Simulated Annealing with Clone Operator

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Abstract

Simulated Annealing has been a very successful general algorithm for the solution of large, complex combinatorial optimization problems. In this paper, we propose a clone operator which imitates the biological cloning process to generate a candidate solution vector for the solution vector in simulated annealing algorithm. The proposed algorithms are tested with various case studies obtained from the literature and the results are presented. The algorithm is capable of getting optima.

AMS subject classification:

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1. Introduction

Simulated annealing (SA) is a compact and robust randomized metaheuristic technique, which provides excellent solutions to single and multiple objective optimization problems with a substantial reduction in computation time. It is a proven method to obtain an optimal solution for a single objective optimization problem and to obtain a Pareto set of solutions for a multi-objective optimization problem. It is based on an analogy of thermodynamics with the way metals cool and anneal. If a liquid metal is cooled slowly,

its atoms form a pure crystal corresponding to the state of minimum energy for the metal. The metal reaches a state with higher energy, if it is cooled quickly. SA has received a lot of attention in the recent times to solve optimization problems. Kirkpatrick et al. [1] and Cerny [2] showed a model for simulating the annealing of solids, based on the theory originally proposed by Metropolis et al. [3]. This model is used for the optimization of problems, where the objective functions to be minimized corresponds to the energy states of the metal. SA has these days become one of the many heuristic approaches designed to give a good, not necessarily optimal solution. Since its introduction, several applications in different fields of engineering, such as integrated circuit placement, VLSI design, pattern recognition, optimal encoding, resource allocation, logic synthesis, have been deploying SA solution. At the same time, theoretical studies have been focusing on the reasons for the excellent behavior of the algorithm. Theoretical developments and researches in simulated annealing are well documented ([4, 5, 6, 7, 8, 9, 10, 11]).

Though, the majority of the theoretical developments and application work with simulated annealing has been for discrete optimization problems, simulated annealing has also been used as a tool to address problems in the continuous domain. We are observing an increased interest in using simulated annealing for global optimization over regions containing several local and global minima. Fabian [12] studies the performance of simulated annealing methods for finding a global minimum of a given objective function. Bohachevsky et al. [13] proposes a generalized simulated annealing algorithm of function optimization for the use in statistical applications and Locatelli [14] presents a proof of convergence for the algorithm. Belisle [15] presents a special simulated annealing algorithm for global optimization, which uses a heuristically motivated cooling schedule. This algorithm provides a reasonable alternative to existing methods and can be implemented easily. Suman ([16, 17, 18]) proposes different simulated annealing-based approach to address constrained multi-objective optimization problems. In [17], a comparative analysis of five simulated annealing algorithms is conducted for the system reliability optimization problem. These methods were aimed to generate a set of efficient solutions that are a good approximation to the entire set of efficient solutions over a fixed period of time.

Belisle et al. [19] discusses convergence properties of simulated annealing algorithms applied to continuous functions and apply these results to hit-and-run algorithms used in global optimization. The convergence properties identified are consistent with those presented in [20] and provide a good contrast between convergence in probability and almost sure convergence. This work is further extended in [21] to an improved hit-and-run algorithm used for global optimization. Several researchers have independently proven the convergence results ([20, 22, 23, 24, 25, 26]).

In this paper, firstly, we give a description of simulated annealing algorithm. Section II gives a description of SA algorithm. In section III, a clone operator which imitates the biological cloning process to generate a candidate solution vector in simulated annealing is proposed. The computational results of test problems are shown in section IV. Finally, section V summarizes the concluding remarks.

2. Simulated Annealing

The basic approach in SA considers the neighbouring state of the current state to reach a stable state for the application. The strategy consists of exploring the solution space starting from an arbitrary selected solution or state and generating a new one produced through conservatively modifying a given state. Every time a new solution is generated, its cost is evaluated to accept or reject the solution according to an acceptance rule. In SA, as we are looking for an actual best solution which would be a global optimum, the algorithm some times even accepts the worse move to avoid being trapped in a local minimum. The acceptance and rejection of the worse move is controlled by a probability function. The probability of accepting a move, which causes an increase Δf in the objective function f , is called the acceptance function. It is normally set to $e^{-\frac{\Delta f}{T}}$, where T is a control parameter, which corresponds to the temperature in analogy with the physical annealing. This acceptance function implies that the small increase in f is more likely to be accepted than a large increase in f . When T is high most uphill moves are accepted, but as T approaches to zero, most uphill moves will be rejected. Therefore, SA algorithm starts with a high temperature to avoid being trapped in local minimum and proceeds by attempting a certain number of moves at each temperature by decreasing the temperature. Thus, the configuration decisions in SA proceed in a logical order.

The overall SA process steps are summarized as follows:

STEP 1 Select an initial temperature T .

STEP 2 Select a randomly generated initial solution vector, X , and generate the objective function.

STEP 3 Select the temperature change counter

STEP 4 Select a temperature cooling schedule

STEP 5 Select a repetition schedule that defines the number of iterations executed at each temperature

STEP 6 Give a random perturbation on X and generate a new solution vector, Y , in the neighbourhood of current solution vector, X , evaluate the objective function

STEP 7 If $f(Y) < f(X)$, replace X with Y . Update the existing optimal solution and go to Step 9.

STEP 8 Else accept Y with the probability $p = e^{-\frac{\Delta f}{T}}$ where $\Delta f = f(Y) - f(X)$. If the solution is accepted, replace X with Y . Update the existing optimal solution and go to Step 9.

STEP 9 Decrease the temperature periodically.

STEP 10 Repeat Steps 6 -9 until stopping criterion is met.

3. Simulated Annealing with Clone Operator

In a previous work [27], we introduced a new operator on a population called Clone Operator. Cloning is carried out with a clone probability. The genes which are not cloned based on probability are mutated ($0 \rightarrow 1$, $1 \rightarrow 0$). It is expected that the new member will have better qualities of the parent member. As an example, consider the string

1	0	1	1	1	0	1	0	0	1
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On applying the operator, each bit may undergo a cloning with a certain probability. If the cloning probability is 0.8, two genes of the string will be mutated randomly and we might get, after cloning, the string-

1	1	1	1	1	0	0	0	0	1
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in which two genes, viz. the second and the seventh, are changed while all the others are cloned.

We propose the clone operator to generate a candidate solution vector for the solution vector in simulated annealing algorithm. The step by step procedure of the new algorithm, which we call SAC, can be summarized as follows:

STEP 1 Select an initial temperature T .

STEP 2 Select a randomly generated binary coded initial solution vector, X , of and generate the objective function.

STEP 3 Select the temperature change counter.

STEP 4 Select a temperature cooling schedule.

STEP 5 Select a repetition schedule that defines the number of iterations executed at each temperature.

STEP 6 Generate a new solution vector, Y , by applying clone operator on X and evaluate the objective function.

STEP 7 If $f(Y) < f(X)$, replace X with Y . Update the existing optimal solution and go to Step 9.

STEP 8 Else accept Y with the probability $p = e^{-\frac{\Delta f}{T}}$ where $\Delta f = f(Y) - f(X)$. If the solution is accepted, replace X with Y . Update the existing optimal solution and go to Step 9.

STEP 9 Decrease the temperature periodically.

STEP 10 Repeat Steps 6-9 until stopping criterion is met.

Fig.1 shows the schematic of the application of SAC algorithm.

Figure 1: Flowchart for SAC algorithm

4. Case studies

We evaluate the performance of the proposed SAC algorithm on a set of 15 benchmark test functions (see table 1 and table 2) in the literature with different complexity. Experiments are conducted on the $2-D$ problems. The results of SAC on $2-D$ problems is compared with simple SA. Parameters used in the implementation of SAC and SA are : Bit length for one variable = 16, probability of clone = 0.6, the initial temperature = 1/10/100/1000 according to the problem, temperature at which to stop = e^{-8} , maximum number of consecutive rejections = 1000, maximum number of tries within one temperature = 300, maximum number of successes within one temperature = 20.

Figure 2: Convergence Graph for the function g_1 using SAC and SA algorithms. Left plot is for SAC algorithm and right plot is for SA algorithm.

We use the MATLAB to implement the algorithms. All the implementations were developed using a computer with processor Core 2 Quad processor of 64 bits that works to a frequency of 2.7 GHz, 4 GB of RAM Memory and Windows XP Professional Version.

In table 3, we can find the experimental results. The table 2 provides the results with 50 Monte-Carlo simulations with the changes in the random sequence. The results are provided in terms of statistical parameters such as average, the median, the best value, the worst value and standard deviation obtained for each test functions. The number of function evaluations for convergence for best and the average number of function evaluations taken by the two methods are also presented. The convergence maps of the algorithm on SAC and SA functions g_1 to g_{15} are plotted in figures 1-15 respectively.

Table 1: Test functions g_1 to g_{12}

Function	Definition
g_1	<p>g_1: Sphere function(De Jong's function 1)</p> $g_1(x) = \sum_{i=1}^D x_i^2$ <p>$-5.12 \leq x_i \leq 5.12$, $x^* = (0, 0, \dots, 0)$, $g_1(x^*) = 0$</p>
g_2	<p>g_2: Rosenbrock function (De Jong's function 2, Banana function)</p> $g_2(x) = \sum_{i=1}^{D-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$ <p>$-2.048 \leq x_i \leq 2.048$, $x^* = (1, 1, \dots, 1)$, $g_2(x^*) = 0$</p>
g_3	<p>g_3:Ackley's function</p> $g_3(x) = 20 + e - 20 \exp\left(-0.2 \sqrt{\frac{1}{D} \sum_{i=1}^D x_i^2}\right) - \exp\left(\frac{1}{D} \sum_{i=1}^D \cos(2\pi x_i)\right)$ <p>$-32.768 \leq x_i \leq 32.768$, $x^* = (0, 0, \dots, 0)$, $g_3(x^*) = 0$</p>
g_4	<p>g_4: Rastrigin function</p> $g_4(x) = 10D + \sum_{i=1}^D x_i^2 - 10 \cos(2\pi x_i)$ <p>$-5.12 \leq x_i \leq 5.12$, $x^* = (0, 0, \dots, 0)$, $g_4(x^*) = 0$</p>
g_5	<p>g_5: Griewank function</p> $g_5(x) = 1 + \sum_{i=1}^D \frac{x_i^2}{4000} - \prod_{i=1}^D \cos\left(\frac{x_i}{\sqrt{i}}\right)$ <p>$-600 \leq x_i \leq 600$, $x^* = (0, 0, \dots, 0)$, $g_5(x^*) = 0$</p>
g_6	<p>g_6: Schwefel's Double Sum function</p> $g_6(x) = \sum_{i=1}^D \left(\sum_{j=1}^i x_j\right)^2$ <p>$-65.536 \leq x_i \leq 65.536$, $x^* = (0, 0, \dots, 0)$, $g_6(x^*) = 0$</p>

Table 1: Continued.

Function	Definition
g_7	<p>g_7: : Schwefel's Max function</p> $g_7(x) = \max_i (x_i : i \in \{1, 2, \dots, n\})$ <p>$-100 \leq x_i \leq 100$, $x^* = (0, 0, \dots, 0)$, $g_7(x^*) = 0$</p>
g_8	<p>g_8: Schwefel's absolute function</p> $g_8(x) = \sum_{i=1}^D x_i + \prod_{i=1}^D x_i $ <p>$-10 \leq x_i \leq 10$, $x^* = (0, 0, \dots, 0)$, $g_8(x^*) = 0$</p>
g_9	<p>g_9: The Step function</p> $g_9(x) = \sum_{i=1}^D (\text{floor}(x_i + 0.5))^2$ <p>$-100 \leq x_i \leq 100$, $x^* = (0, 0, \dots, 0)$, $g_9(x^*) = 0$</p>
g_{10}	<p>g_{10}: The Tenth Power function</p> $g_{10}(x) = \sum_{i=1}^D x_i^{10}$ <p>$-5.12 \leq x_i \leq 5.12$, $x^* = (0, 0, \dots, 0)$, $g_{10}(x^*) = 0$</p>
g_{11}	<p>g_{11}: Easom function</p> $g_{11}(x) = -\cos(x_1) \cos(x_2) \exp(-(x_1 - \pi)^2 - (x_2 - \pi)^2)$ <p>$-100 \leq x_i \leq 100$, $x^* = (\pi, \pi)$, $g_{11}(x^*) = -1$</p>
g_{12}	<p>g_{12}: Shenkel's foxholes function</p> $g_{12}(x) = \left[\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^2 (x_i - a_{ij})^6} \right]^{-1}$ <p>where a_{ij} is the element in the ith row and jth column of $a = \begin{bmatrix} b_0 & \cdots & b_0 \\ b_1 & \cdots & b_1 \end{bmatrix}$</p> $b_0 = [-32 \quad -16 \quad 0 \quad 16 \quad 32]$ $b_i = (16(i - 1) - 32) [1 \quad 1 \quad 1 \quad 1 \quad 1]$ <p>$-65.536 \leq x_i \leq 65.536$, $x^* = (-32, -32)$, $g_{12}(x^*) = 1$</p>

Table 2: Test functions g_{13} to g_{15}

Function	Definition
g_{13}	<p>g_{13}: Goldstein-Price function</p> $g_{13}(x) = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)]$ $\times [30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$ $-2 \leq x_i \leq 2, \quad x^* = (0, -1), \quad g_{13}(x^*) = 3$
g_{14}	<p>g_{14}: Six hump camel back function</p> $g_{14}(x) = [(4 - 2.1x_1^2 + x_1^{4/3})x_1^2 + x_1x_2 + (-4 + 4x_2^2)x_2^2]$ $-3 \leq x_1 \leq 3, -2 \leq x_2 \leq 2, \quad x^* = (-0.0898, 0.7126), (0.0898, -0.7126), \quad g_{14}(x^*) = -1.0316$
g_{15}	<p>g_{15}: Branin's rcos function</p> $g_{15}(x) = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + \left(10 - \frac{5}{4\pi}\right)\cos x_1 + 10$ $-5 \leq x_1 \leq 10, 0 \leq x_2 \leq 15, \quad x^* = (-\pi, 12.275), (\pi, 2.275), (9.42478, 2.475) \quad g_{15}(x^*) = 0.397887$

Table 3: Comparison results of the best value, the median, the worst value, average, and standard deviation of the functions $g_1 - g_{15}$ for 50 independent runs using CGA,CEA and GA methods

F	Method	Best	Median	Worse	Average	S.D	NFCB	ANFC
g_1	SAC	1.1642E-08	2.2935E-06	3.9233E-06	2.2972E-06	1.3876E-06	12063	11492
	SA	1.4136E-10	1.1834E-08	4.9742E-08	1.2281E-09	7.1842E-06	4372	5132
g_2	SAC	2.3516E-08	7.7233E-06	1.3587E-05	7.4902E-06	4.3857E-06	13209	12690
	SA	1.8498E-06	8.1437E-06	5.4235E-05	9.8367E-06	4.0757E-05	10651	11770
g_3	SAC	0.0020	0.0083	0.0130	0.0082	0.0032	3300	4092
	SA	9.7133E-07	4.4241E-05	7.4176E-04	1.7511E-05	3.2534E-05	11959	11554
g_4	SAC	2.3097E-06	1.5013E-04	3.3490E-04	1.6454E-04	1.1561E-04	8386	6749
	SA	2.6316E-08	1.6317E-07	1.6523E-06	4.6412E-07	1.8557E-05	10216	11486
g_5	SAC	5.6607E-04	0.0071	0.0082	0.0057	0.0025	6805	4945
	SA	0.2929	0.2929	0.2929	0.2929	1.0121E-17	4036	4632
g_6	SAC	9.8374E-07	1.3477E-04	2.7643E-04	1.4201E-04	7.9812E-05	12628	12182
	SA	3.5179E-09	6.1342E-09	2.9201E-07	9.3266E-08	1.1271E-05	5826	6164
g_7	SAC	0.0015	0.0107	0.0137	0.0091	0.0034	7980	6496
	SA	3.2158E-06	5.1157E-05	4.9007E-05	1.0211E-05	1.0193E-05	9288	9476
g_8	SAC	3.0520E-04	0.0012	0.0021	0.0014	5.5128E-04	7462	8519
	SA	1.9585E-07	2.6243E-06	6.1208E-06	3.1147E-06	3.1206E-05	7384	8134
g_9	SAC	0	0	0	0	0	3000	2448
	SA	0	0	0	0	0	1710	2236

Table 3: Continued.

F	Method	Best	Median	Worse	Average	S.D	NFCB	ANFC
g_{10}	SAC	3.7755E-33	1.0886E-21	6.9026E-20	8.7412E-21	1.8272E-20	17212	17922
	SA	3.6763E-14	1.3712E-14	7.6102E-13	4.7126E-14	3.1773E-05	2334	1376
g_{11}	SAC	-1.0000	-0.9997	-0.9992	-0.9996	2.5179E-04	4341	4234
	SA	-1	-1	-0.9997	-1	7.1214E-05	3887	4237
g_{12}	SAC	0.9980	0.9980	0.9980	0.9980	3.9318E-10	10867	16128
	SA	0.9980	0.9980	0.9980	0.9980	1.2305E-16	6215	6186
g_{13}	SAC	3.0000	3.0000	3.0001	3.0000	1.9365E-05	3000	6516
	SA	3.0000	3.0000	3.0000	3.0000	1.2614E-05	8967	9154
g_{14}	SAC	-1.0316	-1.0316	-1.0316	-1.0316	3.3893E-07	10030	11387
	SA	-1.0316	-1.0316	-1.0316	-1.0316	1.2164E-08	6088	6854
g_{15}	SAC	0.397887	0.397887	0.3978869	0.397887	1.7308E-06	9298	12389
	SA	0.397887	0.397887	0.397887	0.397887	1.1699E-06	7372	7895

F-Function NFCB-Number of function evaluation for the convergence of best
 ANFC-Average number of function evaluation for the convergence

Figure 3: Convergence Graph for the function g_2 using SAC and SA algorithms. Left plot is for SAC algorithm and right plot is for SA algorithm.

Figure 4: Convergence Graph for the function g_3 using SAC and SA algorithms. Left plot is for SAC algorithm and right plot is for SA algorithm.

Figure 5: Convergence Graph for the function g_4 using SAC and SA algorithms. Left plot is for SAC algorithm and right plot is for SA algorithm.

Figure 6: Convergence Graph for the function g_5 using SAC and SA algorithms. Left plot is for SAC algorithm and right plot is for SA algorithm.

Figure 7: Convergence Graph for the function g_6 using SAC and SA algorithms. Left plot is for SAC algorithm and right plot is for SA algorithm.

Figure 8: Convergence Graph for the function g_7 using SAC and SA algorithms. Left plot is for SAC algorithm and right plot is for SA algorithm.

Figure 9: Convergence Graph for the function g_8 using SAC and SA algorithms. Left plot is for SAC algorithm and right plot is for SA algorithm.

Figure 10: Convergence Graph for the function g_9 using SAC and SA algorithms. Left plot is for SAC algorithm and right plot is for SA algorithm.

Figure 11: Convergence Graph for the function g_{10} using SAC and SA algorithms. Left plot is for SAC algorithm and right plot is for SA algorithm.

Figure 12: Convergence Graph for the function g_{11} using SAC and SA algorithms. Left plot is for SAC algorithm and right plot is for SA algorithm.

Figure 13: Convergence Graph for the function g_{12} using SAC and SA algorithms. Left plot is for SAC algorithm and right plot is for SA algorithm.

Figure 14: Convergence Graph for the function g_{13} using SAC and SA algorithms. Left plot is for SAC algorithm and right plot is for SA algorithm.

Figure 15: Convergence Graph for the function g_{14} using SAC and SA algorithms. Left plot is for SAC algorithm and right plot is for SA algorithm.

Figure 16: Convergence Graph for the function g_{15} using SAC and SA algorithms. Left plot is for SAC algorithm and right plot is for SA algorithm.

5. Conclusion

In this paper a clone operator which imitates the biological cloning process to generate a candidate solution vector in simulated annealing algorithm is proposed. We have simulated the results for the SA algorithm and SA with clone operator with the help of various case studies obtained from literature. The results of the two methods, namely proposed SAC method and SA, indicate that the proposed SAC method is capable of generating better solution quality.

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