Optimization of Welded Beam with Imprecise Load and Stress by Parameterized Intuitionistic Fuzzy Optimization Technique

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Abstract

This paper develops a solution procedure of fuzzy and intuitionistic fuzzy optimization to solve welded beam with inexact co-efficient and resources. Interval approximation method is used here to convert the imprecise co-efficient which is a triangular intuitionistic fuzzy number to an interval number. We transform this interval number to a parametric interval valued functional form and then solve this parametric problem by fuzzy and intuitionistic fuzzy optimization technique. Usually interval valued optimization consist of two level mathematical programs but a parametric interval valued optimization in fuzzy and intuitionistic fuzzy environment is direct approach to find the objective function, this is the main advantage. In this paper we have considered a welded beam design with cost of welding as objectives and the maximum shear stress in the weld group, maximum bending stress in the beam, buckling load of the beam and deflection at the tip of a welded steel beam as constraints Numerical example is given here to illustrate this structural model through this approximation method.

Keywords: Intuitionistic Fuzzy Set; Generalized Fuzzy Number; Generalized Intuitionistic Fuzzy Number; Nearest Interval Approximation; Interval Valued Function; Single Objective Intuitionistic Optimization; Welded Beam Optimization.

1. INTRODUCTION

Welding, a process of joining metallic parts with the application of heat or pressure or the both, with or without added material, is an economical and efficient method for obtaining permanent joints in the metallic parts. These welded joints are generally used as a substitute for riveted joint or can be used as an alternative method for casting or forging. The welding processes can broadly be classified into following two groups, the welding process that uses heat alone to join two metallic parts and the welding process that uses a combination of heat and pressure for joining (Bhandari. V. B). However, above all the design of welded beam should preferably be economical and durable one. Since decades, deterministic optimization has been widely used in practice for optimizing welded connection design. These include mathematical traditional optimization algorithms (Ragsdell & Phillips [1]), GA-based methods (Deb [2], Deb [3], Coello [4], Coello [5]), particle swarm optimization (Reddy [6]), harmony search method (Lee & Geem [7]), and Big-Bang Big-Crunch (BB-BC) algorithm (O. Hasançebi, [8]), subset simulation (Li [9]), improved harmony search algorithm (Mahadavi [10]), as methods to solve this problem. All these deterministic optimizations aim to search the optimum solution under given constraints without consideration of uncertainties. So, while a deterministic optimization approach is unable to handle structural performances such as imprecise stresses and deflection etc. due to the presence of impreciseness, to get rid of such problem fuzzy (Zadeh, [11]), intuitionistic fuzzy (Atanassov,[12]) play great roles.

In IFS theory we usually consider degree of acceptance, and degree of rejection where as we consider only membership function in fuzzy set. A few research work has been done on intuitionistic fuzzy optimization in the field of structural optimization. Sarkar [13] optimize two bar truss design with imprecise load and stress in intuitionistic fuzzy environment calculating total integral values of triangular intuitionistic fuzzy number. Shu [14] applied triangular intuitionistic fuzzy number to fault tree analysis on printed board circuit assembly. P.Grzegorzewski et.al [15], H.B.Mitchell et.al [16], G.Nayagam et.al [17], H.M.Nehi et.al [18], S.Rezvani et.al [19] have been employed concept of intuitionistic fuzzy number in multi-attribute decision making(MADM) problem. So indeterminate information should be considered in decision making process. So to deal with different impreciseness on load, stresses and deflection, we have been motivated to incorporate the concept of intuitionistic fuzzy number in this problem, and have developed fuzzy as well as intuitionistic fuzzy optimization algorithm to optimize the optimum design in imprecise environment. We define generalized triangular fuzzy, intuitionistic number and nearest interval approximation of this number and using parametric interval valued function for approximated interval number of triangular fuzzy and intuitionistic fuzzy number we solve a single objective welded beam design problem

in both fuzzy and intuitionistic fuzzy environment .This paper develops optimization algorithm using max-min operator in fuzzy and intuitionistic fuzzy environment to optimize the cost of welding, while the maximum shear stress in the weld group, maximum bending stress in the beam, and buckling load of the beam have been considered and deflection at the tip of a welded steel beam as constraints. Here parametric interval valued function of generalized triangular fuzzy and intuitionistic fuzzy number have been considered for applied load , stress and deflection .

The present study investigates computational algorithm for solving single-objective nonlinear programming problem by parametric fuzzy and Intuitionistic fuzzy optimization approach. The remainder of this paper is organized in the following manner. In section 2, we discuss about single objective welded beam design. In section 3, we discuss about fuzzy set, fuzzy number, intuitionistic fuzzy set, intuitionistic fuzzy number, α -cut and arithmetic operation on triangular intuitionistic fuzzy number. In section 4, we discuss the solution procedure of single objective nonlinear programming problem by parametric fuzzy and intuitionistic non-linear programming technique. In section 5, we discuss about solution of single objective welded beam optimization Problem by parametric fuzzy and intuitionistic fuzzy optimization technique. In section 6, we discuss about numerical solution of single objective welded beam. Finally, we draw conclusions from the results in section 7.

2. SINGLE-OBJECTIVE STRUCTURAL MODEL

In sizing optimization problems, the aim is to minimize single objective function, usually the cost of the structure under certain behavioural constraints which are displacement or stresses. The design variables are most frequently chosen to be dimensions of the height, length, depth and width of the structures. Due to fabrications limitations the design variables are not continuous but discrete for belongingness of cross-sections to a certain set. A discrete structural optimization problem can be formulated in the following form

$$\text{Minimize } C(X) \tag{1}$$

$$\text{subject to } \sigma_i(X) \leq [\sigma_i], i = 1, 2, \dots, m$$

$$X_j \in R^d, \quad j = 1, 2, \dots, n$$

where $C(X)$ represents cost function, $\sigma_i(X)$ is the behavioural constraints and $[\sigma_i(X)]$ denotes the maximum allowable value , ‘ m ’ and ‘ n ’ are the number of constraints and design variables respectively. A given set of discrete value is expressed by R^d and in this paper objective function is taken as

$$C(X) = \sum_{t=1}^T c_t \prod_{n=1}^m x_n^{t_n}$$

and constraint are chosen to be stress of structures as follows

$$\sigma_i(X) \leq \sigma_i \text{ with allowable tolerance } \sigma_i^0 \text{ for } i=1,2,\dots,m$$

And deflection of the structure as follows

$$\delta(X) \leq \delta_{\max} \text{ with allowable tolerance } \delta_{\max}^0$$

Where c_t is the cost coefficient of t^{th} side and x_n is the n^{th} design variable respectively, m is the number of structural element, σ_i and σ_i^0 $\delta_{\max}^0$ are the i^{th} stress, allowable stress and allowable deflection respectively.

3. MATHEMATICAL PRELIMINARIES

3.1. Fuzzy Set

Let X denotes a universal set. Then the fuzzy subset A in X is a subset of order pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ where $\mu_{\tilde{A}} : X \rightarrow [0,1]$ is called the membership function which assigns a real number $\mu_{\tilde{A}}(x)$ in the interval $[0,1]$ to each element $x \in X$. A is non-fuzzy and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of crisp set. It is clear that the range of membership function is a subset of non-negative real numbers.

3.2. α -Level set or α -cut of a Fuzzy Set

The α -level set of a fuzzy set A of X is a crisp set A_α which contains all the elements of X that have membership values greater than or equal to α i.e. $A = \{x : \mu_A(x) \geq \alpha, x \in X, \alpha \in [0,1]\}$.

3.3. Generalized Fuzzy Number

A generalised fuzzy number $\tilde{A} = (a, b, c, d; w)$ where $0 < w \leq 1$ and a, b, c, d are real numbers. The generalized fuzzy number $\tilde{A} = (a, b, c, d; w)$ is a fuzzy subset of real line $\mathbb{R}$ whose membership function $\mu_{\tilde{A}}(x)$ satisfies the following conditions

- i) $\mu_{\tilde{A}}(x)$ is a continuous mapping from $\mathbb{R}$ to closed interval $[0,1]$.

- ii) $\mu_{\tilde{A}}(x) = 0$ where $-\infty < x \leq a$
- ii) $\mu_{\tilde{A}}(x)$ is strictly increasing in constant rate on $[a, b]$
- iv) $\mu_{\tilde{A}}(x) = w$ where $b \leq x \leq c$.
- v) $\mu_{\tilde{A}}(x)$ is strictly decreasing in constant rate on $[c, d]$
- vi) $\mu_{\tilde{A}} = 0$ where $d \leq x \leq \infty$

Now if $b = c, w = 1$ it is called triangular fuzzy number (TFN) as $\tilde{A} = (a, b, d; w)$ and membership function of fuzzy set $\tilde{A} = (x, \mu_{\tilde{A}}(x))$ can be defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} w \left(\frac{x-a}{b-a} \right) & \text{for } a \leq x \leq b \\ w \left(\frac{d-x}{d-b} \right) & \text{for } b \leq x \leq d \\ 0 & \text{otherwise} \end{cases}$$

3.4. α -Level set or α -cut of a Fuzzy Number

Let $\tilde{A} = (a_1^{\mu}, a_2, a_3^{\mu}; w_a)$ be a triangular fuzzy number then α -cut of this fuzzy number is defined by the crisp set $A_{\alpha} = \{x : \mu_{\tilde{A}}(x) \geq \alpha, x \in X, \alpha \in [0, 1]\}$. A_{α} is nonempty bounded closed interval contained in X and it can be denoted by $A_{\alpha} = [A_L(\alpha), A_R(\alpha)]$, $A_L(\alpha)$ and $A_R(\alpha)$ are the lower and upper bound of the closed interval, respectively.

3.5. Intuitionistic Fuzzy Set

Let $X = \{x_1, x_2, \dots, x_n\}$ be a finite universal set. An intuitionistic fuzzy set (IFS) set $\tilde{A}^i$ in the sense of Atanassov (1986) is given by equation $\tilde{A}^i = \{ \langle x, \mu_{\tilde{A}^i}(x), \nu_{\tilde{A}^i}(x) \rangle \mid x \in X \}$ where the function $\mu_{\tilde{A}^i}(x^i) : X \rightarrow [0, 1]$; $x_i \in X \rightarrow \mu_{\tilde{A}^i}(x_i) \in [0, 1]$ and $\nu_{\tilde{A}^i}(x^i) : X \rightarrow [0, 1]$; $x_i \in X \rightarrow \nu_{\tilde{A}^i}(x_i) \in [0, 1]$ define the degree of membership and degree of non-membership of an element $x_i \in X$ to the set $\tilde{A}^i \subseteq X$, such that they satisfy the condition $0 \leq \mu_{\tilde{A}^i}(x_i) + \nu_{\tilde{A}^i}(x_i) \leq 1, \forall x_i \in X$. For

each IFS $\tilde{A}^i$ in X the amount $\Pi_{\tilde{A}^i}(x_i) = 1 - (\mu_{\tilde{A}^i}(x_i) + \nu_{\tilde{A}^i}(x_i))$ is called the degree of uncertainty (or hesitation) associated with the membership of elements $x_i \in X$ in $\tilde{A}^i$ we call it intuitionistic fuzzy index of $\tilde{A}^i$ with respect of an element $x_i \in X$.

3.6. Generalized Intuitionistic Fuzzy Number

A generalised intuitionistic fuzzy number $\tilde{A}^i$ can be defined as with the following properties

- i) It is an intuitionistic fuzzy subset of real line.
- ii) It is normal i.e there is any $x_0 \in R$ such that $\mu_{\tilde{A}^i}(x_0) = w (\in R)$ and $\nu_{\tilde{A}^i}(x_0) = \tau (\in R)$ for $w + \tau \leq 1$;

- iii) It is a convex set for membership function $\mu_{\tilde{A}^i}(x)$ i.e

$$\mu_{\tilde{A}^i}(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)) \text{ for all } x_1, x_2 \in R, \lambda \in [0, w].$$

- iv) It is a concave set for membership function $\mu_{\tilde{A}^i}(x)$ i.e

$$\mu_{\tilde{A}^i}(\lambda x_1 + (1 - \lambda)x_2) \geq \max(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)) \text{ for all } x_1, x_2 \in R, \lambda \in [\tau, 1].$$

- v) $\mu_{\tilde{A}^i}$ is continuous mapping from R to the closed interval $[0, w]$ and $\nu_{\tilde{A}^i}$ is continuous mapping from R to the closed interval $[\tau, 1]$ and for $x_0 \in R$ the relation $\mu_{\tilde{A}^i} + \nu_{\tilde{A}^i} \leq 1$ holds.

3.7. Generalized Triangular Intuitionistic Fuzzy Number

A generalized triangular intuitionistic fuzzy number $\tilde{A}^i = ((a_1^\mu, a_2, a_3^\mu; w_a)(a_1^\nu, a_2, a_3^\nu; \tau_a))$ is a IFN in R and can be defined with the following membership function and non-membership function as follows

$$\mu_{\tilde{A}^i} = \begin{cases} w_a \frac{x - a_1^\mu}{a_2 - a_1^\mu} & \text{for } a_1^\mu \leq x \leq a_2 \\ w_a & \text{for } x = a_2 \\ w_a \frac{a_3^\mu - x}{a_3^\mu - a_2} & \text{for } a_2 \leq x \leq a_3^\mu \\ 0 & \text{otherwise} \end{cases}$$

$$\nu_{\tilde{A}^i} = \begin{cases} \tau_a \frac{x - a_1^\nu}{a_2 - a_1^\nu} & \text{for } a_1^\nu \leq x \leq a_2 \\ \sigma_a & \text{for } x = a_2 \\ \tau_a \frac{x - a_2}{a_3^\nu - a_2} & \text{for } a_2 \leq x \leq a_3^\nu \\ 1 & \text{otherwise} \end{cases}$$

Where $a_1^\nu \leq a_1^\mu \leq a_2 \leq a_3^\mu \leq a_3^\nu$.

3.8. α -Level set or α -cut of a Intuitionistic Fuzzy Number

Let $\tilde{A}^i = ((a_1^\mu, a_2, a_3^\mu; w_a)(a_1^\nu, a_2, a_3^\nu; \tau_a))$ be a triangular intuitionistic fuzzy number then α -cut of this fuzzy number is defined by the closed interval $[\mu_{\tilde{A}_L^i}(\alpha), \mu_{\tilde{A}_U^i}(\alpha)]$, $\alpha \in (0, 1]$ and $[\nu_{\tilde{A}_L^i}(\alpha), \nu_{\tilde{A}_U^i}(\alpha)]$, $\alpha \in [0, 1)$ where

$$\mu_{\tilde{A}_L^i}(\alpha) = \inf \{x \in R : \mu_{\tilde{A}_L^i}(x) \geq \alpha\}$$

$$\mu_{\tilde{A}_U^i}(\alpha) = \sup \{x \in R : \mu_{\tilde{A}_U^i}(x) \geq \alpha\}, \nu_{\tilde{A}_L^i}(\alpha) = \inf \{x \in R : \nu_{\tilde{A}_L^i}(x) \leq \alpha\}$$

$$\nu_{\tilde{A}_U^i}(\alpha) = \sup \{x \in R : \nu_{\tilde{A}_U^i}(x) \leq \alpha\},$$

3.9. Arithmetic Operations on Triangular Intuitionistic Fuzzy Numbers

Let $\tilde{A}^i = ((a_1^\mu, a_2, a_3^\mu; w_a)(a_1^\nu, a_2, a_3^\nu; \tau_a))$ and $\tilde{B}^i = ((b_1^\mu, b_2, b_3^\mu; w_b)(b_1^\nu, b_2, b_3^\nu; \tau_b))$ be two triangular intuitionistic fuzzy number then the arithmetic operations on these numbers can be defined as follows

(i)

$$\tilde{A}^i + \tilde{B}^i = ((a_1^\mu + b_1^\mu, a_2 + b_2, a_3^\mu + b_3^\mu; \min(w_a, w_b))(a_1^\nu + b_1^\nu, a_2 + b_2, a_3^\nu + b_3^\nu; \max(\tau_a, \tau_b)))$$

$$\begin{aligned}
\text{(ii)} \quad \tilde{A}^i - \tilde{B}^i &= \left((a_1^\mu - b_1^\mu, a_2 - b_2, a_3^\mu - b_3^\mu; \min(w_a, w_b)) (a_1^\nu - b_1^\nu, a_2 - b_2, a_3^\nu - b_3^\nu; \max(\tau_a, \tau_b)) \right) \\
\text{(iii)} \quad k\tilde{A}^i &= \begin{cases} \left((ka_1^\mu, ka_2, ka_3^\mu; w_a) (ka_1^\nu, ka_2, ka_3^\nu; \tau_a) \right) & \text{for } k > 0 \\ \left((ka_3^\mu, ka_2, ka_1^\mu; w_a) (ka_3^\nu, ka_2, ka_1^\nu; \tau_a) \right) & \text{for } k < 0 \end{cases} \\
\text{(iv)} \quad \tilde{A}^i . \tilde{B}^i &= \begin{cases} \left((a_1^\mu b_1^\mu, a_2 b_2, a_3^\mu b_3^\mu; \min(w_a, w_b)) (a_1^\nu b_1^\nu, a_2 b_2, a_3^\nu b_3^\nu; \max(\tau_a, \tau_b)) \right) & \text{for } \tilde{A}^i > 0, \tilde{B}^i > 0 \\ \left((a_1^\mu b_3^\mu, a_2 b_2, a_3^\mu b_1^\mu; \min(w_a, w_b)) (a_1^\nu b_3^\nu, a_2 b_2, a_3^\nu b_1^\nu; \max(\tau_a, \tau_b)) \right) & \text{for } \tilde{A}^i > 0, \tilde{B}^i < 0 \\ \left((a_3^\mu b_3^\mu, a_2 b_2, a_1^\mu b_1^\mu; \min(w_a, w_b)) (a_3^\nu b_3^\nu, a_2 b_2, a_1^\nu b_1^\nu; \max(\tau_a, \tau_b)) \right) & \text{for } \tilde{A}^i < 0, \tilde{B}^i < 0 \end{cases} \\
\text{(v)} \quad \tilde{A}^i / \tilde{B}^i &= \begin{cases} \left((a_1^\mu / b_3^\mu, a_2 / b_2, a_3^\mu / b_1^\mu; \min(w_a, w_b)) (a_1^\nu / b_3^\nu, a_2 / b_2, a_3^\nu / b_1^\nu; \max(\sigma_a, \sigma_b)) \right) & \text{for } \tilde{A}^i > 0, \tilde{B}^i > 0 \\ \left((a_3^\mu / b_3^\mu, a_2 / b_2, a_1^\mu / b_1^\mu; \min(w_a, w_b)) (a_3^\nu / b_3^\nu, a_2 / b_2, a_1^\nu / b_1^\nu; \max(\sigma_a, \sigma_b)) \right) & \text{for } \tilde{A}^i < 0, \tilde{B}^i > 0 \\ \left((a_3^\mu / b_1^\mu, a_2 / b_2, a_1^\mu / b_3^\mu; \min(w_a, w_b)) (a_3^\nu / b_1^\nu, a_2 / b_2, a_1^\nu / b_3^\nu; \max(\sigma_a, \sigma_b)) \right) & \text{for } \tilde{A}^i < 0, \tilde{B}^i < 0 \end{cases}
\end{aligned}$$

4. MATHEMATICAL ANALYSIS

4.1. Nearest Interval Approximation for Fuzzy Number

Here we want to approximate an fuzzy number $\tilde{A} = (a_1^\mu, a_2, a_3^\mu; w_a)$ by a crisp model.

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy number with α -cuts are $[A_L(\alpha), A_U(\alpha)]$ and $[B_L(\alpha), B_U(\alpha)]$ respectively. Then the distance between them can be measured according to Euclidean metric (Dey [22]) as

$$\tilde{d}_E^2 = \int_0^1 (A_L(\alpha) - B_L(\alpha))^2 d\alpha + \int_0^1 (A_U(\alpha) - B_U(\alpha))^2 d\alpha$$

Now we find a closed interval $\tilde{C}_{d_E}(\tilde{A}^i) = [C_L, C_U]$ which is nearest to $\tilde{A}$ with respect to the matrix $\tilde{d}_E$. Again it is obvious that each real interval can also be considered as

an fuzzy number with constant α -cut $(\tilde{C}_{d_E}(\tilde{A}))\alpha = [C_L, C_U]$ for all $\alpha \in [0, 1]$. Now

we have to minimize $\tilde{d}_E^2(\tilde{A}, \tilde{C}_{d_E}(\tilde{A})) = \int_0^1 (A_L(\alpha) - C_L)^2 d\alpha + \int_0^1 (A_U(\alpha) - C_U)^2 d\alpha$ with

respect to C_L and C_U . In order to minimize $\tilde{d}_E(\tilde{A}, \tilde{C}_{d_E}(\tilde{A}))$

It is sufficient to minimize the functions $D(C_L, C_U) (= \tilde{d}_E^2(\tilde{A}, \tilde{C}_{d_E}(\tilde{A})))$. The first

partial derivatives are $\frac{\partial D(C_L, C_U)}{\partial C_L} = -2 \int_0^1 A_L(\alpha) d\alpha + 2C_L$

$$\frac{\partial D(C_L, C_U)}{\partial C_U} = -2 \int_0^1 A_U(\alpha) d\alpha + 2C_U$$

And then we solve the system

$$\frac{\partial D(C_L, C_U)}{\partial C_U} = 0, \frac{\partial D(C_L, C_U)}{\partial C_L} = 0 \text{ The solution is}$$

$$C_L = \int_0^1 A_L(\alpha) d\alpha; C_U = \int_0^1 A_U(\alpha) d\alpha$$

$$\text{Since } \det \begin{pmatrix} \frac{\partial^2 D(C_L, C_U)}{\partial C_L^2} & \frac{\partial^2 D(C_L, C_U)}{\partial C_L \partial C_U} \\ \frac{\partial^2 D(C_L, C_U)}{\partial C_L \partial C_U} & \frac{\partial^2 D(C_L, C_U)}{\partial C_U^2} \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 4 > 0 \text{ then } C_L, C_U \text{ mentioned above minimize } D(C_L, C_U). \text{ The nearest}$$

interval of the intuitionistic fuzzy number $\tilde{A}$ with respect to the matrix $\tilde{d}_E$ is

$$\tilde{C}_{d_E}(\tilde{A}) = \left[\int_0^1 A_L(\alpha) d\alpha, \int_0^1 A_U(\alpha) d\alpha \right] = \left[\frac{(2w-1)a_1 + a_2}{2w}, \frac{(2w-1)a_3 + a_2}{2w} \right]$$

4.2. Nearest Interval Approximation for Intuitionistic Fuzzy Number

Here we want to approximate an intuitionistic fuzzy number

$\tilde{A}^i = ((a_1^\mu, a_2, a_3^\mu; w_a)(a_1^\nu, a_2, a_3^\nu; \eta_a))$ by a crisp model.

Let $\tilde{A}^i$ and $\tilde{B}^i$ be two intuitionistic fuzzy number. Then the distance between them can be measured according to Euclidean matrix as (Dey [22])

$$\begin{aligned} \tilde{d}_E^2 &= \frac{1}{2} \int_0^1 (\mu_{A_L}(\alpha) - \mu_{B_L}(\alpha))^2 d\alpha + \frac{1}{2} \int_0^1 (\mu_{A_U}(\alpha) - \mu_{B_U}(\alpha))^2 d\alpha \\ &+ \frac{1}{2} \int_0^1 (\nu_{A_L}(\alpha) - \nu_{B_L}(\alpha))^2 d\alpha + \frac{1}{2} \int_0^1 (\nu_{A_U}(\alpha) - \nu_{B_U}(\alpha))^2 d\alpha \end{aligned}$$

Now we find a closed interval $\tilde{C}_{d_E}(\tilde{A}^i) = [C_L, C_U]$ which is nearest to $\tilde{A}^i$ with respect to the matrix $\tilde{d}_E$. Again it is obvious that each real interval can also be considered as an intuitionistic fuzzy number with constant α -cut $[C_L, C_U]$ for all $\alpha \in [0, 1]$. Now we have to minimize $\tilde{d}_E(\tilde{A}^i, \tilde{C}_{d_E}(\tilde{A}^i))$ with respect to C_L and C_U , that is to minimize

$$F_1(C_L, C_U) = \int_0^1 (\mu_{A_L}(\alpha) - C_L)^2 d\alpha + \int_0^1 (\mu_{A_U}(\alpha) - C_U)^2 d\alpha \\ + \int_0^1 (\nu_{A_L}(\alpha) - C_L)^2 d\alpha + \int_0^1 (\nu_{A_U}(\alpha) - C_U)^2 d\alpha$$

With respect to C_L and C_U . We define partial derivatives

$$\frac{\partial F_1(C_L, C_U)}{\partial C_L} = -2 \int_0^1 (\mu_{A_L}(\alpha) + \nu_{A_L}(\alpha)) d\alpha + 4C_L$$

$$\frac{\partial F_1(C_L, C_U)}{\partial C_U} = -2 \int_0^1 (\mu_{A_U}(\alpha) + \nu_{A_U}(\alpha)) d\alpha + 4C_U$$

And then we solve the system

$$\frac{\partial F_1(C_L, C_U)}{\partial C_L} = 0, \frac{\partial F_1(C_L, C_U)}{\partial C_U} = 0$$

The solution is

$$C_L = \int_0^1 \frac{\mu_{A_L}(\alpha) + \nu_{A_L}(\alpha)}{2} d\alpha; \quad C_U = \int_0^1 \frac{\mu_{A_U}(\alpha) + \nu_{A_U}(\alpha)}{2} d\alpha$$

$$\det \begin{pmatrix} \frac{\partial^2 F_1(C_L, C_U)}{\partial C_L^2} & \frac{\partial^2 F_1(C_L, C_U)}{\partial C_L \partial C_U} \\ \frac{\partial^2 F_1(C_L, C_U)}{\partial C_U \partial C_L} & \frac{\partial^2 F_1(C_L, C_U)}{\partial C_U^2} \end{pmatrix}$$

Since

$$= \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = 4 > 0 \text{ then } C_L, C_U \text{ mentioned above minimize } F_1(C_L, C_U). \text{ The nearest}$$

interval of the intuitionistic fuzzy number $\tilde{A}^i$ with respect to the matrix $\tilde{d}_E$ is

$$\tilde{C}_{d_E}(\tilde{A}^i) = \left[\int_0^1 \frac{\mu_{A_L}(\alpha) + \nu_{A_L}(\alpha)}{2} d\alpha, \int_0^1 \frac{\mu_{A_U}(\alpha) + \nu_{A_U}(\alpha)}{2} d\alpha \right] \\ = \left[\frac{a_1^\mu + a_2}{2} + \frac{a_2 - a_1^\mu}{4w} + \frac{a_2 - a_1^\nu}{4\eta}, \frac{a_2 + a_3^\mu}{2} + \frac{a_3^\mu - a_2}{4w} + \frac{a_3^\nu - a_2}{4\eta} \right]$$

4.3. Parametric Interval valued Function

If $[m, n]$ be an interval with $m, n > 0$ we can express an interval number by a function. The parametric interval-valued function for the interval $[m, n]$ can be taken as $g(s) = m^{1-s} n^s$ for $s \in [0, 1]$ which is strictly monotone continuous function and its inverse exists. Let ψ be the inverse of $g(s)$ then $s = \frac{\log \psi - \log m}{\log n - \log m}$.

4.4. Formulation of Fuzzy and Intuitionistic Fuzzy Programming with imprecise coefficient in parametric form

A single objective intuitionistic fuzzy non-linear programming problem with imprecise co-efficient can be formulated as

$$\text{Minimize } \tilde{f}(x) = \sum_{t=1}^T \xi_t \tilde{c}_t \prod_{j=1}^n x_j^{a_{tj}}$$

$$\text{Such that } \tilde{f}_i(x) = \sum_{t=1}^{T_i} \xi_{it} \tilde{c}_{it} \prod_{j=1}^n x_j^{a_{ijt}} \leq \xi_i \tilde{b}_i \text{ for } i = 1, 2, \dots, m$$

$$x_j > 0 \quad j = 1, 2, \dots, n$$

Here ξ_t, ξ_{it}, ξ_i are the signum function used to indicate sign of term in the equation. $\tilde{c}_t > 0, \tilde{c}_{it} > 0$. a_{tj}, a_{ijt} are real numbers for all i, t, j .

Here $\tilde{c}_t = (c_t^{1\mu}, c_t^2, c_t^{3\mu}; w_t)$; $\tilde{c}_{it} = (c_{it}^{1\mu}, c_{it}^2, c_{it}^{3\mu}; w_{it})$; $\tilde{b}_i = (b_i^{1\mu}, b_i^2, b_i^{3\mu}; w_i)$ for fuzzy number as coefficients and $\tilde{c}_t = ((c_t^{1\mu}, c_t^2, c_t^{3\mu}; w_t)(c_t^{1\nu}, c_t^2, c_t^{3\nu}; \tau_t))$; $\tilde{c}_{it} = ((c_{it}^{1\mu}, c_{it}^2, c_{it}^{3\mu}; w_{it})(c_{it}^{1\nu}, c_{it}^2, c_{it}^{3\nu}; \tau_{it}))$; $\tilde{b}_i = ((b_i^{1\mu}, b_i^2, b_i^{3\mu}; w_i)(b_i^{1\nu}, b_i^2, b_i^{3\nu}; \tau_i))$ for intuitionistic fuzzy coefficient.

Using nearest interval approximation method for both fuzzy and intuitionistic fuzzy number, we transform all the triangular intuitionistic fuzzy number into interval number i.e. $[c_t^L, c_t^U]$, $[c_{it}^L, c_{it}^U]$, and $[b_i^L, b_i^U]$

Now the intuitionistic single objective programming with imprecise parameter is of the following form

$$\text{Minimize } \hat{f}(x) = \sum_{t=1}^T \xi_{k_0 t} \hat{c}_t \prod_{j=1}^n x_j^{a_{tj}}$$

Such that $\hat{f}_i(x) = \sum_{t=1}^{T_i} \xi_{it} \hat{c}_{it} \prod_{j=1}^n x_j^{a_{ij}} \leq \sigma_i \hat{b}_i$ for $i = 1, 2, \dots, m$

$$x_j > 0 \quad j = 1, 2, \dots, n$$

Here ξ_t, ξ_{it}, ξ_i are the signum function used to indicate sign of term in the equation. $\hat{c}_t > 0, \hat{c}_{it} > 0; \hat{b}_i > 0$ denote the interval component i.e. $\hat{c}_t = [c_t^L, c_t^U]$, $\hat{c}_{it} = [c_{it}^L, c_{it}^U]$, and $\hat{b}_i = [b_i^L, b_i^U]$ and a_{ij}, a_{ij} are real numbers for all i, t, j .

Using parametric interval valued function the above problem transform into

$$\text{Minimize } f(x; s) = \sum_{t=1}^T \xi_t (c_t^L)^{1-s} (c_t^U)^s \prod_{j=1}^n x_j^{a_{ij}}$$

Such that $f_i(x; s) = \sum_{t=1}^{T_i} \xi_{it} (c_{it}^L)^{1-s} (c_{it}^U)^s \prod_{j=1}^n x_j^{a_{ij}} \leq \xi_i (b_i^L)^{1-s} (b_i^U)^s$ for $i = 1, 2, \dots, m$

$$x_j > 0 \quad j = 1, 2, \dots, n \quad s \in [0, 1]$$

Here ξ_t, ξ_{it}, ξ_i are the signum function used to indicate sign of term in the equation.

This is a parametric single objective non-linear programming problem and can be solved by intuitionistic fuzzy optimization technique.

4.5. Fuzzy and Intuitionistic Fuzzy Non-linear Programming (IFNLP) Optimization to solve Parametric Single-Objective Non-linear Programming Problem (PSO NLP)

Let us consider a single-objective parametric nonlinear optimization problem as

$$\text{Minimize } f(x) \tag{2}$$

$$g_j(x; s) \leq b_j(s) \quad j = 1, 2, \dots, m$$

$$x \geq 0; s \in [0, 1]$$

Usually constraints goals are considered as fixed quantity .But in real life problem ,the constraint goal can not be always exact. So we can consider the constraint goal for less than type constraints at least $b_j(s)$ and it may possible to extend to $b_j^0(s)$ so that the maximum allowable tolerance is $b_j^1(s)$ so that $b_j^0(s) = b_j(s) + b_j^1(s)$. This fact seems to take the constraint goal as a intuitionistic fuzzy set and which will be more

realistic descriptions than others. Then the NLP becomes IFO problem with intuitionistic resources, which can be described as follows

$$\text{Minimize } f(x) \quad (3)$$

$$g_j(x; s) \leq \tilde{b}_j^i(s) \quad j = 1, 2, \dots, m$$

$$x \geq 0; s \in [0, 1]$$

To solve the IFO (3), following warner's (1987) and Angelov (1995) we are presenting a solution procedure for single-objective NSO problem (3) as follows

Step-1: Following warner's approach solve the single objective non-linear programming problem without tolerance in constraints (i.e. $g_j(x; s) \leq b_j(s)$), with tolerance of acceptance in constraints (i.e. $g_j(x; s) \leq b_j^0(s)$) by appropriate non-linear programming technique

Here they are

Sub-problem-1

$$\text{Minimize } f(x) \quad (4)$$

$$g_j(x; s) \leq b_j(s) \quad j = 1, 2, \dots, m$$

$$x \geq 0; s \in [0, 1]$$

Sub-problem-2

$$\text{Minimize } f(x) \quad (5)$$

$$g_j(x; s) \leq b_j^0(s), \quad j = 1, 2, \dots, m$$

$$x \geq 0; s \in [0, 1]$$

we may get optimal solutions $x^* = x^1, f(x^*) = f(x^1)$ and $x^* = x^2, f(x^*) = f(x^2)$ for sub-problem 1 and 2 respectively.

Step-2: From the result of step 1 we now find the lower bound and upper bound of objective functions. If $U_{f(x)}^\mu, U_{f(x)}^\nu$ be the upper bounds of truth, indeterminacy, falsity function for the objective respectively and $L_{f(x)}^\mu, L_{f(x)}^\nu$ be the lower bound of membership and non-membership functions of objective respectively then

$$U_{f(x)}^\mu = \max \{f(x^1), f(x^2)\}, L_{f(x)}^\mu = \min \{f(x^1), f(x^2)\},$$

$$U_{f(x)}^\nu = U_{f(x)}^\mu, L_{f(x)}^\nu = L_{f(x)}^\mu + \varepsilon_{f(x)} \text{ where } 0 < \varepsilon_{f(x)} < (U_{f(x)}^\mu - L_{f(x)}^\mu)$$

Step-3: In this step we calculate linear membership for membership and non membership functions of objective as follows

$$\mu_{f(x)}(f(x)) = \begin{cases} 1 & \text{if } f(x) \leq L_{f(x)}^\mu \\ \left(\frac{U_{f(x)}^\mu - f(x)}{U_{f(x)}^\mu - L_{f(x)}^\mu} \right) & \text{if } L_{f(x)}^\mu \leq f(x) \leq U_{f(x)}^\mu \\ 0 & \text{if } f(x) \geq U_{f(x)}^\mu \end{cases}$$

$$\nu_{f(x)}(f(x)) = \begin{cases} 0 & \text{if } f(x) \leq L_{f(x)}^\nu \\ \frac{f(x) - L_{f(x)}^\nu}{U_{f(x)}^\nu - L_{f(x)}^\nu} & \text{if } L_{f(x)}^\nu \leq f(x) \leq U_{f(x)}^\nu \\ 1 & \text{if } f(x) \geq U_{f(x)}^\nu \end{cases}$$

Step-4: In this step using linear function for membership and non-membership functions, we may calculate membership and non membership function for constraints as follows

$$\mu_{g_j(x;s)}(g_j(x;s)) = \begin{cases} 1 & \text{if } g_j(x;s) \leq b_j(s) \\ \left(\frac{b_j^0(s) - g_j(x;s)}{b_j^1(s) - b_j^0(s)} \right) & \text{if } b_j(s) \leq g_j(x;s) \leq b_j^0(s) \\ 0 & \text{if } g_j(x;s) \geq b_j^0(s) \end{cases}$$

$$\nu_{g_j(x;s)}(g_j(x;s)) = \begin{cases} 0 & \text{if } g_j(x;s) \leq b_j(s) + \varepsilon_{g_j(x;s)} \\ \frac{g_j(x;s) - b_j(s) - \varepsilon_{g_j(x;s)}}{b_j^1(s) - \varepsilon_{g_j(x;s)}} & \text{if } b_j(s) + \varepsilon_{g_j(x;s)} \leq g_j(x;s) \leq b_j^0(s) \\ 1 & \text{if } g_j(x;s) \geq b_j^0(s) \end{cases}$$

where and for $j = 1, 2, \dots, m$ $0 < \varepsilon_{g_j(x;s)} < b_j^1(s)$.

$$x \geq 0; s \in [0, 1]$$

Step-5: Now using Fuzzy and IFO for single objective optimization technique(Singh .et.al [23]) the optimization problem (2) can be formulated as

Maximize α

Such that

$$\mu_{f(x)}(x) \geq \alpha; \mu_{g_j}(x; s) \geq \alpha; \quad (6)$$

$$\alpha \in [0, 1]$$

$$x \geq 0; s \in [0, 1]$$

and

Maximize $(\alpha - \beta)$

Suh that

$$\mu_{f(x)}(x) \geq \alpha; \mu_{g_j}(x; s) \geq \alpha; \quad (7)$$

$$\nu_{f(x)}(x) \leq \beta; \nu_{g_j}(x; s) \leq \beta;$$

$$\alpha + \beta \leq 1; \alpha \geq \beta$$

$$\alpha, \beta \in [0, 1] \quad x \geq 0; s \in [0, 1]$$

respectively. Now the above problem (6),(7) can be solved appropriate mathematical programming.

5. SOLUTION OF SINGLE-OBJECTIVE WELDED BEAM DESIGN(SOWBD) BY FUZZY AND INTUITIONISTIC FUZZY OPTIMIZATION TECHNIQUE

The parametric Welded beam design problem can be formulated as

$$\text{Minimize } C(X) \quad (8)$$

$$\text{subject to } \sigma_i(X; s) \leq [\sigma_i(s)], i = 1, 2, \dots, m$$

$$X_j \in R^d, \quad j = 1, 2, \dots, n$$

$$X > 0; s \in [0, 1]$$

where $C(X)$ represents cost function, $\sigma_i(X)$ is the behavioural constraints and $[\sigma_i(X)]$ denotes the maximum allowable value, 'm' and 'n' are the number of constraints and design variables respectively. A given set of discrete value is expressed by R^d and in this paper objective function is taken as

$$C(X) = \sum_{t=1}^T c_t \prod_{n=1}^m x_n^m$$

and constraint are chosen to be stress of structures as follows

$$\sigma_i(X; s) \lesssim \sigma_i(s) \text{ with allowable tolerance } \sigma_i^0(s) \text{ for } i = 1, 2, \dots, m$$

And deflection of the structure as follows

$$\delta(X; s) \lesssim \delta_{\max}(s) \text{ with allowable tolerance } \delta_{\max}^0(s)$$

Where c_t is the cost coefficient of t^{th} side and x_n is the n^{th} design variable

respectively, m is the number of structural element, σ_i and $\sigma_i^0(s)$ $\delta_{\max}^0(s)$ are the i^{th} stress, allowable stress and allowable deflection respectively.

To solve the SOWBP (8), step 1 of 4 is used and we will get optimum solutions of two sub problem as X^1 and X^2 . After that according to step 2 we find upper and lower bound of membership function of objective function as $U_{C(X)}^\mu, U_{C(X)}^\nu$ and

$$L_{C(X)}^\mu, L_{C(X)}^\nu \quad \text{where } U_{C(X)}^\mu = \max \{C(X^1), C(X^2)\}, L_{C(X)}^\mu = \min \{C(X^1), C(X^2)\},$$

$$U_{C(X)}^\nu = U_{C(X)}^\mu, L_{C(X)}^\nu = L_{C(X)}^\mu + \varepsilon_{C(X)} \text{ where } 0 < \varepsilon_{C(X)} < (U_{C(X)}^\mu - L_{C(X)}^\mu)$$

Let the linear membership function for objective be

$$\mu_{C(X)}(C(X)) = \begin{cases} 1 & \text{if } C(X) \leq L_{C(X)}^\mu \\ \left(\frac{U_{C(X)}^\mu - C(X)}{U_{C(X)}^\mu - L_{C(X)}^\mu} \right) & \text{if } L_{C(X)}^\mu \leq C(X) \leq U_{C(X)}^\mu \\ 0 & \text{if } C(X) \geq U_{C(X)}^\mu \end{cases}$$

$$\nu_{C(X)}(C(X)) = \begin{cases} 0 & \text{if } C(X) \leq L_{C(X)}^\mu + \varepsilon_{C(X)} \\ \left(\frac{C(X) - (L_{C(X)}^\mu + \varepsilon_{C(X)})}{U_{C(X)}^\mu - L_{C(X)}^\mu - \varepsilon_{C(X)}} \right) & \text{if } L_{C(X)}^\mu + \varepsilon_{C(X)} \leq C(X) \leq U_{C(X)}^\mu \\ 1 & \text{if } C(X) \geq U_{C(X)}^\mu \end{cases}$$

and constraints be

$$\mu_{g_j(X;s)}(g_j(X;s)) = \begin{cases} 1 & \text{if } g_j(X;s) \leq b_j(s) \\ \left(\frac{b_j^0(s) - g_j(X;s)}{b_j^1(s)} \right) & \text{if } b_j(s) \leq g_j(X;s) \leq b_j^0(s) \\ 0 & \text{if } g_j(X;s) \geq b_j^0(s) \end{cases}$$

$$\nu_{g_j(X;s)}(g_j(X;s)) = \begin{cases} 0 & \text{if } g_j(X;s) \leq b_j(s) + \varepsilon_{g_j(X;s)} \\ \frac{g_j(X;s) - b_j(s) - \varepsilon_{g_j(X;s)}}{b_j^0(s) - \varepsilon_{g_j(X;s)}} & \text{if } b_j(s) + \varepsilon_{g_j(X;s)} \leq g_j(X;s) \leq b_j^0(s) \\ 1 & \text{if } g_j(X;s) \geq b_j^0(s) \end{cases}$$

where and for $g_i(X;s) = \sigma_i(X;s)$ or $\delta_i(X;s)$ or $\tau_i(X;s)$, $0 < \varepsilon_{g_j(X;s)} < b_j^0(s)$

then parametric fuzzy and intuitionistic fuzzy optimization problem can be formulated as

$$\text{Maximize } \alpha \tag{9}$$

such that

$$\mu_{C(X;s)}(C(X;s)) \geq \alpha; \quad \mu_{g_j(X;s)}(g_j(X;s)) \geq \alpha;$$

$$g_i(X;s) \leq [g_i(s)]; \quad \alpha \in [0,1] \quad X \geq 0; s \in [0,1]$$

$$g_i(X;s) = \sigma_i(X;s) \text{ or } \delta_i(X;s) \text{ or } \tau_i(X;s), 0 < \varepsilon_{g_j(X;s)} < b_j^1(s)$$

and

$$\text{Maximize } (\alpha - \beta) \tag{10}$$

such that

$$\mu_{C(X;s)}(C(X;s)) \geq \alpha; \mu_{g_j(X;s)}(g_j(X;s)) \geq \alpha;$$

$$\nu_{C(X;s)}(C(X;s)) \leq \beta; \nu_{g_j(X;s)}(g_j(X;s)) \leq \beta$$

$$g_i(X;s) \leq [g_i(s)]; \alpha + \beta \leq 1; \alpha \geq \beta; \alpha, \beta \in [0,1] \quad X \geq 0; s \in [0,1]$$

$$g_i(X;s) = \sigma_i(X;s) \text{ or } \delta_i(X;s) \text{ or } \tau_i(X;s), 0 < \varepsilon_{g_j(X;s)} < b_j^l(s)$$

respectively. Now the above problems can be solved by appropriate mathematical programming.

6. NUMERICAL ILLUSTRATION

A welded beam (Ragsdell and Philips 1976, Fig. 2) has to be designed at minimum cost whose constraints are shear stress in weld (τ), bending stress in the beam (σ), buckling load on the bar (P), and deflection of the beam (δ). The design variables are

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} h \\ l \\ t \\ b \end{bmatrix} \text{ where } h \text{ is the weld size, } l \text{ is the length of the weld, } t \text{ is the depth of the}$$

welded beam, b is the width of the welded beam.

Fig.2. Design of the welded beam

Cost Function

The performance index appropriate to this design is the cost of weld assembly. The major cost components of such an assembly are (i) set up labour cost, (ii) welding labour cost, (iii) material cost.

$C(X) = C_0 + C_1 + C_2$ where, $f(X)$ = cost function; C_0 = set up cost; C_1 = welding labour cost; C_2 = material cost;

Set up cost C_0 : The company has chosen to make this component a weldment, because of the existence of a welding assembly line. Furthermore, assume that fixtures for set up and holding of the bar during welding are readily available. The cost C_0 can therefore be ignored in this particular total cost model.

Welding labour cost C_1 : Assume that the welding will be done by machine at a total cost of \$10/hr (including operating and maintenance expense). Furthermore suppose that the machine can lay down a cubic inch of weld in 6 min. The labour cost is then

$$C_1 = \left(10 \frac{\$}{hr}\right) \left(\frac{1}{60} \frac{hr}{min}\right) \left(6 \frac{min}{in^3}\right) V_w = 1 \left(\frac{\$}{in^3}\right) V_w. \text{Where } V_w = \text{weld volume, } in^3$$

Material cost C_2 : $C_2 = C_3 V_w + C_4 V_B$. Where C_3 = cost per volume per weld material, \$/in³ = (0.37)(0.283) ; C_4 = cost per volume of bar stock, \$/in³ = (0.37)(0.283) ; V_B = volume of bar, in³. From geometry $V_w = h^2 l$; volume of the weld material (in³) $V_{weld} = x_1^2 x_2$ and $V_B = tb(L+l)$; volume of bar (in³) $V_{bar} = x_3 x_4 (L + x_2)$. Therefore cost function become

$$C(X) = h^2 l + C_3 h^2 l + C_4 tb(L+l) = 1.10471 x_1^2 x_2 + 0.04811 x_3 x_4 (14.0 + x_2)$$

Engineering Relationship

Fig 3. Shear stresses in the weld group.

Maximum shear stress in weld group:

To complete the model it is necessary to define important stress states

$$\text{Direct or primary shear stress } \tau_1 = \frac{\text{Load}}{\text{Throat area}} = \frac{P}{A} = \frac{P}{\sqrt{2}hl} = \frac{P}{\sqrt{2}x_1x_2}$$

Since the shear stress produced due to turning moment $M = P.e$ at any section is proportional to its radial distance from centre of gravity of the joint 'G', therefore stress due to M is proportional to R and is in a direction at right angles to R . In

$$\text{other words } \frac{\tau_2}{R} = \frac{\tau}{r} = \text{constant. Therefore } R = \sqrt{\left(\frac{l}{2}\right)^2 + \left(\frac{h+t}{2}\right)^2} = \sqrt{\frac{x_2^2}{4} + \frac{(x_1+x_3)^2}{4}}$$

Where, τ_2 is the shear stress at the maximum distance R and τ is the shear stress at any distance r . Consider a small section of the weld having area dA at a distance r from 'G'. Therefore shear force on this small section $= \tau \times dA$ and turning moment of the shear force about centre of gravity $dM = \tau \times dA \times r = \frac{\tau_2}{R} \times dA \times r^2$. Therefore total

turning moment over the whole weld area $M = \frac{\tau_2}{R} \int dA \times r^2 = \frac{\tau_2}{R} J$. where J = polar moment of inertia of the weld group about centre of gravity. Therefore shear stress due to the turning moment i.e. secondary shear stress, $\tau_2 = \frac{MR}{J}$. In order to find the

resultant stress, the primary and secondary shear stresses are combined vectorially. Therefore the maximum resultant shear stress that will be produced at the weld group,

$$\tau = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2 \cos \theta}, \quad \text{where, } \theta = \text{Angle between } \tau_1 \text{ and } \tau_2. \text{ As}$$

$$\cos \theta = \frac{l/2}{R} = \frac{x_2}{2R}; \quad \tau = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2 \frac{x_2}{2R}}.$$

Now the polar moment of inertia of the throat area (A) about the centre of gravity is obtained by parallel axis theorem,

$$J = 2 \left[I_{xx} + A \times x^2 \right] = 2 \left[\frac{A \times l^2}{12} + A \times x^2 \right] = 2A \left(\frac{l^2}{12} + x^2 \right) = 2 \left\{ \sqrt{2}x_1x_2 \left[\frac{x_2^2}{12} + \frac{(x_1+x_3)^2}{2} \right] \right\}$$

Where, A = throat area $= \sqrt{2}x_1x_2$, l = Length of the weld, x = Perpendicular distance

$$\text{between two parallel axes} = \frac{t}{2} + \frac{h}{2} = \frac{x_1+x_3}{2}.$$

Maximum bending stress in beam:

Now Maximum bending moment = PL , Maximum bending stress = $\frac{T}{Z}$, where

$$T = PL;$$

Z = section modulus = $\frac{I}{y}$; I = moment of inertia = $\frac{bt^3}{12}$; y = distance of extreme fibre

from centre of gravity of cross section = $\frac{t}{2}$; Therefore $Z = \frac{bt^2}{6}$. So bar bending stress

$$\sigma(x) = \frac{T}{Z} = \frac{6PL}{bt^2} = \frac{6PL}{x_4 x_3^2}.$$

Maximum deflection in beam:

$$\text{Maximum deflection at cantilever tip} = \frac{PL^3}{3EI} = \frac{PL^3}{3E \frac{bt^3}{12}} = \frac{4PL^3}{Ebt^3}$$

Buckling load of beam:

$$\text{buckling load can be approximated by } P_c(x) = \frac{4.013\sqrt{EIC}}{l^2} \left(1 - \frac{a}{l} \sqrt{\frac{El}{C}} \right)$$

where, I = moment of inertia = $\frac{bt^3}{12}$; torsional rigidity $C = GJ = \frac{1}{3}tb^3G$; $l = L$; $a = \frac{t}{2}$;

$$= \frac{4.013\sqrt{E \frac{t^2 b^6}{36}}}{L^2} \left(1 - \frac{t}{2L} \sqrt{\frac{E}{4G}} \right) = \frac{4.013\sqrt{EGx_3^6 x_4^6 / 36}}{L^2} \left(1 - \frac{x_3}{2L} \sqrt{\frac{E}{4G}} \right);$$

The single-objective optimization problem can be stated as follows

$$\text{Minimize } C(X) \equiv 1.10471x_1^2 x_2 + 0.04811(14 + x_2)x_3 x_4 \quad (11)$$

Such that

$$g_1(x) \equiv \tau(x) - \tau_{\max} \leq 0;$$

$$g_2(x) \equiv \sigma(x) - \sigma_{\max} \leq 0;$$

$$g_3(x) \equiv x_1 - x_4 \leq 0;$$

$$g_4(x) \equiv 0.10471x_1^2x_2 + 0.04811x_3x_4(14 + x_2) - 5 \leq 0;$$

$$g_5(x) \equiv 0.125 - x_1 \leq 0;$$

$$g_6(x) \equiv \delta(x) - \delta_{\max} \leq 0;$$

$$g_7(x) \equiv P - P_c(x) \leq 0;$$

$$0.1 \leq x_1, x_4 \leq 2.0$$

$$0.1 \leq x_2, x_3 \leq 2.0$$

$$\text{where } \tau(x) = \sqrt{\tau_1^2 + 2\tau_1\tau_2 \frac{x_2}{2R} + \tau_2^2} \quad ; \quad \tau_1 = \frac{P}{\sqrt{2}x_1x_2}; \tau_2 = \frac{MR}{J}; M = P\left(L + \frac{x_2}{2}\right);$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}; J = \left\{ \frac{x_1x_2}{\sqrt{2}} \left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2 \right] \right\}; \sigma(x) = \frac{6PL}{x_4x_3^2}; \delta(x) = \frac{4PL^3}{Ex_4x_3^2};$$

$$P_c(x) = \frac{4.013\sqrt{EGx_3^6x_4^6/36}}{L^2} \left(1 - \frac{x_3}{2L} \sqrt{\frac{E}{4G}} \right); \quad P = \text{Force on beam}; L = \text{Beam length}$$

beyond weld; x_1 = Height of the welded beam; x_2 = Length of the welded beam; x_3 = Depth of the welded beam; x_4 = Width of the welded beam; $\tau(x)$ = Design shear stress; $\sigma(x)$ = Design normal stress for beam material; M = Moment of P about the centre of gravity of the weld, J = Polar moment of inertia of weld group; G = Shearing modulus of Beam Material; E = Young modulus; $\tau_{\max}$ = Design Stress of the weld; $\sigma_{\max}$ = Design normal stress for the beam material; $\delta_{\max}$ = Maximum deflection; τ_1 = Primary stress on weld throat. τ_2 = Secondary torsional stress on weld. Input data are given in table 1 and 5.

Table 1: Input data for crisp model (11)

Applied load P (lb)	Beam length beyond weld L (in)	Young Modulus E (psi)	Value of G (psi)	Maximum allowable shear stress $\tau_{\max}$ (psi)	Maximum allowable deflection $\delta_{\max}$ (in)	Maximum allowable normal stress $\sigma_{\max}$ (psi)
6000 $\equiv (5900, 6000, 6100; w_p^{14})$		3×10^6	12×10^6	13550 $\equiv (135400, 13550, 13560; w_\tau)$ Maximum allowable value	0.25 $\equiv (0.24, 0.25, 0.26; w_\delta)$ Maximum allowable value	3000 $\equiv (2900, 3000, 3100; w_\sigma)$ Maximum allowable value

				13600 $\equiv (13500, 13600, 13700; w_\tau^1)$	$0.2\tilde{6}$ $\equiv (0.25, 0.26, 0.27; w_\delta^1)$	3100 $\equiv (3000, 3100, 3200; w_\sigma^1)$
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where $w_p, w_\sigma, w_\delta, w_\tau$ and $w_p^1, w_\sigma^1, w_\delta^1, w_\tau^1$ are degree of acceptance or aspiration level and maximum degree of acceptance or aspiration level of applied load, normal stress, deflection and allowable shear stress respectively.

Now parameterized value of interval valued function can be calculated as

$$\hat{p} = \left(\left(\frac{(2w_p - 1)5900 + 6000}{2w_p} \right)^{1-s} \left(\frac{(2w_p - 1)6100 + 6000}{2w_p} \right)^s \right);$$

$$\hat{\tau}^{\max} = \left(\left(\frac{(2w_\tau - 1)13400 + 13500}{2w_\tau} \right)^{1-s} \left(\frac{(2w_\tau - 1)13600 + 13500}{2w_\tau} \right)^s \right);$$

Allowable value of $\hat{\tau}^{\max}$

$$\hat{\tau}_1^{\max} = \left(\left(\frac{(2w_{\tau_1} - 1)13500 + 13600}{2w_{\tau_1}} \right)^{1-s} \left(\frac{(2w_{\tau_1} - 1)13700 + 13600}{2w_{\tau_1}} \right)^s \right);$$

$$\hat{\delta}^{\max} = \left(\left(\frac{(2w_\delta - 1)0.24 + 0.25}{2w_\delta} \right)^{1-s} \left(\frac{(2w_\delta - 1)0.26 + 0.25}{2w_\delta} \right)^s \right);$$

Allowable value of $\hat{\delta}^{\max}$

$$\hat{\delta}_1^{\max} = \left(\left(\frac{(2w_{\delta_1} - 1)0.25 + 0.26}{2w_{\delta_1}} \right)^{1-s} \left(\frac{(2w_{\delta_1} - 1)0.27 + 0.26}{2w_{\delta_1}} \right)^s \right);$$

$$\hat{\sigma}^{\max} = \left(\left(\frac{(2w_\sigma - 1)2900 + 3000}{2w_\sigma} \right)^{1-s} \left(\frac{(2w_\sigma - 1)3100 + 3000}{2w_\sigma} \right)^s \right);$$

Allowable value of $\hat{\sigma}^{\max}$

$$\hat{\sigma}_1^{\max} = \left(\left(\frac{(2w_{\sigma_1} - 1)3000 + 3100}{2w_{\sigma_1}} \right)^{1-s} \left(\frac{(2w_{\sigma_1} - 1)3200 + 3100}{2w_{\sigma_1}} \right)^s \right);$$

Table. 2. The Upper and lower value of objective for different values of w pessimistic value of s

The pessimistic value of $s=0.2$			
Aspiration level $w_p = w_\sigma = w_\delta = w_\tau$ $= w_p = w_\sigma^1 = w_\delta^1 = w_\tau^1 = w$	$w = 0.3$	$w = 0.5$	$w = 0.7$
Upper and lower value of objective	$L_{c(x)}^\mu = 0.1264788,$ $U_{c(x)}^\mu = 0.1433088$	$L_{c(x)}^\mu = 0.1258191,$ $U_{c(x)}^\mu = 0.1426818$	$L_{c(x)}^\mu = 0.1255331,$ $U_{c(x)}^\mu = 0.1424090$

Table.3 The Upper and lower value of objective for different values of w , moderate value of value of s

The pessimistic value of $s=0.5$			
Aspiration level $w_p = w_\sigma = w_\delta = w_\tau$ $= w_p = w_\sigma^1 = w_\delta^1 = w_\tau^1 = w$	$w = 0.3$	$w = 0.5$	$w = 0.7$
Upper and lower value of objective	$L_{c(x)}^\mu = 0.1426734,$ $U_{c(x)}^\mu = 0.1438970$	$L_{c(x)}^\mu = 0.1426818,$ $U_{c(x)}^\mu = 0.1439054$	$L_{c(x)}^\mu = 0.1426802,$ $U_{c(x)}^\mu = 0.1439054$

Table.4 The Upper and lower value of objective for different values of w optimistic value of s

The pessimistic value of $s=0.8$			
Aspiration level $w_p = w_\sigma = w_\delta = w_\tau$ $= w_p = w_\sigma^1 = w_\delta^1 = w_\tau^1 = w$	$w = 0.3$	$w = 0.5$	$w = 0.7$

Upper and lower value of objective	$L_{C(x)}^\mu = 0.1420414,$ $U_{C(x)}^\mu = 0.1432617$	$L_{C(x)}^\mu = 0.1426818,$ $U_{C(x)}^\mu = 0.1439054$	$L_{C(x)}^\mu = 0.1429521,$ $U_{C(x)}^\mu = 0.1441772$
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Again when input parameters are as like as table 5

Table 5: Input data for crisp model (9)

Applied load P (lb)	Beam length beyond weld L (in)	Young Modulus E (psi)	Value of G (psi)	Maximum allowable shear stress $\tau_{\max}$ (psi)	Maximum allowable deflection $\delta_{\max}$ (in)	Maximum allowable normal stress $\sigma_{\max}$ (psi)
6000 $\equiv \left((5900, 6000, 6100; w_p), (5800, 6000, 6200; \eta_p) \right)$	14	3×10^6	12×10^6	13550 $\equiv \left((13540, 13550, 13560; w_\tau), (13530, 13550, 13570; \eta_\tau) \right)$ Maximum allowable value 13600 $\equiv \left((13500, 13600, 13700; w_\tau^1), (13400, 13600, 13800; \eta_\tau^1) \right)$	0.25 $\equiv \left((0.24, 0.25, 0.26; w_\delta), (0.23, 0.25, 0.27; \eta_\delta) \right)$ Maximum allowable value 0.26 $\equiv \left((0.25, 0.26, 0.27; w_\delta^1), (0.24, 0.26, 0.28; \eta_\delta^1) \right)$	3000 $\equiv \left((2900, 3000, 3100; w_\sigma), (2800, 3000, 3200; \eta_\sigma) \right)$ Maximum allowable value 3100 $\equiv \left((3000, 3100, 3200; w_\sigma^1), (2900, 3100, 3300; \eta_\sigma^1) \right)$

where $w_p, w_\sigma, w_\delta, w_\tau$ and $w_p^1, w_\sigma^1, w_\delta^1, w_\tau^1$ are degree of acceptance or aspiration level and maximum degree of acceptance or aspiration level of applied load, normal stress, deflection and allowable shear stress respectively.

and $\eta_p, \eta_\sigma, \eta_\delta, \eta_\tau$ and $\eta_p^1, \eta_\sigma^1, \eta_\delta^1, \eta_\tau^1$ are degree of rejection or desperation level and maximum degree of rejection or desperation level of applied load, normal stress, deflection and allowable shear stress respectively.

Now parameterized value of interval valued function can be calculated as

$$\hat{P} = \left(\left(5950 + \frac{25}{w_p} + \frac{50}{\eta_p} \right)^{1-s} \left(6050 + \frac{25}{w_p} + \frac{50}{\eta_p} \right)^s \right);$$

$$\hat{\tau}^{\max} = \left(\left(13545 + \frac{2.5}{w_\tau} + \frac{5}{\eta_\tau} \right)^{1-s} \left(13555 + \frac{2.5}{w_\tau} + \frac{50}{\eta_\tau} \right)^s \right);$$

Allowable value of $\hat{\tau}^{\max}$

$$\hat{\tau}_1^{\max} = \left(\left(13550 + \frac{25}{w_{\tau_1}} + \frac{50}{\eta_{\tau_1}} \right)^{1-s} \left(13650 + \frac{25}{w_{\tau_1}} + \frac{50}{\eta_{\tau_1}} \right)^s \right);$$

$$\hat{\delta}^{\max} = \left(\left(0.145 + \frac{.0025}{w_{\delta}} + \frac{.005}{\eta_{\delta}} \right)^{1-s} \left(0.255 + \frac{.0025}{w_{\delta}} + \frac{.005}{\eta_{\delta}} \right)^s \right);$$

Allowable value of $\hat{\delta}^{\max}$

$$\hat{\delta}_1^{\max} = \left(\left(0.255 + \frac{.0025}{w_{\delta_1}} + \frac{.005}{\eta_{\delta_1}} \right)^{1-s} \left(0.265 + \frac{.0025}{w_{\delta_1}} + \frac{.005}{\eta_{\delta_1}} \right)^s \right);$$

$$\hat{\sigma}^{\max} = \left(\left(2950 + \frac{25}{w_{\sigma}} + \frac{50}{\eta_{\sigma}} \right)^{1-s} \left(3050 + \frac{25}{w_{\sigma}} + \frac{50}{\eta_{\sigma}} \right)^s \right);$$

Allowable value of $\hat{\sigma}^{\max}$

$$\hat{\sigma}_1^{\max} = \left(\left(3050 + \frac{25}{w_{\sigma_1}} + \frac{50}{\eta_{\sigma_1}} \right)^{1-s} \left(3150 + \frac{25}{w_{\sigma_1}} + \frac{50}{\eta_{\sigma_1}} \right)^s \right);$$

Table. 6 The upper and lower value of objective for different values of w, η
pessimistic value of s

The pessimistic value of $s=0.2$			
Aspiration level $w_p = w_{\sigma} = w_{\delta} = w_{\tau}$ $= w_{\sigma}^1 = w_{\delta}^1 = w_{\tau}^1 = w$ and Desperation level $\eta_p = \eta_{\sigma} = \eta_{\delta} = \eta_{\tau}$ $= \eta_{\sigma}^1 = \eta_{\delta}^1 = \eta_{\tau}^1 = \eta$	$w = 0.3, \eta = 0.6$	$w = 0.5, \eta = 0.5$	$w = 0.7, \eta = 0.2$

Upper and lower value of objective	$L_{c(x)}^{\mu} = 0.1448289,$ $U_{c(x)}^{\mu} = 0.1466027$	$L_{c(x)}^{\mu} = 0.1445683,$ $U_{c(x)}^{\mu} = 0.1461895$	$L_{c(x)}^{\mu} = 0.1466777,$ $U_{c(x)}^{\mu} = 0.1495614$
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Table.7 The upper and lower value of objective for different values of w, η , moderate value of s

The moderate value of $s=0.5$			
Aspiration level $w_p = w_{\sigma} = w_{\delta} = w_{\tau}$ $= w_{\sigma}^1 = w_{\delta}^1 = w_{\tau}^1 = w$ and Desperation level $\eta_p = \eta_{\sigma} = \eta_{\delta} = \eta_{\tau}$ $= \eta_{\sigma}^1 = \eta_{\delta}^1 = \eta_{\tau}^1 = \eta$	$w = 0.3, \eta = 0.6$	$w = 0.5, \eta = 0.5$	$w = 0.7, \eta = 0.2$
Upper and lower value of objective	$L_{c(x)}^{\mu} = 0.1452953,$ $U_{c(x)}^{\mu} = 0.1468469$	$L_{c(x)}^{\mu} = 0.1450354,$ $U_{c(x)}^{\mu} = 0.1464910$	$L_{c(x)}^{\mu} = 0.1493870,$ $U_{c(x)}^{\mu} = 0.1471385$

Table.8 The upper and lower value of objective for different values of w, η optimistic value of s

The optimistic value of $s=0.8$			
Aspiration level $w_p = w_{\sigma} = w_{\delta} = w_{\tau}$ $= w_{\sigma}^1 = w_{\delta}^1 = w_{\tau}^1 = w$ and Desperation level	$w = 0.3, \eta = 0.6$	$w = 0.5, \eta = 0.5$	$w = 0.7, \eta = 0.2$

$\eta_p = \eta_\sigma = \eta_\delta = \eta_\tau$ $= \eta_\sigma^1 = \eta_\delta^1 = \eta_\tau^1 = \eta$			
Upper and lower value of objective	$L_{c(x)}^\mu = 0.1457635,$ $U_{c(x)}^\mu = 0.1470916$	$L_{c(x)}^\mu = 0.1455044,$ $U_{c(x)}^\mu = 0.1467933$	$L_{c(x)}^\mu = 0.1476010,$ $U_{c(x)}^\mu = 0.1492130$

Now using membership and non membership function as mentioned in section 5 fuzzy and intuitionistic optimization problem can be formulated as similar as (9) and (10) and solving these for different values of s, w, τ can be obtained as follows.

Table.9 The optimum values of design variables for different values of s, w, η

The pessimistic value of $s=0.2$						
Value of $\varepsilon_1, \varepsilon_2,$ $w_p = w_\sigma = w_\delta = w_\tau$ $= w_\sigma^1 = w_\delta^1 = w_\tau^1 = w$ and Desperation level $\eta_p = \eta_\sigma = \eta_\delta = \eta_\tau$ $= \eta_\sigma^1 = \eta_\delta^1 = \eta_\tau^1 = \eta$	$w = 0.3$	$w = 0.3,$ $\eta = 0.6$ $\varepsilon_i =$ $(U_i - L_i) \times 0.1$	$w = 0.5,$	$w = 0.5,$ $\eta = 0.5$ $\varepsilon_i =$ $(U_i - L_i) \times 0.1$	$w = 0.7,$	$w = 0.7,$ $\eta = 0.2$ $\varepsilon_i =$ $(U_i - L_i) \times 0.1$
$x_1(in)$	0.3230761	0.3230720	0.3226488	0.3228958	0.3224620	0.3243104
$x_2(in)$	0.9823865	0.9839329	0.9800923	0.9829856	0.9790931	0.9906277
$x_3(in)$	1.806978	1.814461	1.802845	1.814461	1.801052	1.828308
$x_4(in)$	0.6244300	0.6246603	0.6245602	0.6246603	0.6246173	0.6242285
$C(X) (\$)$	0.1433088	0.1461895	0.1426818	0.1461895	0.1424090	0.1495614

Where U_i and L_i are upper and lower bound of respective objective and constraints

Table. 10 The optimum values of design variables for different values of s, w, η

The pessimistic value of $s=0.5$

Value of $\varepsilon_1, \varepsilon_2$, $w_p = w_\sigma = w_\delta = w_\tau$ $= w_\sigma^1 = w_\delta^1 = w_\tau^1 = w$ and Desperation level $\eta_p = \eta_\sigma = \eta_\delta = \eta_\tau$ $= \eta_\sigma^1 = \eta_\delta^1 = \eta_\tau^1 = \eta$	$w = 0.3$	$w = 0.3$, $\eta = 0.6$ $\varepsilon_i =$ $(U_i - L_i) \times 0.1$	$w = 0.5$	$w = 0.5$, $\eta = 0.5$ $\varepsilon_i =$ $(U_i - L_i) \times 0.1$	$w = 0.7$	$w = 0.7$, $\eta = 0.2$ $\varepsilon_i =$ $(U_i - L_i) \times 0.1$
$x_1(in)$	0.3216023	0.3233862	0.3216085	0.3232112	0.3216074	0.3246158
$x_2(in)$	0.9760721	0.9856260	0.9761017	0.9846832	0.9760963	0.9922898
$x_3(in)$	1.802017	1.819238	1.802058	1.817532	1.802050	1.831334
$x_4(in)$	0.6250552	0.6245107	0.6250529	0.6245641	0.6250534	0.6241353
$C(X) (\$)$	0.1426734	0.1468469	0.1426818	0.1464910	0.1426802	0.1493870

Where U_i and L_i are upper and lower bound of respective objective and constraints.

Table.11 The optimum values of design variables for different values of s, w, η

The pessimistic value of $s=0.8$						
Value of $\varepsilon_1, \varepsilon_2$, $w_p = w_\sigma = w_\delta = w_\tau$ $= w_\sigma^1 = w_\delta^1 = w_\tau^1 = w$ and Desperation level $\eta_p = \eta_\sigma = \eta_\delta = \eta_\tau$ $= \eta_\sigma^1 = \eta_\delta^1 = \eta_\tau^1 = \eta$	$w = 0.3$	$w = 0.3$, $\eta = 0.6$ $\varepsilon_i =$ $(U_i - L_i) \times 0.1$	$w = 0.5$	$w = 0.5$, $\eta = 0.5$ $\varepsilon_i =$ $(U_i - L_i) \times 0.1$	$w = 0.7$	$w = 0.7$, $\eta = 0.2$ $\varepsilon_i =$ $(U_i - L_i) \times 0.1$
$x_1(in)$	0.3211664	0.3237005	0.3216085	0.3235267	0.3217943	0.3249214
$x_2(in)$	0.9737554	0.9873225	0.9761018	0.9863837	0.9770912	0.9939550
$x_3(in)$	1.797855	1.822309	1.802057	1.820608	1.803837	1.834365
$x_4(in)$	0.6251880	0.6244147	0.6250529	0.6244678	0.6249963	0.6240418
$C(X) (\$)$	0.1420414	0.1470916	0.1426818	0.1467933	0.1429521	0.1492130

Where U_i and L_i are upper and lower bound of respective objective and constraints

From the above results it is clear whenever we chose fuzzy number and intuitionistic number for imprecise load and stress, deflection in welded beam design model, cost of welding is more optimized in fuzzy environment than the case of consideration of intuitionistic fuzzy environment.

7. CONCLUSIONS

In this paper, fuzzy and IFO approach have been used to solve IFNLPP with intuitionistic fuzzy resources as well as intuitionistic fuzzy numbers as coefficients which may be considered as triangular fuzzy and triangular intuitionistic fuzzy numbers. The proposed method is utilized with linear membership function to a welded beam design. At first step triangular fuzzy number and intuitionistic fuzzy number has been transformed into an interval number by interval approximation method. After that parametric interval valued functional form are created from that interval numbers so that the solution of the undertaken numerical problem can be solved by fuzzy and intuitionistic fuzzy optimization method given by Angelov [21]. As it has been seen that there is a few method in literature for solving IFNLP and the proposed method has been applied to a fully IFNLP problem so the method will be very beneficial and applicable for solving NLPP arising in other field of engineering in intuitionistic fuzzy environment .

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