Skew Chromatic Index of Cycle Related Graphs

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Abstract

A skew edge coloring of a graph G is defined as a set of two edge colorings such that no two edges are assigned the same unordered pair of colors. The skew chromatic index $s(G)$ is the minimum number of colors required for a skew edge coloring of G . In this paper, we develop algorithms for skew edge coloring of certain cycle related graphs. Also the skew chromatic index of those cycle related graphs is solved in polynomial time.

Keywords: skew edge coloring; skew chromatic index; cycle related graphs; NP - complete.

1. INTRODUCTION

Interconnection networks are becoming increasingly pervasive in many different applications. The advent of very large scale integrated circuits has enabled the construction of complex interconnection networks. Graph theory is a fundamental and powerful mathematical tool for designing and analyzing topological structure of interconnection networks. An interconnection network can be modeled as a simple graph whose vertices represent components of a network and whose edges represent physical communication links. Topologically, graphs and interconnection networks are the same [12].

Graph coloring problems are fundamental and important problems of graph theory. It has a wide range of applications in scheduling, register allocation in compilers,

bandwidth allocation, timetable designing and pattern matching, and so on. Edge coloring problems have a variety of applications and are well studied in both computer science and mathematics [5]. Edge coloring problems are used to model various scheduling problems. In cellular communication, frequency reusing is done by modeling it as an edge coloring problem to avoid co-channel interference [9].

Let $G = (V, E)$ be a finite, simple, connected graph with vertex set V and edge set E . A proper edge coloring of a graph G is an assignment of colors to the edges of G so that no two adjacent edges are assigned the same color [1]. The minimum number of colors required for such an edge coloring of G is the edge chromatic number or the chromatic index and is denoted by $\chi'(G)$. The degree of a vertex of a graph G is the number of edges incident to the vertex [2]. The maximum degree of a graph G is denoted by $\Delta(G)$. Vizing [11] has shown that for any graph G , $\chi'(G)$ is either $\Delta(G)$ or $\Delta(G) + 1$. The problem of determining the chromatic index of an arbitrary graph is a difficult task. Holyer [7] has proved that edge coloring problem is *NP* - complete.

2. AN OVERVIEW OF THE PAPER

In this paper, we consider skew edge coloring problems that are inspired from the study of skew Room squares by R. A. Brualdi [6]. The concept of skew chromatic index was introduced by Marsha. F. Foregger and better upper bounds for $s(G)$ in terms of its order, was discussed when G is cyclic, cubic or bipartite [6].

A skew edge coloring of a graph G is defined to be a set of two edge colorings such that no two edges are assigned the same unordered pair of colors. Equivalently, a skew edge coloring of G is an assignment of an ordered pair of colors (a_i, b_i) to each edge e_i of G such that

- (i) the a_i 's form an edge coloring of G ,
- (ii) the b_i 's form an edge coloring of G , and
- (iii) the pairs $\{a_i, b_i\}$ are all distinct.

The two edge colorings are referred to as component colorings of the skew edge coloring. The skew chromatic index $s(G)$ is the minimum number of colors required for a skew edge coloring of G . We see that $s(K_3) = 3$ where the two component colorings of K_3 are 1, 2, 3 and 2, 3, 1 respectively and not less than three colors can be used for skew edge coloring of K_3 . See Figure 1.

Figure 1: (a) First component coloring of K_3 (b) Second component coloring of K_3 (c) skew edge coloring of K_3

The terminologies used in this paper are as in [3]. In this paper, algorithms are determined for skew edge coloring of a cycle C_n of length of n with parallel chords, a cycle C_n with parallel P_l chords, where P_l denotes a path of order l , $3 \leq l \leq 10$, a double headed circular fan, a circular fan with two chords, a circular fan with four chords. We have also solved the skew chromatic index of those cycle related graphs in polynomial time.

3. LOWER BOUND ON $s(G)$

Skew chromatic index, $s(G)$ is defined as the minimum number of colors used in two edge colorings of G such that no two edges are assigned the same unordered pair of colors. Since each component coloring of a skew edge coloring is itself an edge coloring, it is obvious that $s(G) \geq \chi'(G)$. By Vizing's theorem, $\Delta(G) \leq \chi'(G) \leq \Delta(G) + 1$, where $\Delta(G)$ is the maximum degree of vertices in G . Hence we have $s(G) \geq \Delta(G)$. If ' k ' colors are used for skew edge coloring, then there are $\binom{k+1}{2}$ unordered pairs of colors [4] and this number must be at least as large as the number of edges in G . Let $k(m)$ denote the smallest integer ' k ' satisfying $\binom{k+1}{2} \geq m$ where ' m ' denotes the number of edges in G . Thus we have $s(G) \geq k(|E(G)|)$. Hence the best lower bound for $s(G)$ is $s(G) \geq \max \{ \Delta(G), k(|E(G)|) \}$ as stated in [6]. In this paper, we prove that the bound on skew chromatic index given here is sharp for the certain cycle related graphs like cycle C_n of length of n with parallel chords, a cycle C_n with parallel P_l chords, where P_l denotes a path of order l , $3 \leq l \leq 10$, a double headed circular fan, a circular fan with two chords and a circular fan with four chords.

4. CYCLE RELATED GRAPHS

In this section, we determine the skew chromatic index of a cycle C_n with parallel chords and with parallel P_l chords for $3 \leq l \leq 10$, a double headed circular fan, a circular fan with two chords and a circular fan with four chords. Here C_n denote a cycle of length n and P_l denotes a path of order l .

(a) Cycle with parallel chords

In this section, we determine the skew chromatic index of cycle C_n with parallel chords for $n \geq 8$.

Definition 4.1: [10] A chord of a cycle C is an edge joining two non-adjacent vertices of a cycle.

Definition 4.2: [10] A graph G is called a cycle with parallel chords if G is obtained from the cycle $C_n : v_0 v_1 v_2 \dots v_{n-1} v_0$, ($n \geq 6$) by adding the chords between each pair of non-adjacent vertices $v_1 v_{n-1}, v_2 v_{n-2}, \dots, v_\alpha v_\beta$, $\alpha = \left\lfloor \frac{n}{2} \right\rfloor - 1$, $\beta = \left\lfloor \frac{n}{2} \right\rfloor + 1$, when n is even and $\alpha = \left\lfloor \frac{n}{2} \right\rfloor - 1$, $\beta = \left\lfloor \frac{n}{2} \right\rfloor + 2$, when n is odd. See Figure 2.

Figure 2: (a) A cycle C_n with parallel chords when n is even

(b) A cycle C_n with parallel chords when n is odd.

Remark 4.1: It is a graph with n vertices and $(3n - 2)/2$ edges if n is even and $(3n - 3)/2$ edges if n is odd. It is observed that $v_\beta = v_{n-\alpha}$, and $v_{\beta-1} = v_{\alpha+1}$ or $v_{\alpha+2}$ depending on whether n is even or odd.

We now give an algorithm for skew edge coloring of a cycle C_n with parallel chords, $n \geq 8$.

Algorithm 4.1

Input: A cycle C_n with parallel chords, $n \geq 8$.

Step 1: Find the smallest positive integer ' k ' satisfying $\binom{k+1}{2} \geq m$, where m denotes the number of edges of the graph.

Step 2: Consider the set of colors $\{1, 2, 3, \dots, k\}$, then there are $\binom{k+1}{2}$ unordered pairs of colors available for skew edge coloring of the graph.

Step 3: Consider the vertex v_0 . The edge (v_0, v_1) is colored first, followed by the coloring of the edge (v_0, v_{n-1}) . Next the chord (v_1, v_{n-1}) is colored. In this manner, the three edges viz., (v_i, v_{i+1}) , (v_j, v_{j-1}) and the chord (v_{i+1}, v_{j-1}) , $i = 1, 2, 3, \dots, \alpha-1$, $j = n-i$ are colored. If n is even, the edges $(v_\alpha, v_{\alpha+1})$ and $(v_\beta, v_{\beta-1})$ are colored taken in order. If n is odd, the edges $(v_\alpha, v_{\alpha+1})$, $(v_\beta, v_{\beta-1})$ and $(v_{\alpha+1}, v_{\beta-1})$ are colored taken in order. Assignment of colors in this step is done using the following **Algorithms A or B** depending upon the nature of ' k ' which may be odd or even.

Algorithm A: k is odd.

Case (i): $m \equiv 0 \pmod{k}$

In the aforementioned method, the first ' k ' edges are assigned colors from the set $\{(c, c), c = 1, 2, \dots, k\}$ taken in order. The remaining edges are assigned colors from the set $\{(c, c + d), c = 1, 2, \dots, (k-d)\} \cup \{((k-d) + i, i), i = 1, 2, \dots, d\}$ for each $d = 1, 2, 3, \dots, \left\lfloor \frac{k}{2} \right\rfloor$

taken in order. Here all the $\binom{k+1}{2}$ pairs of colors are used.

Case (ii): $m \equiv r \pmod{k}$, $r = 1, 2, 3, \dots, k-1$.

In the aforementioned method, the first ' k ' edges are assigned colors from the set $\{(c, c), c = 1, 2, \dots, k\}$ taken in order. The remaining edges are assigned colors from the set

$\{(c, c + d), c = 1, 2, \dots, (k-d)\} \cup \{((k-d) + i, i), i = 1, 2, \dots, d\}$ for each $d = 1, 2, 3, \dots, \left\lfloor \frac{k}{2} \right\rfloor$

taken in order. Here the last $(k-r)$ pairs of colors are left unused.

Algorithm B: k is even.

Case (i): $m \equiv r \pmod{(k+1)}$, $r = 0, 1$.

In the aforementioned method, the first ' k ' edges are assigned colors from the set $\{(c, c), c = 1, 2, \dots, k\}$ taken in order. The remaining edges are assigned colors from the set

$\{(c, c + d), c = 1, 2, \dots, (k-d)\} \cup \{((k-d) + i, i), i = 1, 2, \dots, d\}$ for each $d = 1, 2, 3, \dots, \frac{k}{2}$ taken in order. Here all the $\binom{k+1}{2}$ pairs of colors are used.

Case (ii): $m \equiv r \pmod{(k+1)}$, $r = 2, 3, \dots, k$.

In the aforementioned method, the first ' k ' edges are assigned colors from the set $\{(c, c), c = 1, 2, \dots, k\}$ taken in order. The remaining edges are assigned colors from the set

$\{(c, c + d), c = 1, 2, \dots, (k-d)\} \cup \{((k-d) + i, i), i = 1, 2, \dots, d\}$ for each $d = 1, 2, 3, \dots, \frac{k}{2}$ taken in order. Here the last $((k+1)-r)$ pairs of colors are left unused.

Output: Skew edge coloring of cycle C_n with parallel chords.

Proof of correctness: As there are m edges, fix ' k ' in such a way that $\binom{k+1}{2} \geq m$, so

that $\binom{k+1}{2}$ pairs are available for skew edge coloring of the graph. First ' k ' edges are assigned the colors in such a way that all ordered pairs of the form (c, c) , $c = 1, 2, 3, \dots, k$ are used. In the second set of ' k ' edges, the first $k-1$ edges are assigned the colors of the form $(c, c+1)$, $c = 1, 2, 3, \dots, k-1$. The k^{th} edge is assigned the color $(k, 1)$ as the ordered pair $(k, k+1)$ is not permissible. In the third set of ' k ' edges, the first $k-2$ edges are assigned the colors of the form $(c, c+2)$, $c = 1, 2, 3, \dots, k-2$. The $(k-1)^{\text{th}}$ and the k^{th} edge are assigned the colors $((k-2) + i, i)$, $i = 1, 2$ respectively as the ordered pairs $(k-1, k+1)$ and $(k, k+2)$ are not permissible. This process is continued till all the edges are colored and the above method of edge coloring results in skew edge coloring of the given graph. See Figure 3.

Figure 3: (a) Skew edge coloring of C_{10} with parallel chords and $s(G) = 5$.

(b) Skew edge coloring of C_{13} with parallel chords and $s(G) = 6$.

Theorem 4.1: Let G be a cycle C_n , $n \geq 8$ with parallel chords and $m = \frac{3n-2}{2}$ edges, n is even. Then $s(G) = k = \left\lceil \frac{-1 + \sqrt{12n-7}}{2} \right\rceil$.

Proof: As there are m edges, at least m pairs of colors are required for skew edge coloring of G . If ' k ' colors are used for coloring, then there are $\binom{k+1}{2}$ unordered pairs. This $\binom{k+1}{2}$ must be at least as large as the number of edges in G . Therefore fix ' k ' in such a way that $\binom{k+1}{2} \geq m$. i.e. $\frac{(k+1)k}{2} \geq \frac{3n-2}{2}$. It follows that $k^2 + k \geq 3n-2$. i.e. $k^2 + k - (3n-2) \geq 0$. Thus solving for ' k ' and taking the positive root, we obtain $k = \frac{-1 + \sqrt{12n-7}}{2} \geq 0$, and its greatest integer $k = \left\lceil \frac{-1 + \sqrt{12n-7}}{2} \right\rceil$ will be minimum number of colors used in skew edge coloring. Hence $s(G) = \left\lceil \frac{-1 + \sqrt{12n-7}}{2} \right\rceil$.

Theorem 4.2: Let G be a cycle C_n , $n \geq 8$ with parallel chords and $m = \frac{3n-3}{2}$ edges, n is odd. Then $s(G) = k = \left\lceil \frac{-1 + \sqrt{12n-11}}{2} \right\rceil$.

Proof. Proof is similar to theorem 4.1.

Thus based on the lower bound for $s(G)$, we obtain an optimal solution for skew chromatic index of cycle C_n with parallel chords for $n \geq 8$.

(b) Cycle with parallel P_l chords

In this section, we determine the skew chromatic index of cycle C_n with parallel P_l chords for $3 \leq l \leq 10$, where C_n denotes a cycle of length n and P_l denotes a path of order l .

Definition 4.3: [10] A cycle with parallel P_l chords is defined as a graph obtained from a cycle C_n , $n \geq 6$ with consecutive vertices $v_0, v_1, v_2, \dots, v_{n-1}, v_n$ by adding disjoint paths(chords) of order l namely P_l , $l \geq 3$ between each pair of non-adjacent vertices $(v_1, v_{n-1}), (v_2, v_{n-2}), \dots, (v_\alpha, v_\beta)$, $\alpha = \left\lfloor \frac{n}{2} \right\rfloor - 1$, $\beta = \left\lfloor \frac{n}{2} \right\rfloor + 1$, when n is even and $\alpha = \left\lfloor \frac{n}{2} \right\rfloor - 1$, $\beta = \left\lfloor \frac{n}{2} \right\rfloor + 2$, when n is odd. See Figure 4.

Figure 4: (a) A cycle C_n with parallel P_l chords when n is even
(b) A cycle C_n with parallel P_l chords when n is odd.

Remark 4.2: It is a graph with $\frac{(n-2)l+4}{2}$ vertices and $\frac{n+l(n-2)+2}{2}$ edges if n is even and $\frac{(n-3)l+6}{2}$ vertices and $\frac{n+l(n-3)+3}{2}$ edges if n is odd.

Parallel P_l chord coloring procedure

Procedure 4.1: The parallel P_l chords joining vertices v_i and v_{n-i} of the cycle C_n are colored in the following manner.

The chord joining vertices v_i and v_{n-i} , $i=1, 2, 3, \dots, \alpha$ is a path on l vertices. Let the internal vertices of path P_l be labeled as $v_{i,1}, v_{i,2}, v_{i,3}, \dots, v_{i,(l-2)}$. In the parallel P_l chords, the right most edge $(v_{i,1}, v_i)$ is colored first, followed by the coloring of the leftmost edge $(v_{n-i}, v_{i,(l-2)})$. Next the second rightmost edge $(v_{i,2}, v_{i,1})$ is colored, followed by the coloring of the second leftmost edge $(v_{i,(l-2)}, v_{i,(l-3)})$. This manner of coloring is continued till all the edges of the chord are colored.

We now give an algorithm for skew edge coloring of a cycle C_n with parallel P_l chords, $3 \leq l \leq 10$.

Algorithm 4.2

Input: A cycle C_n with parallel P_l chords.

Step 1: Find the smallest positive integer ' k ' satisfying $\binom{k+1}{2} \geq m$, where m denotes the number of edges of the given graph.

Step 2: Consider the set of colors $\{1, 2, 3, \dots, k\}$ so that $\binom{k+1}{2}$ unordered pairs of colors are available for skew edge coloring of the given graph.

Step 3: Consider the vertex v_0 . The edge (v_0, v_1) is colored first, followed by the coloring of the edge (v_0, v_{n-1}) . Next the P_l chord (v_1, v_{n-1}) is colored as per the procedure 4.1. In this manner, for each $i=1, 2, 3, \dots, \alpha$, $j=n-i$, the edges (v_i, v_{i+1}) , (v_j, v_{j-1}) and the P_l chord (v_{i+1}, v_{j-1}) are colored taken in order. Assignment of colors in this step is done using the **Algorithms A or B** of step 3 of **Algorithm 4.1** depending upon the nature of ' k ' which may be odd or even.

Output: Skew edge coloring of the cycle C_n with parallel P_l chords.

Remark 4.3: The above algorithm holds good for all cycles C_n with parallel P_l chords where $l = 2i + 1$ and $l = 2i + 2$, for $n \geq 2i + 8, i = 1, 2, 3, 4$. See Figure 5.

Figure 5: (a) A cycle C_{10} with parallel P_4 chords and $s(G) = 7$.

(b) A cycle C_{11} with parallel P_3 chords and $s(G) = 6$.

Theorem 4.3: Let G be a cycle C_n with parallel P_l chords, $3 \leq l \leq 10$ and n is even.

$$\text{Then } s(G) = \left\lceil \frac{-1 + \sqrt{4n - 4l(2-n) + 9}}{2} \right\rceil.$$

Theorem 4.4: Let G be a cycle C_n with parallel P_l chords, $3 \leq l \leq 10$ and n is odd. Then

$$s(G) = \left\lceil \frac{-1 + \sqrt{4n - 4l(3-n) + 13}}{2} \right\rceil.$$

(c) Double Headed Circular Fan

In this section, we obtain the skew chromatic index of double headed circular fan $DF(n)$.

Definition 4.4: [8] Let $C: x_1x_2x_3 \dots x_nx_1$ be a cycle on n vertices. For $x_i \in V(C)$, the graph obtained by adding edges $(x_i, u), i = 1, 2, \dots, n-3$ and $(x_i, v), i = n-2, n-1, n$ to C is called a double headed circular fan. It is denoted by $DF(n)$. The new edges are called spokes of $DF(n)$. See Figure 6.

Figure 6: (a) $DF(n)$, (b) skew edge colored $DF(10)$

Now, we give an algorithm for skew edge coloring of $DF(n)$ for $n \geq 9$.

Algorithm 4.3:

Input: A double headed circular fan $DF(n)$, $n \geq 9$.

Step 1: Find $\deg(u)$. Let it be ' k ' (say).

Step 2: Consider the list of colors $\{1, 2, 3, \dots, k\}$ so that $\binom{k+1}{2}$ pairs are available for skew edge coloring of $DF(n)$.

Step 3: Assign the pairs of colors of the form (i, i) to the spokes $(x_i, u), i = 1, 2, 3, \dots, n-3$.

Step 4: Color the edges of the cycle $C_n: x_1x_2x_3 \dots x_nx_1$.

The first $k-1$ edges of the cycle $C_n: x_{n-1}x_nx_1x_2x_3 \dots x_{n-3}x_{n-2}x_{n-1}$ are assigned the colors of the form $(i, i+1), i = 1, 2, 3, \dots, k-1$. The next edge is assigned the color $(k, 1)$. The

remaining $n - k$ edges of the cycle C_n are assigned the colors of the form $(i, i + 2)$, $i = 1, 2, 3$.

Step 5:

Case (i) If $k = 6$

The spokes (x_i, v) , $i = n, n - 1, n - 2$ are assigned the next consecutive colors taken in order of the form $(4, 6)$, $(5, 1)$, $(6, 2)$ respectively.

Case (ii) If $k = 7$

The spokes (x_i, v) , $i = n, n - 1, n - 2$ are assigned the next consecutive colors taken in order of the form $(4, 6)$, $(5, 7)$, $(6, 1)$ respectively.

Case (iii) If $k \geq 8$

The spokes (x_i, v) , $i = n, n - 1, n - 2$ are assigned the next consecutive colors taken in order of the form $(i, i + 2)$, $i = 4, 5, 6$ respectively.

Output: Skew edge coloring of $DF(n)$.

Proof of correctness: As ' k ' edges are incident on the vertex u , at least ' k ' distinct colors are needed for skew edge coloring of $DF(n)$. Thus colors of the form (i, i) are assigned to the spokes (x_i, u) , $i = 1, 2, 3, \dots, k$. The edges (x_i, x_{i+1}) , $i = 1, 2, 3, \dots, n - 4$ of the cycle C_n are adjacent to the spokes (x_i, u) and (x_{i+1}, u) , $i = 1, 2, 3, \dots, n - 4$. Hence (x_i, x_{i+1}) , $i = 1, 2, 3, \dots, n - 4$ are assigned the colors $(i, i + 1)$, $i = 3, 4, \dots, k - 1$. The next edge (x_{n-4}, x_{n-5}) is assigned the color $(k, 1)$ which is adjacent to the spokes (x_{n-5}, u) and (x_{n-4}, u) . The remaining $n - k$ edges of the cycle C_n are assigned the colors $(i, i + 2)$, $i = 1, 2, 3$ respectively. As ' k ' colors are used, depending on the value of ' k ', the spokes (x_i, v) , $i = n, n - 1, n - 2$ are assigned the colors of the form $(i, i + 2)$, $i = 4, 5, 6$ taken in order for $k \geq 8$ thus resulting in a skew edge coloring of $DF(n)$.

Theorem 4.3: Let G be a $DF(n)$, $n \geq 9$. Then $s(DF(n)) = n - 3$.

Proof: Proof follows directly from the algorithm.

(d) Circular Fan with Two Chords

In this section, we obtain the skew chromatic index of circular fan with two chords $F(n, 2)$.

Definition 4.5: [8] Let $C: x_1x_2x_3 \dots x_nx_1$ be a cycle on n vertices. Let u be a new vertex. The graph obtained by adding edges $(x_i, u), i = 1, 2, \dots, n-4$ to C along with two chords (x_{n-1}, x_{n-3}) and (x_n, x_{n-2}) is called a circular fan with two chords and is denoted by $F(n, 2)$. The new edges are called spokes of $F(n, 2)$. See Figure 7.

Figure 7: (a) $F(n, 2)$ (b) skew edge colored $F(11, 2)$

Now, we give an algorithm to skew edge color $F(n, 2)$ for $n \geq 12$.

Algorithm 4.4:

Input: A circular fan with two chords $F(n, 2)$, $n \geq 12$.

Step 1: Find $\deg(u)$. Let it be ' k ' (say).

Step 2: Consider the list of colors $\{1, 2, 3, \dots, k\}$ so that $\binom{k+1}{2}$ pairs are available for skew edge coloring of $F(n, 2)$.

Step 3: Assign the pairs of colors of the form (i, i) to the spokes (x_i, u) , $i = 1, 2, 3, \dots, n-4$.

Step 4: Color the edges of the cycle $C_n: x_1x_2x_3 \dots x_nx_1$.

The first $k-1$ edges of the cycle $C_n: x_{n-1}x_nx_1x_2x_3 \dots x_{n-3}x_{n-2}x_{n-1}$ are assigned the colors of the form $(i, i+1)$, $i = 1, 2, 3, \dots, k-1$. The next edge is assigned the color $(k, 1)$. The remaining $n-k$ edges of the cycle C_n are assigned the colors of the form $(i, i+2)$, $i = 1, 2, 3, 4$.

Step 5: The two chords (x_{n-1}, x_{n-3}) and (x_n, x_{n-2}) are assigned the pairs of colors $(i, i + 2)$, $i = 5, 6$ respectively.

Output: Skew edge coloring of $F(n, 2)$.

Proof of correctness: Proof follows directly from the algorithm.

Theorem 4.4: Let G be a $F(n, 2)$, $n \geq 12$. Then $s(G) = n - 4$.

(e) Circular Fan with Four Chords

In this section, we obtain the skew chromatic index of circular fan with four chords $F(n, 4)$.

Definition 4.6: [8] Let $C: x_1x_2x_3 \dots x_nx_1$ be a cycle on n vertices. Let u be a new vertex. The graph obtained by adding edges (x_i, u) , $i = 1, 2, \dots, n-8$ to C along with four chords (x_n, x_{n-2}) , (x_{n-1}, x_{n-4}) , (x_{n-3}, x_{n-6}) and (x_{n-5}, x_{n-7}) is called a circular fan with four chords and is denoted by $F(n, 4)$. The new edges are called spokes of $F(n, 4)$. See Figure 8.

Figure 8: (a) $F(n, 4)$ (b) skew edge colored $F(17, 4)$

Now, we give an algorithm to skew edge color $F(n, 4)$ for $n \geq 17$.

Algorithm 4.5:

Input: A circular fan with four chords $F(n, 4)$, $n \geq 17$.

Step 1: Find $\deg(v)$. Let it be ' k ' (say).

Step 2: Consider $\{1, 2, 3, \dots, k\}$ as the list of colors so that $\binom{k+1}{2}$ pairs are available for skew edge coloring of $F(n, 4)$.

Step 3: Assign the pairs of the form (i, i) to the spokes (x_i, u) , $i = 1, 2, 3, \dots, n - 8$.

Step 4: Color the edges of the cycle $C_n : x_1 x_2 x_3 \dots x_n x_1$.

The first $k - 1$ edges of the cycle $C_n : x_{n-1} x_n x_1 x_2 x_3 \dots x_{n-3} x_{n-2} x_{n-1}$ are assigned the colors of the form $(i, i + 1)$, $i = 1, 2, 3, \dots, k - 1$. The next edge (x_{n-6}, x_{n-7}) is assigned the color $(k, 1)$. The remaining $n - k$ edges of the cycle C_n are assigned the colors of the form $(i, i + 2)$, $i = 1, 2, 3, \dots, 8$.

Step 5: The four chords (x_{n-7}, x_{n-5}) , (x_{n-6}, x_{n-3}) , (x_{n-4}, x_{n-1}) and (x_{n-2}, x_n) are assigned the pairs of colors (k, i) , $i = 2, 3, 4, 5$ respectively.

Output: Skew edge coloring of $F(n, 4)$.

Theorem 4.5: Let G be a $F(n, 4)$, $n \geq 17$. Then $s(G) = n - 4$.

Time Complexity analysis of algorithm:

The time complexity analysis of the above mentioned algorithms depends on the size of the graph. The algorithm may be implemented to run in $O(m)$, where m is the number of edges of G .

CONCLUSION

In this paper we have developed algorithms for certain cycle related graphs and obtained an optimal solution for $s(G)$. We have already obtained the same for comb, ladder, Mobius ladder and Circular ladder graphs. Finding efficient algorithms to determine skew edge coloring for other interconnection networks is quite challenging.

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