all $r > 0$ or zero for all $r < 0$.

Figure 1. Phase line portrait of exponential model given by Eq. (1): phase trajectory reveals the linear rate of change in growth as a function of N .

Figure 2. Dynamics of exponential growth given by Eq. (2): exponential growth for a set of arbitrary positive growth rates r .

Figure 3. Dynamics of exponential growth described by Eq. (1): exponential decay for a set of arbitrary negative growth rates r .

Assumptions of the Model :

The Malthus model is one of the simplest models of growth for any reproducing population; however, it is too simple to be useful in most circumstances. As such, it makes the following assumptions: the population is homogeneous (i.e., all members are identical); the population inhabits a uniform and unvarying environment; an infinite supply of nutrients is available; there are no spatial limitations, and growth is density-independent. Realistically, population growth is limited by various factors from resource availability to predation. Additional limitations arise from the system's internal dynamics, such as overcrowding. The Malthus model may accurately describe the growth of a population for a limited period of time; however, unrestrained growth is never sustainable, and thus additional components are necessary to obtain a more realistic model.

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