Unsteady Peristaltic Transport of MHD Fluid through an Inclined Stenosed Artery with Slip Effect

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Abstract:

The current theoretical study deals with effect of slip velocity on unsteady generalized non-Newtonian blood flow through an inclined artery with permeable wall. In this mathematical model, a constant transverse magnetic field is applied on peristaltic flow of blood and we are treating blood as an elastico-viscous, incompressible and electrically conducting fluid. To solve unsteady non-Newtonian axially symmetric momentum equation, appropriate transformation methods are adopted in cylindrical polar co-ordinate system with suitably prescribed conditions and to validate the applicability of the proposed study, analytical expressions for axial velocity, fluid acceleration, volumetric flow rate and wall shear stress are computed. Obtained numerical results are depicted graphically for different values of flow variables of interest, to analyze the influence in axial velocity, wall shear stress and volumetric flow rate of streaming blood.

Keywords: Peristalsis, elastico-viscous, slip velocity, selective permeability and stenosed artery

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NOMENCLATURE:

p_0 , amplitude of pressure gradient steady ($\frac{N}{m^2}$);
 p_1 , amplitude of pressure gradient oscillations ($\frac{N}{m^2}$);
 B , magnetic flux intensity (T);
 B_0 , applied magnetic field;
 $R(z)$, radius of artery in stenosed region (m);
 R_0 , radius of normal artery (m);
 H Hartmann number;
 $\vec{J}$, current density vector ($\frac{A}{m^2}$);
 l_0 , length of stenosis (m);
 s , stenosis shape parameter ($s \geq 2$);
 μ , particle concentration parameter;
 d , stenosis location;

r radius (m);
 t time (s);
 u velocity component of streaming blood ($\frac{m}{s}$);
 $\vec{v}$ velocity vector;
 $G(t)$ body acceleration ($\frac{m}{s^2}$);
 α Womersley frequency parameter;
 ρ density of blood ($\frac{kg}{m^3}$);
 μ dynamic viscosity of blood ($\frac{kg}{ms}$);
 σ electrical conductivity ($\frac{S}{m}$);
 φ inclined angle of arterial segment(degrees);
 ω_p $2\pi f_p$ heart pressure frequency;
 f_p pulse rate frequency(Hz);
 τ shear stress ($\frac{N}{m^2}$).

1. INTRODUCTION

Recently rheology of streaming blood in human arteries is a very interesting issue of research for haematologists and has attracted much attention. Peristaltic pumping means to supply nutrients through oxygen rich blood to each and every cell of human body under a sufficient pressure induced by a progressive wave of regular contraction and expansion of heart and then blood or (bodily fluid) returns along with cellular waste. But this transport of blood is not always so regular since it depends on hydrodynamic behaviour and mechanical properties of blood vessel walls. It may the cause of development of many cardiovascular diseases particularly atherosclerosis(athero means gruel or paste and sclerosis means hardness), medically called stenosis, is a common disease developed by narrowing of artery's lumen, due to deposition of arteriosclerotic plaque or other types of abnormal tissues along the wall of blood vessel. This results disorder in circulatory system by reducing or occluding blood supply which may develop cardiovascular diseases especially heart attack, hypertension, hypotension, stroke etc. Eventually

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necrosis may exit, when blood supply to a tissue reduces or blocks. Therefore the analysis of blood flow in stenosed arteries is of great importance for scientific investigation in the present scenario.

To explore the flow characteristics of blood through constricted vessel, Beaver and Joseph [1] employed boundary conditions at a naturally permeable vessel wall and it would help to judge that why no-slip condition at vessel wall should be replaced by slip velocity. Oka and Murata [14] studied hydro- dynamical theory for flow of blood through a blood vessel with permeable wall and investigated the exchange of fluid as a steady slow motion across the permeable wall upon the motion of the fluid within rigid circular tube. Saffman [16] discussed boundary conditions for flow of fluid at the surface of a porous medium. Popel et al. [15] have investigated a continuum approach for blood flow with couple stresses. Steady flow of blood through modelled vascular stenosis has been investigated by McDonald [10]. Shukla et al. [17] discussed the effect of stenosis on non-Newtonian flow of blood in an artery but later on they took into account the effect of radial distribution of cells and the existence of the peripheral plasma layer near the wall and studied the flow of blood through an artery with mild stenosis by assuming blood as a power law fluid. Blood flow through an artery with mild stenosis have been investigated by Sinha and Singh [18] and couple stresses effects on blood flow also discussed by them. Srivastava [20] studied the flow through stenotic blood vessel and he considered blood as couple stress fluid. A numerical study on flow of fluid have been developed by Lee [8] and fluid passed through tubes with double constrictions. The flow characteristics in the neighbourhood of double constrictions were studied numerically in a circular cylindrical tube. Misra et al.[12] have presented a non-Newtonian model in which they have discussed flow of blood through arteries under stenotic conditions. Halder and Ghosh [6] have studied blood flow under the effect of magnetic field through an indented artery in the presence of erythrocytes taking blood as Newtonian fluid and discussed the expressions for blood velocity, pressure and flow rate. Chakravarty and Mandal [2] investigated a two dimensional flow of blood under stenotic conditions through tapered artery. Srivastava [19] has investigated the flow of blood flow in a mild stenosed artery taking central layer (core region) is as couple stress fluid as it is the suspension of erythrocytes (RBC) and a peripheral layer of plasma as Newtonian fluid. Vajravelu et al.[22] have studied the peristaltic transport of Herschley-Bulkley fluid through an inclined tube.

A large number of investigations have been explored by several researchers to understand disorders in blood flow in an inclined vessel due to development of stenosis, by considering blood as Newtonian or non-Newtonian fluid. When our body experiences a sudden a change in velocity, flow of blood is disturbed. Although our body has remarkable adaptation to such type of sudden change, even then when our body accelerates while driving vehicles (car, bus, motorcycle etc.), travelling in aircraft, spacecraft, jogging etc., leads to many health issues like increasing abdominal pain, pulse rate, headache and losing vision. Varshney et al. [23] have also presented the effect of magnetic field on blood flow in a multiple stenosed artery and they have analysed that all the flow variables are affected by the intensity of magnetic field in the presence of multiple stenosis. Verma and Srivastava [24] have presented a theoretical study of hydro- magnetic pulsatory blood flow in a channel which is constricted and porous. They have considered blood as Power-law fluid and provided much evidences for the vital role of hydro-dynamic factors in the development and the progression of stenosis in a artery. Kumar et al. [7] have designed a model to investigate flow of non-Newtonian blood through an artery with multiple stenosis. Nadeem and Ijaz [13] presented a theoretical analysis of metallic nanoparticles on flow of blood through a curved channel with permeable walls in the presence of stenosis. The unsteady slip flow of blood through constricted artery has been investigated by Gaur and Gupta [5] and obtained results have been plotted, showing the variation in flow characteristics along the axial distance passage of time. Malek and Horque [11] have recently presented a theoretical study for flow of blood through a stenosed artery with permeable wall taking consideration of hematocrit level. Tripathi and Sharma [21] have focused on study of effects of heat and mass transfer on an arterial blood (with variable viscosity)flow in the presence of applied magnetic field with chemical reaction.

Motivated from above investigations, a mathematical model is developed to study the effect of inclination angle on constricted unsteady axial flow of blood and one more step close to realism of phenomenon, we have made an attempt to present theoretical analysis to explore the significant influence of slip velocity on flow of non-Newtonian blood through an inclined stenosed artery with permeable wall in the presence of externally applied magnetic field. In order to simulate the constricted artery flow problem, non-Newtonian rheology of streaming blood is characterized by its elastico-viscosity. Since selective permeability of the arterial wall permits for applying slip condition in place of no-slip condition, it is one

more close step to the real situation of flow of blood through wall of narrow artery as the arterial permeable walls allow the fluid particles to slip at boundary. Extensive quantitative analysis is set up by applying Laplace and finite Hankel transformations to estimate quantitative effects of slip velocity and applied magnetic field with appropriate scientific analysis so as to substantiate the utility of the present study.

2. FORMULATION OF PROBLEM

In this paper, we consider one dimensional motion of blood in a straight and rigid cylindrical inclined stenosed artery through a porous medium by considering blood as non-Newtonian elastico-viscous, incompressible and electrically conducting fluid under the influence of transversely applied magnetic field. It is well known that when magnetic field is applied on an electrically conducting fluid like blood, an electromagnetic force will produce due to the interaction of current with magnetic field. The electromotive force is given by the proportionality relation to the speed of motion and the magnetic field intensity by Ohm's law, which is given by

$$\vec{J} = \sigma(\vec{E} + \vec{u} \times \vec{B}) \quad (1)$$

where $\vec{E}$ is the electric field intensity vector, σ is the electrical conductivity, $\vec{u}$ is the velocity vector, $\vec{B} = \vec{B}_0 + \vec{B}_1$ is the total magnetic flux intensity vector in which $\vec{B}_1$ is the induced magnetic field vector of very small amount (negligibly small) in comparison with external applied magnetic field vector $\vec{B}_0$ for magneto-hydrodynamic (MHD) flow. We also consider that the electric field intensity vector $\vec{E}$, due to the polarization of charge is also negligible. Now electromagnetic force is defined and included in the momentum equation as

$$\vec{F} = \vec{J} \times \vec{B} = -\sigma B_0^2 \vec{u}, \quad \text{where } |\vec{B}_0| = B_0 \quad (2)$$

Further we consider that the blood flow is laminar, unsteady, axially-symmetric and fully developed. We restrict our study to one-dimensional blood flow taking place along axial direction of cylindrical small inclined artery. Now the geometry of inclined arterial segment with symmetric shape of stenosis is

$$\frac{R(z)}{R_0} = 1 - A[l_0^{s-1}(z-d) - (z-d)^s], \quad d \leq z \leq d + l_0 \quad (3)$$

Figure 1: Geometrical representation of inclined arterial segment with stenosis.

where

$$A = \frac{\epsilon}{R_0 l_0^s} \frac{s}{(s-1)} \quad (4)$$

$s \geq 2$ is a stenosis shape parameter, d is position of stenosis, l_0 is the length of stenosis, $R(z)$ is the radius of stenosed vessel, R_0 is the radius of normal artery.

3. GOVERNING EQUATION

The Navier-Stokes equation for blood flow including Lorenz's force with above assumptions, in cylindrical polar co-ordinates is

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial z} + (\mu + \mu_1 \frac{\partial}{\partial t}) \nabla^2 u - \frac{\mu}{k} u - \sigma B_0^2 u + \rho g \sin \phi \quad (5)$$

where

$$\nabla^2 \cong \left(\frac{1}{r} \frac{\partial}{\partial r} \right) \left(r \frac{\partial}{\partial r} \right) \quad (6)$$

$$-\frac{\partial p}{\partial z} = p_0 + p_1 \cos(w_p t); t \geq 0 \quad (7)$$

where p_0 is amplitude of steady part of pressure gradient, p_1 is the amplitude of the oscillatory of pressure gradient, $w_p = 2\pi f_p$ with f_p is the heart pulse frequency, z is axial

direction. Then equation (5) is written as

$$\rho \frac{\partial u}{\partial t} = p_0 + p_1 \cos(w_p t) + (\mu + \mu_1 \frac{\partial}{\partial t}) (\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r}) - \frac{\mu}{K} u - \sigma B_0^2 u + \rho g \sin \varphi \quad (8)$$

where

$u(r, t)$ is the component of velocity in axial direction

ρ is the density of blood

μ is the viscosity coefficient of blood

μ_1 is elastic-viscosity coefficient of blood

σ is the electrical conductivity of blood

K is porosity coefficient

r is radial co-ordinate

t is time variable

g is gravitational acceleration

φ is inclination angle of arterial segment with horizontal

Let us introduce following non-dimensional quantities:

$$\begin{aligned} u' &= \frac{u}{w_p R_0}, \quad r' = \frac{r}{R_0}, \quad p'_0 = \frac{R_0}{w_p \mu} p_0, \\ p'_1 &= \frac{R_0}{w_p \mu} p_1, \quad z' = \frac{z}{R_0}, \quad t' = t w_p, \quad g' = \frac{g}{w_p^2 R_0}, \\ K' &= \frac{K}{R_0^2}. \end{aligned}$$

After applying respectively non-dimensional terms and on dropping primes, we have a simplified form of equation (9) as

$$\alpha^2 \frac{\partial u}{\partial t} = p_0 + p_1 \cos t + (1 + \beta \frac{\partial}{\partial t}) (\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r}) - (H^2 + M^2) u + g_0 \quad (9)$$

where $\alpha = R_0 \sqrt{\frac{\rho w}{\mu}}$ is the Womersley parameter, $H = B_0 R_0 \sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number, $M = \sqrt{\frac{1}{K}}$ is the parameter for permeability and $\beta = \frac{\mu_1}{\mu}, g_0 = \frac{R_0^2}{\mu} g \sin \varphi$

Further we assume that at $t \leq 0$, only heart's pumping action is present and at $t = 0$, blood flow in the constricted artery corresponds to the instantaneous pressure gradient. i.e.

$$-\frac{\partial p}{\partial z} = p_0 + p_1 \quad (10)$$

4. THE INITIAL AND BOUNDARY CONDITIONS

When intensity of permeability is small, the boundary condition proposed by Beaver and Joseph [1] is applicable which was further simplified by Saffman [4] (also known as the Saffman's Slip condition) as $\frac{du}{dr} = \frac{\eta}{\sqrt{K}} u$ where η is constant depending on the porous material properties and on its structure, K is the permeability parameter (or Darcy number) of

porous material of the wall. Also this condition is applicable in case of unsteady flows as well as MHD fluids flow. Now the initial and boundary conditions are prescribed as follows:

$$\frac{\partial u}{\partial r} = -hu \quad (\text{slip condition})$$

or

$$u' = -hu$$

or

$$(i) \quad u' + hu = 0 \quad \text{when } r = a \quad \text{and } t \geq 0 \quad (11)$$

where

$$h = -\frac{\eta}{R_0 \sqrt{K}} \quad \text{and} \quad a = \frac{R(z)}{R_0}$$

$$(ii) \quad u(r, 0) = \sum_{n=1}^{\infty} \frac{2h}{a} \left(\frac{p_0 + g_0 + p_1}{(H^2 + M^2 + \lambda_n^2)(\lambda_n^2 + h^2)} \right) \frac{J_0(r\lambda_n)}{J_1(a\lambda_n)} \quad (12)$$

$$\text{when when } 0 \leq r \leq a$$

$$(iii) \quad u(0, t) \text{ is finite}$$

5. REQUIRED INTEGRAL TRANSFORM

If $g(t)$ is a continuous function of t and is of exponential order for $t \geq 0$, then its Laplace transformation is defined as

$$g(s) = \int_0^{\infty} e^{-st} g(t) dt, \quad s > 0 \quad (13)$$

If $f(t)$ satisfies the Dirichlet condition in finite interval $[0, a]$ then its finite Hankel transformation defined as

$$f(\lambda_n) = \int_0^a r f(r) J_0(r\lambda_n) dr \quad (14)$$

where λ_n are the roots of the equation $\lambda J'_0(a\lambda) + h J_0(a\lambda) = 0$ and $J_0(r), J_1(r)$ are the Bessel's functions of first kind, then $f(r)$ is given by

$$f(r) = \frac{2}{a^2} \sum_{n=1}^{\infty} \frac{\lambda_n^2}{(h^2 + \lambda_n^2)} \frac{J_0(r\lambda_n)}{J_0^2(a\lambda_n)} f(\lambda_n) \quad (15)$$

6. ANALYSIS

Applying Laplace and Hankel transformation on eq.(11) in light of eq.(15) and eq.(16), we obtain;

$$\begin{aligned} & (\alpha^2 s + H^2 + M^2 + \lambda_n^2 + \beta s \lambda_n^2) \overline{u(\lambda_n, s)} \\ & - (\beta \lambda_n^2 + \alpha^2) u(\lambda_n, 0) \\ & = \left(\frac{1}{s} (p_0 + g_0) + \frac{s}{1+s^2} p_1 \right) \frac{a}{\lambda_n} J_1(a \lambda_n) \end{aligned} \quad (16)$$

or

$$\begin{aligned} \overline{u(\lambda_n, s)} &= \left[\frac{1}{s(s+l)} (p_0 + g_0) + \frac{s}{(s+l)(1+s^2)} p_1 \right. \\ &+ \left. \frac{1}{l(s+l)} (p_0 + p_1) \right] \frac{a}{\lambda_n (\alpha^2 + \beta \lambda_n^2)} J_1(a \lambda_n) \end{aligned} \quad (17)$$

where

$$l = \frac{H^2 + M^2 + \lambda_n^2}{\alpha^2 + \beta \lambda_n^2} \quad (18)$$

To obtain the expression for fluid velocity, we apply inverse Laplace transformation and inverse Hankel transformation on eq.(19) and we get

$$\begin{aligned} u(r, t) &= \frac{2h}{a} \sum_{n=1}^{\infty} \frac{J_0(r \lambda_n)}{(\lambda_n^2 + h^2) J_0(a \lambda_n)} [(p_0 + g_0) \\ &+ e^{-lt} \left(\frac{1}{l} (p_1 - g_0) \right. \\ &- \left. \frac{l}{1+l^2} p_1 \right) + (l \cos t + \sin t) \frac{p_1}{1+l^2}] \end{aligned} \quad (19)$$

The expression for fluid acceleration is obtained by

$$F(r, t) = \frac{\partial u}{\partial t} \quad (20)$$

or

$$\begin{aligned} F(r, t) &= \frac{2h}{a} \sum_{n=1}^{\infty} \frac{J_0(r \lambda_n)}{(\lambda_n^2 + h^2) J_0(a \lambda_n)} [(-l) e^{-lt} \left(\frac{1}{l} (p_1 - g_0) \right. \\ &- \left. \frac{l}{1+l^2} p_1 \right) + (l \cos t - \sin t) \frac{p_1}{1+l^2}] \end{aligned} \quad (21)$$

Similarly we can obtain the expression for volumetric flow rate, which is given by

$$Q(z, t) = 2\pi \int_0^a r u(r, t) dr \quad (22)$$

or

$$\begin{aligned} Q(z, t) &= \frac{4\pi h}{a} \sum_{n=1}^{\infty} \frac{a J_1(a \lambda_n)}{(\lambda_n^2 + h^2) \lambda_n J_0(a \lambda_n)} [(p_0 + g_0) \\ &+ e^{-lt} \left(\frac{1}{l} (p_1 - g_0) \right. \\ &- \left. \frac{l}{1+l^2} p_1 \right) + (l \cos t + \sin t) \frac{p_1}{1+l^2}] \end{aligned} \quad (23)$$

Also we can obtain the expression for shear stress, which is given by

$$\tau(r, t) = \mu \frac{\partial u}{\partial r} \quad (24)$$

or

$$\begin{aligned} \tau(r, t) &= \frac{-2\mu h}{a} \sum_{n=1}^{\infty} \frac{\lambda_n J_1(r \lambda_n)}{(\lambda_n^2 + h^2) J_0(a \lambda_n)} [(p_0 + g_0) \\ &+ e^{-lt} \left(\frac{1}{l} (p_1 - g_0) \right. \\ &- \left. \frac{l}{1+l^2} p_1 \right) + (l \cos t + \sin t) \frac{p_1}{1+l^2}] \end{aligned} \quad (25)$$

7. GRAPHICAL RESULTS AND DISCUSSION

The axial velocity profile is plotted by using the velocity expression in eq.(21) for different values of Hartmann number H , slip parameter h , the permeability parameter of porous medium M and have been revealed through figures 2 to 5. It is observed that from Figure 2 that as the Hartmann number increases, the axial velocity decreases with small amount. Figures 3 to 5 depict that by increasing slowly the values of slip parameter and inclination angle φ , the axial velocity increases maximum at throat of stenosis. The volumetric flow rate exhibits the same profile in figures 10 to 13. Flow rate slightly improves when artery is inclined perpendicularly to the horizontal axis in comparison to the non-inclined artery. The variation of wall shear stress reduces with increasing slip velocity in an inclined artery. Also from the plots in figures 6 to 9, we observe that the shear stress decreases maximum at stenosis's throat.

In this paper, the effects of wall slip velocity on unsteady pulsatile blood flow through a constricted, porous and inclined artery in the presence of magnetic field are analysed. It is noticed that the axial blood velocity increases with increasing slip velocity at stenotic region of the inclined artery. Also fluid velocity increases with increasing of Womersley parameter and the permeability parameter of porous medium whereas it decreases with increasing the Hartmann number and inclination angle. The shear stress decreases with

increasing the permeability parameter of porous medium, Womersley parameter and slip parameter whereas increases with increasing Hartmann number and inclination angle. Finally, the fluid acceleration increases with an increase in the slip parameter whereas decreases the magnitude of the wall shear stress. In opposition, the flow acceleration caused by slippage at the stenotic throat reduces the wall shears stress but increases flow rate with increasing the permeability parameter

of porous medium, Womersley parameter and slip parameter whereas decreases with increasing Hartmann number and the frequency of body acceleration. It is also noted that effective viscosity and wall shear stress decreases as our objective but velocity and flow rate increase due to the wall slip velocity in an inclined artery. The current study concludes that slip velocity plays a significant role in modelling of blood flow in a stenosed artery.

Figure 2: Variation in Axial Velocity with Variation in Hartmann number.
 $\alpha = 1, b = 2, p_0 = 2, p_1 = 4, t = 1, \varphi = 30^\circ, M = 2, h = 0.25$

Figure 3: Variation in Axial Velocity with Variation in slip parameter.
 $\alpha = 1, b = 2, p_0 = 2, p_1 = 4, t = 1, \varphi = 30^\circ, M = 2, H = 4$

Figure 4: Variation in Axial Velocity with Variation in inclination angle.
 $\alpha = 1, b = 2, p_0 = 2, p_1 = 4, t = 1, H = 4, M = 2, h = 0.25$

Figure 5: Three Dimensional Variation in Axial Velocity.
 $\alpha = 1, b = 2, p_0 = 2, p_1 = 4, t = 4, \varphi = 60^\circ, M = 2, h = 0.25, H = 4$

Figure 6: Variation in Shear Stress with Variation in Hartmann number
 $\alpha = 1, b = 2, p_0 = 2, p_1 = 4, t = 1, \varphi = 60^\circ, M = 2, h = 0.25$

Figure 7: Variation in Shear Stress with Variation in Slip parameter.
 $\alpha = 1, b = 2, p_0 = 2, p_1 = 4, t = 1, \varphi = 60^\circ, M = 2, H = 4$

Figure 8: Variation in Shear Stress with Variation in inclination angle.
 $\alpha = 1, b = 2, p_0 = 2, p_1 = 4, t = 1, H = 4, M = 2, h = 0.25$

Figure 9: Three Dimensional Variation in Shear Stress.
 $\alpha = 1, b = 2, p_0 = 2, p_1 = 4, t = 3, \varphi = 45^\circ, M = 2, H = 4, h = 0.50$

Figure 10: Variation in Volumetric Flow Rate with Variation in Hartmann number.
 $\alpha = 1, b = 2, p_0 = 2, p_1 = 4, t = 2, \varphi = 45^\circ, M = 2, h = 0.25$

Figure 11: Variation in Volumetric Flow Rate with Variation in Slip parameter.
 $\alpha = 1, b = 2, p_0 = 2, p_1 = 4, t = 3, \varphi = 60^\circ, M = 2, H = 4$

Figure 12: Variation in Volumetric Flow Rate with Variation in inclination angle.
 $\alpha = 1, b = 2, p_0 = 2, p_1 = 4, t = 1, h = 0.25, M = 2, H = 4$

Figure 13: Three Dimensional Variation in Volumetric Flow Rate.
 $\alpha = 1, b = 2, p_0 = 2, p_1 = 4, t = 1, \varphi = 60^\circ, M = 2, H = 4, h = 0.50$

8. CONCLUSION

In the present mathematical model, it is concluded that

- (i) The estimation of axial velocity and volumetric flow rate increases considerably with the increase of slip velocity parameters in the presence of magnetic field.
- (ii) There is a considerable decline in wall shear stress at the throat of stenosis with the increase of slip velocity in the presence of magnetic field.
- (iii) The application of the magnetic field is helpful in controlling the axial blood flow velocity since it causes a decrease in axial speed of blood.
- (iv) A significant growth in velocity slip leads to a considerable growth in the streaming blood velocity and volumetric flow rate with the applied magnetic field and porosity. This suggests that the blood flow through stenosed inclined arteries can be enhanced by increasing application of slip inducing drugs.
- (v) When the blood flow rises i.e. the heart has to pump blood at a faster rate. Hence cardio-vascular diseased patients should be advised to avoid strenuous activities in order to avoid unjust pressure on the heart.
- (vi) Evidently the application of a suitable slip inducing drugs and magnetic field can control the wall shear stress within the stenotic section of the inclined porous artery. Further the mean wall shear stress decreases as the relative effectiveness of velocity slip parameter in the presence of magnetic field. A proper understanding of interactions of inclination angle and slip velocity with rheological properties of streaming blood may lead to a remedial use in cardiovascular diseases. Such a combined investigation of inclination angle and slip velocity could also be useful in the diagnosis and curative treatment of some health problems like vision loss, joint's pain and vascular problems. By applying proper magnetic field, it is practicable to control blood pressure and also it is desirable for conditions such as poor circulation (necrosis), headaches, joint's pain, travel sickness, muscle pain. The slip condition plays a significant role in spurt, hysteresis effects and shear skin. The fluids with boundary slip have useful technological and clinical applications such as in polishing internal cavities and artificial heart's valves. Hoping that the present investigation may have an improvement for further studies in the field of clinical research and also the obtained results can be applied to the clinical situations of streaming blood in coronary stenosed arteries.

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