An EOQ Form amid Cubic Demand Carry Forwarded with Three Parameter Weibull Distribution Deteriorating Item, Exclusive of Scarcity and Salvage Value

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Abstract

In this work, an EOQ form has been established on behalf of deteriorating objects where scarcity overruled and retrieve worth has been included with the above specified objects. This analysis has been expressed the demand rate measuring cubic utility of time and time reliant holding cost. The very deteriorating rate beholds three parameter Weibull utility on reducing the whole inventory time, methodically the form has also been resolved out. The usefulness of the aforesaid model as well as the sensitivity investigation of the finest resolution corresponding to a mixture of features has been observed considering a mathematical illustration into account. Ultimately the model significance has been arrived through a wholesome entire inventory cost.

Keywords: EOQ models, Deteriorating objects, Cubic demand, Salvage value, Weibull distribution

1. INTRODUCTION

Deteriorating significance implies weakening or worsening of the object utility for any other application. Matters concern to the domain of electronic, nuclear, photoelectric, etc. weakens concern to potentiality and service with regard to time. Therefore degradation or declining of matters in an inventory is purely one natural attribute and hence inventory analysts experience the reality of this essence. Illustratively the conventional goods which includes fruits, meat, food grains (potato, paddy, onion, wheat, etc.) damage, spoilage, dryness, vaporization, perfumes, alcohol, gasoline, etc which deteriorate during their normal storage period Goyal and Giri [14]. Ghosh and Chaudhuri [12] presented as well as justified the significance of the worsening object's immediate supply, quadratic time-varying order and scarcity. Chang [6] developed an inventory frame with diminishing objects corresponding to price rises while trader acknowledgment associated to demand a mount on behalf of the aforesaid objects through time-varying order and worsening rates. Alamri and Balkhi [1] has published the consequences of knowledge as well as wavering out the most favorable building up lot amount in favor of diminishing objects considering time changeable order and diminishing rates. According to Dye et al. [5] most favorable selling price and bulk amount considering a changeable rate of diminishing and

exponential fractional backlogging. Ajanta Roy [2] has also established an inventory model on behalf of diminishing objects considering time changeable holding cost as well as price dependent requirement. Biswaranjan-Mandal [4] has outlined the EOQ model on behalf of Weibull distributed diminishing objects holding ramp kind requirement and scarcity. Sahoo et al. [10] has also formulated an EOQ model for deteriorating rate and time-changeable holding worth. Hung [8] has also been implemented the most profound sort order, worsening as well as backorder rates. Mishra and Singh [13] has observed an EOQ inventory model in favor of ramp kind order, and time reliant diminishing objects considering retrieve worth and scarcities and apart from that the same work has also been propounded that an EOQ model for diminishing objects perpetuated with steady refill rate holding power form order as well as cubic polynomial deteriorating rate. Jain & Kumar [7] presented on deteriorating objects holding worth reliant order as well as Weibull distribution three features diminishing rate and stating with shortage. Patel [11] have used different demand rate and instantaneous deteriorating items with partial back logging. Karthikeyan & Santhi [9] has also been put forwarded an inventory formulation considering cubic order rate along with time reliant asset value proposing diminishing objects retrieve worth. In the recent paper, Sahoo, Paul & Kumar [15] emphasized upon inventory model possessing two warehouses inventory model has been developed with exponentially diminishing order rate with limited suspension price including salvages. Singh et. al. [16] propounded their observation for diminishing objects considering seasonal & stock reliant order through scarcity. Recently Aliyu & Sani [3] has also been explicitly expressed an EOQ formulation on behalf of diminishing objects considering time changeable worsening value as well as exponential reducing requirement.

In the current study, it has been established an EOQ formulation for diminishing objects holding requirement rate as cubic purpose of instant as well as the deterioration rate guided through three parameter Weibull distribution. This article has established scarcity over ridden as well as retrieve worth incorporated with diminishing objects. Notations & assumptions are listed in the next section. Outcomes have also been elaborated through a suitable illustration. Sensitivity

analysis against the most favorable outcome has exclusively made to justify the aforesaid model. The fundamental aim of this analysis is to establish a most suitable demand size reducing the entire inventory time.

2. NOTATIONS AND ASSUMPTIONS

Corresponding to the assessment of the projected model the underneath notations as well as assumptions are scheduled:

Notations:

$D(t)$: Demand rate is cubic in time.
 θ : Three characteristics Weibull distribution rate.
 HC : Holding cost per unit time, $H(t) = p + qt$.
 C_1 : The unit cost of an object.
 DC : Worsening cost per order.
 SC : The ordering cost per order.
 SV : Salvage value per item per unit time.
 $I(t)$: The inventory range accordance to time.
 Q : The maximum inventory level during $[0, T]$
 T : The span of cycle time.
 TC : The whole cost of an inventory arrangement.

Assumptions:

- The inventory formulation considers only one item.
- The demand rate is time reliant cubic function, i.e. $D(t) = a + bt + ct^2 + dt^3$, $a \geq 0, b \neq 0, c \neq 0, d \neq 0$. Here a is the commencement demand rate, b is the initial rate of change of demand, c is the rate at which the demand rate augments and d is the rate at which the change in the demand rate itself increases.
- Deterioration rate $\theta = \alpha\beta(t - \mu)^{\beta-1}$
 $\alpha > 0, \beta > 1, \mu (0 < \mu < 1)$
- Lead time is negligible.
- Scarcity over ridden.
- Replacement rate is infinite.
- The time horizon is finite.
- The holding cost $H(t)$ per unit time is time dependent and it is assumed $H(t) = p + qt$, $p > 0, q > 0$
- The salvage value ηC_1 , $0 \leq \eta \leq 1$ is associated to deteriorated units during a cycle time.
- T is the length of cycle time.

3. MATHEMATICAL FORMULATION

Let $I(t)$ be the inventory level at any time t , the differential equation that describes the inventory range in the interval $(0, T)$ is appeared.

$$\frac{dI(t)}{dt} + \theta I(t) = -D(t), 0 \leq t \leq T$$

Where $D(t) = a + bt + ct^2 + dt^3$

$$\theta = \alpha\beta(t - \mu)^{\beta-1}$$

$$\Rightarrow \frac{dI(t)}{dt} + \alpha\beta(t - \mu)^{\beta-1}I(t) = -(a + bt + ct^2 + dt^3), 0 \leq t \leq T \quad (1)$$

The general solution of equation (1) with boundary condition $I(T) = 0$ is

$$I(t) = a(T - t) + \frac{b}{2}(T^2 - t^2) + \frac{c}{3}(T^3 - t^3) + \frac{d}{4}(T^4 - t^4) + \alpha \left\{ a \frac{(T-\mu)^{\beta+1}}{\beta+1} + b \left(\frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) + c \left(\frac{(T-\mu)^{\beta+3}}{\beta+3} + 2\mu \frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu^2 \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) + d \left(\frac{(T-\mu)^{\beta+4}}{\beta+4} + 3\mu \frac{(T-\mu)^{\beta+3}}{\beta+3} + 3\mu^2 \frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu^3 \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) \right\} e^{-\alpha(t-\mu)^{\beta}} - \alpha \left\{ a \frac{(t-\mu)^{\beta+1}}{\beta+1} + b \left(\frac{(t-\mu)^{\beta+2}}{\beta+2} + \mu \frac{(t-\mu)^{\beta+1}}{\beta+1} \right) + c \left(\frac{(t-\mu)^{\beta+3}}{\beta+3} + 2\mu \frac{(t-\mu)^{\beta+2}}{\beta+2} + \mu^2 \frac{(t-\mu)^{\beta+1}}{\beta+1} \right) + d \left(\frac{(t-\mu)^{\beta+4}}{\beta+4} + 3\mu \frac{(t-\mu)^{\beta+3}}{\beta+3} + 3\mu^2 \frac{(t-\mu)^{\beta+2}}{\beta+2} + \mu^3 \frac{(t-\mu)^{\beta+1}}{\beta+1} \right) \right\} e^{-\alpha(t-\mu)^{\beta}} \quad (2)$$

At $I(0) = Q$ then the equation (2) reduces to

$$Q = aT + \frac{bT^2}{2} + \frac{cT^3}{3} + \frac{dT^4}{4} + \alpha \left\{ a \frac{(T-\mu)^{\beta+1}}{\beta+1} + b \left(\frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) + c \left(\frac{(T-\mu)^{\beta+3}}{\beta+3} + 2\mu \frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu^2 \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) + d \left(\frac{(T-\mu)^{\beta+4}}{\beta+4} + 3\mu \frac{(T-\mu)^{\beta+3}}{\beta+3} + 3\mu^2 \frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu^3 \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) \right\} e^{-\alpha(-\mu)^{\beta}} - \alpha \left\{ a \frac{(-\mu)^{\beta+1}}{\beta+1} + b \left(\frac{(-\mu)^{\beta+2}}{\beta+2} + \mu \frac{(-\mu)^{\beta+1}}{\beta+1} \right) + c \left(\frac{(-\mu)^{\beta+3}}{\beta+3} + 2\mu \frac{(-\mu)^{\beta+2}}{\beta+2} + \mu^2 \frac{(-\mu)^{\beta+1}}{\beta+1} \right) + d \left(\frac{(-\mu)^{\beta+4}}{\beta+4} + 3\mu \frac{(-\mu)^{\beta+3}}{\beta+3} + 3\mu^2 \frac{(-\mu)^{\beta+2}}{\beta+2} + \mu^3 \frac{(-\mu)^{\beta+1}}{\beta+1} \right) \right\} e^{-\alpha(-\mu)^{\beta}} \quad (3)$$

Total cost (TC) per unit time consists of the underneath components:

(1) Inventory holding cost per cycle $[0, T]$ is given by,

$$HC = \frac{1}{T} \int_0^T H(t) I(t) dt$$

$$HC = \left(p + \frac{qT}{2} \right) \left[\left(aT + \frac{bT^2}{2} + \frac{cT^3}{3} + \frac{dT^4}{4} \right) + \alpha \left\{ a \frac{(T-\mu)^{\beta+1}}{\beta+1} + b \left(\frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) + c \left(\frac{(T-\mu)^{\beta+3}}{\beta+3} + 2\mu \frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu^2 \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) + d \left(\frac{(T-\mu)^{\beta+4}}{\beta+4} + 3\mu \frac{(T-\mu)^{\beta+3}}{\beta+3} + 3\mu^2 \frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu^3 \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) \right\} \right] e^{-\alpha(-\mu)^{\beta}} \quad (4)$$

(2) The number of diminishing units (NDU) during each cycle time $[0, T]$ is described by,

$$\begin{aligned}
 NDU &= Q - \int_0^T D(t)dt \\
 &= \left[\alpha \left\{ a \frac{(T-\mu)^{\beta+1}}{\beta+1} + b \left(\frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) + \right. \right. \\
 &\quad c \left(\frac{(T-\mu)^{\beta+3}}{\beta+3} + 2\mu \frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu^2 \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) + d \left(\frac{(T-\mu)^{\beta+4}}{\beta+4} + \right. \\
 &\quad \left. \left. 3\mu \frac{(T-\mu)^{\beta+3}}{\beta+3} + 3\mu^2 \frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu^3 \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) \right\} \right] e^{-\alpha(-\mu)^\beta} - \\
 &\quad \alpha \left\{ a \frac{(-\mu)^{\beta+1}}{\beta+1} + b \left(\frac{(-\mu)^{\beta+2}}{\beta+2} + \mu \frac{(-\mu)^{\beta+1}}{\beta+1} \right) + c \left(\frac{(-\mu)^{\beta+3}}{\beta+3} + \right. \right. \\
 &\quad \left. \left. 2\mu \frac{(-\mu)^{\beta+2}}{\beta+2} + \mu^2 \frac{(-\mu)^{\beta+1}}{\beta+1} \right) + d \left(\frac{(-\mu)^{\beta+4}}{\beta+4} + 3\mu \frac{(-\mu)^{\beta+3}}{\beta+3} + \right. \right. \\
 &\quad \left. \left. 3\mu^2 \frac{(-\mu)^{\beta+2}}{\beta+2} + \mu^3 \frac{(-\mu)^{\beta+1}}{\beta+1} \right) \right\} e^{-\alpha(-\mu)^\beta} \quad (5)
 \end{aligned}$$

(3) The cost of deterioration (DC) for the cycle $[0, T]$ is given by,

$$\begin{aligned}
 DC &= \frac{\text{the unit cost of item}}{T} (NDU) \\
 &= \frac{c_1}{T} \left[Q - \int_0^T D(t)dt \right] \\
 &= \frac{c_1}{T} \left[\alpha \left\{ \begin{aligned} &a \frac{(T-\mu)^{\beta+1}}{\beta+1} + b \left(\frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) \\ &+ c \left(\frac{(T-\mu)^{\beta+3}}{\beta+3} + 2\mu \frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu^2 \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) \\ &+ d \left(\frac{(T-\mu)^{\beta+4}}{\beta+4} + 3\mu \frac{(T-\mu)^{\beta+3}}{\beta+3} + 3\mu^2 \frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu^3 \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) \end{aligned} \right\} e^{-\alpha(-\mu)^\beta} \right. \\
 &\quad \left. - \alpha \left\{ \begin{aligned} &a \frac{(-\mu)^{\beta+1}}{\beta+1} + b \left(\frac{(-\mu)^{\beta+2}}{\beta+2} + \mu \frac{(-\mu)^{\beta+1}}{\beta+1} \right) \\ &+ c \left(\frac{(-\mu)^{\beta+3}}{\beta+3} + 2\mu \frac{(-\mu)^{\beta+2}}{\beta+2} + \mu^2 \frac{(-\mu)^{\beta+1}}{\beta+1} \right) \\ &+ d \left(\frac{(-\mu)^{\beta+4}}{\beta+4} + 3\mu \frac{(-\mu)^{\beta+3}}{\beta+3} + 3\mu^2 \frac{(-\mu)^{\beta+2}}{\beta+2} + \mu^3 \frac{(-\mu)^{\beta+1}}{\beta+1} \right) \end{aligned} \right\} e^{-\alpha(-\mu)^\beta} \right] \quad (6)
 \end{aligned}$$

(4) Ordering cost (SC) per cycle $[0, T]$ is given by

$$SC = \frac{A}{T} \quad (7)$$

(5) Salvage value (SV) per cycle $[0, T]$ is given by,

$$\begin{aligned}
 SV &= \frac{\eta c_1}{T} \left[Q - \int_0^T D(t)dt \right] \\
 &= \frac{\eta c_1}{T} \left[\alpha \left\{ a \frac{(T-\mu)^{\beta+1}}{\beta+1} + b \left(\frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) + \right. \right. \\
 &\quad c \left(\frac{(T-\mu)^{\beta+3}}{\beta+3} + 2\mu \frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu^2 \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) + d \left(\frac{(T-\mu)^{\beta+4}}{\beta+4} + \right. \\
 &\quad \left. \left. 3\mu \frac{(T-\mu)^{\beta+3}}{\beta+3} + 3\mu^2 \frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu^3 \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) \right\} \right] e^{-\alpha(-\mu)^\beta} - \\
 &\quad \alpha \left\{ a \frac{(-\mu)^{\beta+1}}{\beta+1} + b \left(\frac{(-\mu)^{\beta+2}}{\beta+2} + \mu \frac{(-\mu)^{\beta+1}}{\beta+1} \right) + c \left(\frac{(-\mu)^{\beta+3}}{\beta+3} + \right. \right. \\
 &\quad \left. \left. 2\mu \frac{(-\mu)^{\beta+2}}{\beta+2} + \mu^2 \frac{(-\mu)^{\beta+1}}{\beta+1} \right) + d \left(\frac{(-\mu)^{\beta+4}}{\beta+4} + 3\mu \frac{(-\mu)^{\beta+3}}{\beta+3} + \right. \right. \\
 &\quad \left. \left. 3\mu^2 \frac{(-\mu)^{\beta+2}}{\beta+2} + \mu^3 \frac{(-\mu)^{\beta+1}}{\beta+1} \right) \right\} \quad (8)
 \end{aligned}$$

Total Cost (TC) = Ordering cost + Holding cost

+Deterioration cost + Setup cost - salvage value

$$\begin{aligned}
 &= \frac{A}{T} + \left[\left(p + \frac{qT}{2} \right) \left(\frac{aT + \frac{bT^2}{2}}{+ \frac{cT^3}{3} + \frac{dT^4}{4}} \right) \right] \\
 &+ \left(p + \frac{qT}{2} + \frac{c_1}{T} + \frac{\eta c_1}{T} \right) e^{-\alpha(-\mu)^\beta} \\
 &+ \alpha \left\{ \begin{aligned} &a \frac{(T-\mu)^{\beta+1}}{\beta+1} + b \left(\frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) \\ &+ c \left(\frac{(T-\mu)^{\beta+3}}{\beta+3} + 2\mu \frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu^2 \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) \\ &+ d \left(\frac{(T-\mu)^{\beta+4}}{\beta+4} + 3\mu \frac{(T-\mu)^{\beta+3}}{\beta+3} + 3\mu^2 \frac{(T-\mu)^{\beta+2}}{\beta+2} + \mu^3 \frac{(T-\mu)^{\beta+1}}{\beta+1} \right) \end{aligned} \right\} \\
 &- \alpha \left\{ \begin{aligned} &a \frac{(-\mu)^{\beta+1}}{\beta+1} + b \left(\frac{(-\mu)^{\beta+2}}{\beta+2} + \mu \frac{(-\mu)^{\beta+1}}{\beta+1} \right) \\ &+ c \left(\frac{(-\mu)^{\beta+3}}{\beta+3} + 2\mu \frac{(-\mu)^{\beta+2}}{\beta+2} + \mu^2 \frac{(-\mu)^{\beta+1}}{\beta+1} \right) \\ &+ d \left(\frac{(-\mu)^{\beta+4}}{\beta+4} + 3\mu \frac{(-\mu)^{\beta+3}}{\beta+3} + 3\mu^2 \frac{(-\mu)^{\beta+2}}{\beta+2} + \mu^3 \frac{(-\mu)^{\beta+1}}{\beta+1} \right) \end{aligned} \right\} e^{-\alpha(-\mu)^\beta} \quad (9)
 \end{aligned}$$

Now we minimize TC, then it must satisfy the minimization condition i.e. $\frac{\partial(TC)}{\partial T} = 0$ and $\frac{\partial^2(TC)}{\partial T^2} > 0$ and for all $T > 0$.

$$\frac{\partial(TC)}{\partial T} = \frac{-A}{T^2} + [p_1 + \alpha(p_2 + p_3 + p_4 + p_5)]e^{-\alpha(-\mu)^\beta} \quad (10)$$

Where

$$P_1 = \left\{ p(a + bT + cT^2 + dT^3) + q \left(aT + \frac{3}{4}bT^2 + \frac{2}{3}cT^3 + \frac{5}{8}dT^4 \right) \right\} e^{-\alpha(-\mu)^\beta}$$

$$p_2 = a \left\{ p(T - \mu)^\beta + \frac{q}{2} \left(\frac{(T - \mu)^{\beta+1}}{\beta+1} + T(T - \mu)^\beta - \frac{(-\mu)^{\beta+1}}{\beta+1} \right) + (\eta c_1 + c_1) \left(\frac{-1}{T^2} \frac{(T - \mu)^{\beta+1}}{\beta+1} + \frac{1(T - \mu)^\beta}{T} + \frac{1}{T^2} \frac{(-\mu)^{\beta+1}}{\beta+1} \right) \right\}$$

$$p_3 = b$$

$$\left\{ p((T - \mu)^{\beta+1} + \mu(T - \mu)^\beta) + \frac{q}{2} \left(\frac{(T - \mu)^{\beta+2}}{\beta+2} + T(T - \mu)^{\beta+1} + \mu \frac{(T - \mu)^{\beta+1}}{\beta+1} + \mu T(T - \mu)^\beta - \frac{(-\mu)^{\beta+2}}{\beta+2} - \mu \frac{(-\mu)^{\beta+1}}{\beta+1} \right) + (\eta c_1 + c_1) \left(\frac{-1}{T^2} \frac{(T - \mu)^{\beta+2}}{\beta+2} + \frac{1}{T} (T - \mu)^{\beta+1} - \frac{\mu}{T^2} \frac{(T - \mu)^{\beta+1}}{\beta+1} + \frac{\mu}{T} (T - \mu)^\beta + \frac{1}{T^2} \frac{(-\mu)^{\beta+2}}{\beta+2} + \frac{\mu}{T^2} \frac{(-\mu)^{\beta+1}}{\beta+1} \right) \right\}$$

$$p_4 = c \left\{ p((T - \mu)^{\beta+2} + 2\mu(T - \mu)^{\beta+1} + \mu^2(T - \mu)^\beta) + \frac{q}{2} \left(\frac{(T - \mu)^{\beta+3}}{\beta+3} + T(T - \mu)^{\beta+2} + 2\mu \frac{(T - \mu)^{\beta+2}}{\beta+2} + 2\mu T(T - \mu)^{\beta+1} + \mu^2 \frac{(T - \mu)^{\beta+1}}{\beta+1} + T\mu^2(T - \mu)^\beta - \frac{(-\mu)^{\beta+3}}{\beta+3} - 2\mu \frac{(-\mu)^{\beta+2}}{\beta+2} - \mu^2 \frac{(-\mu)^{\beta+1}}{\beta+1} \right) + (\eta c_1 + c_1) \left(-\frac{1}{T^2} \frac{(T - \mu)^{\beta+3}}{\beta+3} + \frac{1}{T} (T - \mu)^{\beta+2} - \frac{2\mu}{T^2} \frac{(T - \mu)^{\beta+2}}{\beta+2} + \frac{2\mu}{T} (T - \mu)^{\beta+1} - \frac{\mu^2}{T^2} \frac{(T - \mu)^{\beta+1}}{\beta+1} + \frac{\mu^2}{T} (T - \mu)^\beta + \frac{1}{T^2} \frac{(-\mu)^{\beta+3}}{\beta+3} + \frac{2\mu}{T^2} \frac{(-\mu)^{\beta+2}}{\beta+2} + \frac{\mu^2}{T^2} \frac{(-\mu)^{\beta+1}}{\beta+1} \right) \right\}$$

$$p_5 = d \left\{ p((T - \mu)^{\beta+3} + 3\mu(T - \mu)^{\beta+2} + 3\mu^2(T - \mu)^{\beta+1} + \mu^3(T - \mu)^\beta) + \frac{q}{2} \left(\frac{(T - \mu)^{\beta+4}}{\beta+4} + T(T - \mu)^{\beta+3} + 3\mu \frac{(T - \mu)^{\beta+3}}{\beta+3} + 3\mu T(T - \mu)^{\beta+2} + 3\mu^2 \frac{(T - \mu)^{\beta+2}}{\beta+2} + 3\mu^2 T(T - \mu)^{\beta+1} + \mu^3 \frac{(T - \mu)^{\beta+1}}{\beta+1} + T\mu^3(T - \mu)^\beta - \frac{(-\mu)^{\beta+4}}{\beta+4} - 3\mu \frac{(-\mu)^{\beta+3}}{\beta+3} - 3\mu^2 \frac{(-\mu)^{\beta+2}}{\beta+2} - \mu^3 \frac{(-\mu)^{\beta+1}}{\beta+1} \right) + (\eta c_1 + c_1) \left(-\frac{1}{T^2} \frac{(T - \mu)^{\beta+4}}{\beta+4} + \frac{1}{T} (T - \mu)^{\beta+3} - \frac{3\mu}{T^2} \frac{(T - \mu)^{\beta+3}}{\beta+3} + \frac{3\mu}{T} (T - \mu)^{\beta+2} - \frac{3\mu^2}{T^2} \frac{(T - \mu)^{\beta+2}}{\beta+2} + \frac{3\mu^2}{T} (T - \mu)^{\beta+1} - \frac{\mu^3}{T^2} \frac{(T - \mu)^{\beta+1}}{\beta+1} + \frac{\mu^3}{T} (T - \mu)^\beta + \frac{1}{T^2} \frac{(-\mu)^{\beta+4}}{\beta+4} + \frac{3\mu}{T^2} \frac{(-\mu)^{\beta+3}}{\beta+3} + \frac{3\mu^2}{T^2} \frac{(-\mu)^{\beta+2}}{\beta+2} + \frac{\mu^3}{T^2} \frac{(-\mu)^{\beta+1}}{\beta+1} \right) \right\}$$

$$\frac{\partial^2(TC)}{\partial T^2} = \frac{2A}{T^3} + [Q_1 + \alpha(Q_2 + Q_3 + Q_4 + Q_5)]e^{-\alpha(-\mu)^\beta} > 0$$

Where

$$Q_1 = \left\{ p(b + 2cT + 3dT^2) + \frac{q}{2} \left(a + \frac{3}{2}bT + 2cT^2 + \frac{5}{4}dT^3 \right) \right\}$$

$$Q_2 = a \left\{ p\beta(T - \mu)^{\beta-1} + \frac{q}{2} (2(T - \mu)^\beta + \beta T(T - \mu)^{\beta-1}) (\eta c_1 + c_1) \left(\frac{2}{T^3} \frac{(T - \mu)^{\beta+1} - (-\mu)^{\beta+1}}{\beta+1} - \frac{2}{T^2} (T - \mu)^\beta + \frac{\beta}{T} (T - \mu)^{\beta-1} \right) \right\}$$

$$Q_3 = b \left\{ p((\beta + 1)(T - \mu)^\beta + \mu\beta(T - \mu)^{\beta+1}) + \frac{q}{2} (2(T - \mu)^{\beta+1} + ((\beta + 1)T + 2\mu)(T - \mu)^\beta + \beta\mu T(T - \mu)^{\beta-1}) + (\eta c_1 + c_1) \left(\frac{2}{T^3} \frac{(T - \mu)^{\beta+2} - (-\mu)^{\beta+2}}{\beta+2} + \frac{2\mu}{T^3} \frac{(T - \mu)^{\beta+1} - (-\mu)^{\beta+1}}{\beta+1} - \frac{2}{T^2} (T - \mu)^{\beta+1} + \frac{\beta\mu}{T} (T - \mu)^{\beta-1} + \left(\frac{\beta+1}{T} - \frac{2\mu}{T^2} \right) (T - \mu)^\beta \right) \right\}$$

$$Q_4 = c \left\{ p((\beta + 2)(T - \mu)^{\beta+1} + 2\mu(\beta + 1)(T - \mu)^\beta + \beta\mu^2(T - \mu)^{\beta-1}) + \frac{q}{2} \left(2(T - \mu)^{\beta+2} + ((\beta + 2)T + 4\mu)(T - \mu)^{\beta+1} + (2\mu(\beta + 1)T + 2\mu^2)((T - \mu)^\beta + \beta\mu^2 T(T - \mu)^{\beta-1}) + (\eta c_1 + c_1) \left(\frac{2}{T^3} \frac{(T - \mu)^{\beta+3} - (-\mu)^{\beta+3}}{\beta+3} + \frac{4\mu}{T^3} \frac{(T - \mu)^{\beta+2} - (-\mu)^{\beta+2}}{\beta+2} + \frac{2\mu^2}{T^3} \frac{(T - \mu)^{\beta+1} - (-\mu)^{\beta+1}}{\beta+1} - \frac{2}{T^2} (T - \mu)^{\beta+2} + \left(\frac{\beta+2}{T} - \frac{4\mu}{T^2} \right) (T - \mu)^{\beta+1} + \left(\frac{2\mu(\beta+1)}{T} - \frac{2\mu^2}{T^2} \right) (T - \mu)^\beta + \frac{\beta\mu^2}{T} (T - \mu)^{\beta-1} \right) \right\}$$

$$Q_5 = d \left\{ p((\beta + 3)(T - \mu)^{\beta+2} + 3\mu(\beta + 2)(T - \mu)^{\beta+1} + 3\mu^2(\beta + 1)(T - \mu)^\beta + \beta\mu^2(T - \mu)^{\beta-1}) + \frac{q}{2} (2(T - \mu)^{\beta+3} + ((\beta + 3)T + 6\mu)(T - \mu)^{\beta+2} + (3\mu(\beta + 2)T + 6\mu^2)(T - \mu)^{\beta+1} + (3\mu^2(\beta + 1)T + 2\mu^3)(T - \mu)^\beta) + (\eta c_1 + c_1) \left(\frac{2}{T^3} \frac{(T - \mu)^{\beta+4} - (-\mu)^{\beta+4}}{\beta+4} + \frac{6\mu}{T^3} \frac{(T - \mu)^{\beta+3} - (-\mu)^{\beta+3}}{\beta+3} + \frac{6\mu^2}{T^3} \frac{(T - \mu)^{\beta+2} - (-\mu)^{\beta+2}}{\beta+2} + \frac{2\mu^3}{T^3} \frac{(T - \mu)^{\beta+1} - (-\mu)^{\beta+1}}{\beta+1} - \frac{2}{T^2} (T - \mu)^{\beta+3} + \left(\frac{\beta+3}{T} - \frac{6\mu}{T^2} \right) (T - \mu)^{\beta+2} + \left(\frac{3\mu(\beta+2)}{T} - \frac{6\mu^2}{T^2} \right) (T - \mu)^{\beta+1} + \left(\frac{3\mu^2(\beta+1)}{T} - \frac{2\mu^3}{T^2} \right) (T - \mu)^\beta + \frac{\beta\mu^3}{T} (T - \mu)^{\beta-1} \right) \right\}$$

Equation (10) is highly non linear and complicated to resolve by logical method. However the example below we use Mathematica version 12.0.

4. NUMERICAL EXAMPLES

To illustrate the Model developed an example is demonstrated considering the underneath factors;

Let $A=500$, $a=20$, $b=15$, $c=10$, $d=5$, $\alpha=0.5$, $\beta=0.6$, $\mu=0.7$, $p=0.2$, $q=0.4$, $\eta=0.1$, $c_1=5$

Replacing and resolving the aforesaid factors into the corresponding equations, we get;

$T=0.4579$, $Q=326.1462$, $TC=572.5623$.

Table-1

% variation in a	a	T	TC	Q
50	30	0.2614	487.235	305.1729
25	25	0.3291	501.873	312.4527
0	20	0.4579	572.5623	326.1462
-25	15	0.5893	683.1432	418.274
-50	10	0.6521	781.2359	537.629

Table-2

% variation in b	b	T	TC	Q
50	22.5	0.5124	583.2156	318.5462
25	18.75	0.4713	578.5362	323.4891
0	15	0.4579	572.5623	326.1462
-25	11.25	0.3921	564.2392	329.1438
-50	7.5	0.3463	558.1562	334.5231

Table-3

% variation in c	c	T	TC	Q
50	15	0.2315	675.2341	269.4742
25	12.5	0.3346	591.2597	292.7992
0	10	0.4579	572.5623	326.1462
-25	7.5	0.5692	516.2814	352.0644
-50	5	0.6923	453.6891	389.3165

Table-4

% variation in d	d	T	TC	Q
50	7.5	0.5325	628.638	478.5612
25	6.25	0.4826	596.920	439.2853
0	5	0.4579	572.5623	326.1462
-25	3.75	0.2315	516.541	280.2166
-50	2.5	0.1956	483.947	253.2021

Table-5

% variation in α	α	T	TC	Q
50	0.75	0.2935	753.1952	392.5254
25	0.625	0.3851	685.2358	356.2054
0	0.5	0.4579	572.5623	326.1462
-25	0.375	0.5216	492.2561	308.0678
-50	0.25	0.5923	413.7999	283.5718

Table-6

% variation in β	β	T	TC	Q
50	0.9	0.8391	729.5042	281.1604
25	0.75	0.6301	657.1192	258.0461
0	0.6	0.4579	572.5623	326.1462
-25	0.45	0.2547	487.6218	359.0548
-50	0.3	0.1028	413.8213	378.0571

Table-7

% variation in μ	μ	T	TC	Q
50	1.05	0.4216	619.1347	312.2859
25	0.875	0.4429	592.6617	318.9202
0	0.7	0.4579	572.5623	326.1462
-25	0.525	0.4596	546.6285	331.6285
-50	0.35	0.4631	532.8033	336.8628

Table-8

% variation in p	p	T	TC	Q
50	0.3	0.2561	752.1346	392.1962
25	0.25	0.3692	682.9217	379.2315
0	0.2	0.4579	572.5623	326.1462
-25	0.15	0.5891	478.2369	313.4291
-50	0.1	0.6327	413.2891	216.1382

Table-9

% variation in q	q	T	TC	Q
50	0.6	0.2316	448.2316	412.3691
25	0.5	0.3259	483.1692	368.5893
0	0.4	0.4579	572.5623	326.1462
-25	0.3	0.5691	593.2167	308.1723
-50	0.2	0.6234	621.6215	289.7913

Table-10

% variation in η	η	T	TC	Q
50	0.15	0.4623	515.2318	305.4387
25	0.125	0.4592	538.2156	314.3076
0	0.1	0.4579	572.5623	326.1462
-25	0.075	0.4532	589.7285	338.9567
-50	0.05	0.4502	603.7368	346.9584

Table-11

% variation in C_1	C_1	T	TC	Q
50	7.5	0.2081	620.1793	289.2529
25	6.25	0.3279	592.9853	312.7580
0	5	0.4579	572.5623	326.1462
-25	3.75	0.5753	546.2515	338.2162
-50	2.5	0.6804	531.1966	346.0639

5. SENSITIVITY ANALYSIS

Under this resolution sensitivity analysis against the system factors; $a, b, c, d, \alpha, \beta, \mu, c_1, p, q$ upon the most suitable entire cost. The sensitivity analysis has also been employed to investigate the influence of alteration in every factors through 50%, 25%, -25%, -50% and considering a single factor at once and holding the other factors unaffected.

Discussion of outcomes

The discussion is based on the numerical example and the outcomes have been reflected in the table mentioned above.

1. As of Table- 1, with raise in the amount of the factor a , the value of T, TC and Q decreases. In this case we observe that while a raises, demand raises consequently the formulation forces a decreases in T, TC and Q .
2. From Table- 2, shows that as the amount of the factor b , the amount of T & TC increase while Q decrease. This is clear since if the demand is high therefore the total variable cost will be higher.
3. As per Table- 3, with raise in the amount of factor C , amount of T and Q decrease while TC increase. As a result of this raise, TC the formulation then reduces to optimum quantity and cycle time.
4. From Table- 4, shows that with raise in the amount of factor d , the value of T, TC as well as Q increases. In this case we observe that when d increase; the requirement for that particular also rises keeping the total cost rise.
5. As of Table-5, with increase in the amount of factor α , the amount of T and Q raise whereas T reduces. Here, when α increases the optimum quantity and total cost are increases.

6. From Table-6, with increases in the amount of the factor β , the values of T , TC rises whereas Q diminishes. Here, when β rises, the total cost of the system increases with the increment in period time ' T '.
7. As of Table-7, corresponding to the rise in the amount of the factor μ , the amount of T, Q reduces while TC rises. In this case when μ is rises, total cost rise however the period time and most favorable amount reduces.
8. From Table-8, it is observed that with rise in the amount of the factor ' p ', the values of TC, Q rises whereas T reduces. Here, the total cost of the arrangement rises with the rise in holding cost ' p '.
9. From Table-9, it observed that rise in the amount of the factor ' q ', the amount of T, TC & Q rises. Here, the total cost of the arrangement rises with the rise in holding cost ' q '.
10. From Table-10, shows that with rise in the amount of the factor ' η ', the amount of TC, Q reduces whereas T rises.
11. As of Table-11, with rise in the amount of the factor ' c_1 ', the amount of T, Q reduces whereas TC rises. Here, the total cost of the arrangement rises with the rise in the unit cost of an object ' c_1 '.

6. CONCLUSION

In this paper we developed an inventory model which determines the deterioration rate is three parameter Weibull function and the holding cost is a time dependent. The demand rate is assumed as cubic function of time. Retailer in super market faces this difficulty during trading products whose importance go down with each passing moment. The pattern in which the basic independent factors influencing the total average cost has also been projected through sensitivity analysis column within this model. This very model can be highly appreciable for the industries in which the demand rate depends upon the time. This model can be used for items like fruits and vegetable. The objective of this paper is to reduction the total time T .

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