Statistical Analysis of Cropland Area in Canada using the Autoregressive Hidden Markov Time Series Model

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Abstract: - Crop production and other agricultural activities are as old as human existence and becoming increasingly intensive, spatially concentrated and specialized. However, diversification in economic activities and recent development in technology in many developed countries have led to significant increase in land use. Thereby, resulting to huge reduction in the total land area available for agricultural activities especially crop production. This study examines the distribution of cropland area in Canada in relation to three contributing factors using the Autoregressive Hidden Markov time series Model (AR-HMM) due to the limitations of the ordinary Autoregressive model in the accuracy of its parameter estimation. Expectation-Maximization (E-M) algorithm method was used to estimate the model parameters so as to investigate the effects of the factors on cropland distribution using secondary data from Food and Agriculture Organisation (FAO). Jarque-Bera and D'Agostino normality tests were carried out to examine the normality of the series. Augmented Dickey Fuller (ADF) and the KPSS tests established the stationarity of the series. The ideal stationary probability distribution for transition was at AR (3)-HMM with the minimum Bayesian Information Criterion (BIC) of 16270.62. The prior transition states for the HMM are 0.462, 0.260 and 0.278 respectively. In conclusion, this study suggests that deforestation and other land use activities as a result of commercial and technological advancements should be minimized to ensure more available cropland area.

Key-Words: - Crop production, Contributing factors, cropland area, Hidden Markov Model (HMM), Bayesian Information, Criterion (BIC)

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1 Introduction

Agricultural development is becoming increasingly rigorous, spatially concentrated and more specialized [4]. Growth and increase in use of off-farm inputs such as machinery, chemical substances, crossbreeds and technological advancement have helped in achieving higher per hectare output and intensive capital as a substitute for labour. Farmers who are unable to employ these new technologies or whose land do not have necessary nutrient requirements usually experience low output in terms of production and income generation and may eventually go out of business as over time. [13] identifies a strong causal relationship between the use of modern agricultural inputs and crop yields and subsequently, yields and economic growth. Extensive empirical research on the causes, effects and implications of agricultural restructuring in

Canada has been examined, which includes identification of the challenges of agriculture in physically marginal areas in the 1950s and 1960s, according to [5]. As a good guide to this study, [18] also discussed Policy distortions, farm size, and the overuse of agricultural chemicals in China stating that reduction in the use of agricultural chemicals to a peak level is a critical challenge for the sustainable agricultural development. Moreover, further studies like [6] and [15] discussed extensively on urban expansion into the countryside and rural sustainability.

We should also recall that some of the carbon dioxide released into the atmosphere by fossil fuels combustion would be taken in by plant. [1] discussed extensively on the influence of agricultural land use and economic growth on nitrous oxide emissions with evidence from developed and developing countries. Hence,

increasing deforestation and other changes in land use would lead to reduction in the carbon dioxide concentration capacity of these areas over time. Thereby resulting to huge reduction in the total area of land available for agricultural activities especially crop production. Changes in the amount of atmospheric CO₂ absorption are some of the ripple effects of gaseous discharges from fossil fuel combustion, production of cement, and carbon interchange between the seas and terrestrial organisms. Variations in land use and land-cover usually influence the amount of carbon available to terrestrial environment (e.g., [9], [10], and [17]) and influence the dispersal of terrestrial carbon sources and sinks ([7] and [8]).

Greater availability of CO₂ usually increases photosynthesis, which potentially hinders the growth plant. It declines the reduction that crops lose through transpiration. In general, increase in temperatures across the globe will definitely bring about increase in daily rate of evaporation and annual evaporation levels.

As marginal producers become disadvantaged and find it difficult to compete in global markets, decline in agricultural output in those areas where they are clustered is inevitable. Technological developments greatly play a major role in this process as maximum productivity income occurs in areas with level topography and moderate climate.

2 Methodology

2.1 Hidden Markov Models (HMM)

This is a class of probabilistic time series models that makes it possible for us to predict a group of unidentified (hidden) variables in a sequence of observed variables. The concepts and theories behind HMM were presented by [2] as well as [3]. HMMs have become increasingly popular in various statistical applications as a result of their usefulness and practical mathematical structure; [11] and [12].

The two major assumptions of HMM are as follows:

1. Markov Assumption: This states that the probability of generating the next state at any time t , is dependent on the current state only which implies that;

$$P(X_{t+1}|X_t, X_{t-1}, \dots, X_0) = p(X_{t+1}|X_t) \text{ for all } t \quad (1)$$

2. Independency Assumption: The likelihood of obtaining current observation symbol depends only on the current state. That is;

$$P(Y|X, \lambda) = \prod_{t=1}^T P(Y_t|X_t, \lambda), \quad (2)$$

in which $Y = \{Y_1, Y_2, \dots, Y_T\}$ and $X = \{X_1, X_2, \dots, X_T\}$ denote the observation sequence and state sequence, respectively.

Sometimes, a set of time series may comprise observations generated by different mechanisms at different intervals of time. In such situation, the observations of the series would behave in a switching to and fro manner between a set of different states. During the state to state transition, the series may experience a drastic change in means, frequencies or breadths. The Autoregressive HMM (ARHMM) are usually employed to address this type of behavior in the series.

2.1.1 Expected-Maximization (E-M) Algorithm

This algorithm was initially proposed to address the issue of missing observations in the analysis of time series data set. It involves using α , and β , the forward and backward probabilities obtained from the forward-backward algorithm and its derivation presented by [14]. The unidentified or unknown states in the HMM could be taken as missing observations in the E-M algorithm. Given a set of observation sequence Y , the unknown state sequence X and the parameter set $\lambda = [\pi, A, B]$. We shall maximize equation (2) by choosing appropriate λ , according to [16]. Mathematically, the observation probability in (2) corresponds to the likelihood function of Y with unknown parameter set λ as stated

$$P(Y|\lambda) = \int_X P(Y, X|\lambda) \quad \text{in;} \\ = \sum_{X_1=1}^N \sum_{X_2=1}^N \dots \sum_{X_T=1}^N P(Y, X_1, \dots, X_T|\lambda) \quad (3)$$

which gives the observation likelihood in terms of $P(Y, X|\lambda)$.

It would be necessary to specify another function for the expected log-likelihood, $Q(\lambda_{t+1}; \lambda_t, Y)$ where the log-likelihood is denoted by λ_{t+1} and the expectation taken wrt λ_t , a new parameter set :

$$Q(\lambda_{t+1}; \lambda_t, Y) = \int_X \log(P(Y, X|\lambda_{t+1}))P(Y, X|\lambda_t) \quad (4)$$

The entire procedure usually begins with an initial estimate of parameter set $\hat{\lambda}_0$. Next, $\hat{\lambda}_{l+1}$ ($l = 0, 1, \dots$) would be solved iteratively until

the function that gives the maximum value for $Q(\lambda_{l+1}; \lambda_l, Y)$ is obtained as follows;

$$\int_X \frac{\partial \log P(Y, X | \lambda_{l+1})}{\partial \lambda_{l+1}} \Big|_{\lambda_{l+1} = \hat{\lambda}_{l+1}} \bullet Q(\lambda_{l+1}; \lambda_l, Y) \quad (5)$$

2.1.2 E-M Formula for ARHMM

We could recall that the Autoregressive (AR) structure of the observation vectors could be defined as:

$$Y_t = S_t' \beta^{(X_t)} + \varepsilon_t \quad (6)$$

where;

$$S_t = (1, Y_{t-1}, Y_{t-2}, \dots, Y_{t-p})$$

$$\beta^{(X_t)} = (\beta_1^{(X_t)}, \beta_2^{(X_t)}, \dots, \beta_p^{(X_t)})$$

$$\varepsilon_t \sim i.i.d \quad N(0, \sigma^2).$$

Hence, the conditional probability density function of Y_t is given by:

$$P(Y_t | Z_t; \lambda) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(Y_t - S_t' \beta^{(X_t)})^2}{2\sigma^2}\right] \quad (7)$$

To obtain the precise estimation formula, we differentiate (7) wrt β_j and σ^{-2}

$$\frac{\partial \log P(Y_t | Z_t; \theta)}{\partial \beta_j} = \begin{cases} \frac{(Y_t - S_t' \beta_j) S_t}{\sigma^2}, & X_t = j \\ 0, & \text{otherwise} \end{cases}$$

$$\frac{\partial \log P(Y_t | Z_t; \theta)}{\partial \sigma^{-2}} = \begin{cases} \frac{\sigma^2}{2} - \frac{(Y_t - S_t' \beta_j)^2}{\sigma^2}, & X_t = j \\ 0, & \text{otherwise} \end{cases}$$

Then,

$$\sum_{t=p+1}^T \frac{(Y_t - S_t' \beta_j^{(l+1)}) S_t}{\sigma_{(l+1)}^2} P(X_t = j | Y; \lambda_l) = 0 \quad (8)$$

$$\sum_{t=p+1}^T \sum_{j=1}^N \left[\frac{\sigma_{(l+1)}^2}{2} - \frac{(Y_t - S_t' \beta_j^{(l+1)})^2}{2} \right] P(X_t = j | Y; \lambda_l) = 0 \quad (9)$$

The estimate of $\beta_j^{(l+1)}$ which gives the solution to (8) can be obtained using the Ordinary Least Square (OLS) regression of $\tilde{Y}_t(j)$ on $\tilde{S}_t(j)$:

$$\beta_j^{(l+1)} = \left[\sum_{t=p+1}^T [\tilde{S}_t(j)] [\tilde{S}_t(j)]' \right]^{-1} \left[\sum_{t=p+1}^T [\tilde{S}_t(j)] Y_t(j) \right] \quad (10)$$

where;

$$\tilde{Y}_t(j) = Y_t \sqrt{P(X_t = j | Y; \lambda_l)}$$

$$\tilde{S}_t(j) = S_t \sqrt{P(X_t = j | Y; \lambda_l)}$$

This implies that the estimation of $\sigma_{(l+1)}^2$ is:

$$\sigma_{(l+1)}^2 = \sum_{t=p+1}^T \sum_{j=1}^N \frac{(\tilde{Y}_t(j) - \tilde{S}_t(j) \beta_j^{(l+1)})^2}{T - p} \quad (11)$$

The estimate of the transition probabilities $A = [a_{ij}]$ and $\pi = [\pi_j]$, the initial probabilities are given by:

$$a_{ij}^{(l+1)} = \frac{\sum_{t=p+1}^T P(X_t = j, X_{t-1} = i | Y; \lambda_l)}{\sum_{t=p+1}^T P(X_{t-1} = j | Y; \lambda_l)} \quad (12)$$

and

$$\pi_j^{(l+1)} = P(X_p = j | Y; \lambda_l), j = 1, 2, \dots, N-1$$

$$\pi_N^{(l+1)} = 1 - \pi_1^{(l+1)} - \pi_2^{(l+1)} - \dots - \pi_{N-1}^{(l+1)} \quad (13)$$

2.1.3 The Smoothed Probabilities

Each iteration of the re-estimation equation in the previous section would involve the calculation of the smoothed probabilities which include $P(X_t, X_{t-1} | Y)$ and $P(X_t | Y)$. When it comes to ARHMM, procedures for the iteration and calculation of the smoothed probability $P(X_t, X_{t-1}, \dots, X_{t-p} | Y)$ are as follows, where p represents degree of the Autoregressive (AR) process as usual.

1. The initial stage requires the initialization of;

$$P(Y_{p+1} | Y^{(p)}) = \sum \sum \dots \sum P(X_{p+1} | X_p) \cdot P(Y_{p+1} | Z_{p+1}) \pi_{X_p, \dots, X_1} \quad (14)$$

and

$$P(X_{p+1}, \dots, X_1 | Y^{(p+1)})$$

$$= \frac{P(X_{p+1} | X_p) P(Y_{p+1} | Z_{p+1}) \pi_{X_p, \dots, X_1}}{P(Y_{p+1} | Y^{(p)})} \quad (15)$$

where: $Y^{(t)} = (Y_1, Y_2, \dots, Y_t)'$

$$Z_t = \{X_t, X_{t-1}, \dots, X_{t-p}, Y_{t-1}, Y_{t-2}, \dots, Y_{t-p}\}'$$

$$\pi_{X_p, X_{p-1}, \dots, X_1} = P(X_p, X_{p-1}, \dots, X_1 | Y)$$

2. Estimate $P(Y_t | Y^{(t-1)})$ and

$P(X_t, X_{t-1}, \dots, X_{t-p} | Y^{(t)})$ for $t = p+2, \dots, T$ using;

$$P(Y_t|Y^{(t-1)}) = \sum_{X_t=1}^N \sum_{X_{t-1}=1}^N \sum_{X_{t-p-1}=1}^N P(X_t|X_{t-1})P(Y_t|Z_t)P(X_{t-1},...,X_{t-p-1}|Y^{(t-1)}) \quad (16)$$

$$P(X_t, ..., X_{t-p}|Y^{(t)}) = \frac{\sum_{X_{t-p-1}=1}^N P(X_t|X_{t-1})P(Y_t|Z_t)P(X_{t-1}, ..., X_{t-p-1}|Y^{(t-1)})}{P(Y_t|Y^{(t-1)})} \quad (17)$$

3. Calculate the advanced probability for $\tau = t+1, t+2, ..., t+p$; for a given t , as follows;

$$P(X_\tau, ..., X_{t-p}|Y^{(\tau)}) = \frac{P(X_\tau|X_{\tau-1})P(Y_\tau|Z_\tau)P(X_{\tau-1}, ..., X_{t-p}|Y^{(\tau-1)})}{P(Y_\tau|Y^{(\tau-1)})} \quad (18)$$

4. The inference for $\tau = t+p+1, t+p+2, ..., P$ brought forward as;

$$P(X_\tau, ..., X_{t-p}, X_t, ..., X_{t-p}|Y^{(\tau)}) = \frac{\sum_{X_{t-p-1}=1}^N P(X_\tau|X_{\tau-1})P(Y_\tau|Z_\tau)P(X_{\tau-1}, ..., X_{t-p-1}, X_t, ..., X_{t-p}|Y^{(\tau-1)})}{P(Y_\tau|Y^{(\tau-1)})} \quad (19)$$

5. Finally, we would sum up the last p states;

$$P(X_t, X_{t-1}, ..., X_{t-p}|Y) = \sum_{X_T=1}^N \sum_{X_{T-1}=1}^N \cdots \sum_{X_{T-p}=1}^N P(X_T, ..., X_{T-p}, X_t, ..., X_{t-p}|Y^{(T)}) \quad (20)$$

In general, the total number of steps expected to go through during the algorithm is of order $N^{2(p+1)}T^2$ due to the fact that N and p are usually fairly small.

3 Results and Discussion

3.1 The Time plot for the factors

3.2 Descriptive Statistics and analysis

Fig.1: The Time plots for Implied emission factor for (in tonnes C/ha) and Net stock change (in gigagrammes) and Net emissions/removal (CO₂) (in gigagrammes) respectively.

From the time plots in fig.1, the implied emission factor measured (in tonnes C/ha) started with the highest level of 4.9631 in 1990 in contributing to the Area (ha) of land. However, from 1990 to around 1992, there was a drastic continual drop of implied emission factor to a limit point of 4.9615, before a continual increment in contributing to emission factor back to an apex point of 4.9631. It continued in zigzag bounded phenomena of (4.9631, 4.9615) until it reaches 2016. In a similar manner, the net stock change (in gigagrammes) started with an upper limit bound of 5350 in gigagrammes between 1991 and 1992 and gradually geared down its quality to reach a value of 5290. Similarly to implied emission factor, the net stock changed in a zigzag manner and bounded by via (5350, 5290), but with a based factor around 5315.

In a similar vein, the net emission (CO₂) measured in gigagrammes started its contribution to the area of land via an upper limit bound of 16900 in gigagrammes between 1991 and 1992, and gradually geared down its quality to reach point 16400. Similarly to the net stock change, the net emission (CO₂) in a zigzag manner and bounded by via (16900, 16400), but with a based factor around 16480.

Table 1. Stationary Tests, Skewed Coefficient of Implied emission factor for implied emission (in tonnes C/ha)

Indexes	Min.	1 st Qu.	Median	Mean	3 rd Qu.	Max.	S.E	skewness	kurtosis
Estimates	4.961	4.961	4.963	4.962	4.963	4.963	0.0007	-0.5131	-1.5519
Tests	Jarque-Bera Normality Test		D'Agostino skewness test		Augmented Dickey Fuller.test		Phillips-Perron Unit Root Test		Box-Pierce test
Coefs	3.894		-1.2509		-2.2078		-4.5385		23.488
P-Values	0.1427		0.211		0.4931		0.8415		1.257e-06

The minimum and maximum emitted factor in tonnes (C/ha) that contributed to the Canadian area of land mass were 4.961 and 4.963 respectively over the twenty-seven (27) years. However, the range is 0.002, while the emitted factor for improving the area of hectares was clustered around 4.962. The variation among the emitted factors in contribution to the area was estimated to be 0.0007 (0.07%), which suggested a lesser (in fact a negligible Coefficient of Variation) dispersed variation among the factors. The value of the median conjoined with the value of the mean approximately, which indicated a normally formed whisker in the QQ-plot, but might be of leptokurtic or platykurtic. In addition, since the kurtosis, -1.5519 is less than 3, the distribution of the emitted factors has a lighter tails (less in the tails) than the Gaussian. The skewness is -0.5131. In this case, the Left Hand (LH) tail is usually longer than the Right Hand (RH) tail, but if skewness lies between -1 and -0.5 or 0.5 and 1, the series is said to be moderately skewed. The Augmented Dickey Fuller (ADF) test value is -2.2078 and the corresponding one-sided P-value is 0.4931, it is to be noted that the level of significance (critical values) is greater than the P-value. So, there is no sufficient proof to reject the Null hypothesis of trend stationarity. The KPSS Test for level of unit root test statistic value is -1.2691 and the associated

one-sided P-value = 0.01, revealed that there is constancy in the variance in the long run (homoscedasticity via residual error), that is, no problem of heteroscedasticity. While the Phillips-Perron unit root test questioned the accuracy of to be estimated or modified parameters via Autoregressive alone because P-value = 0.8415 is greater than 5% level of significant.

Table 2. Stationary Tests, Skewed Coefficient of Net stock change (in gigagrammes)

Indexes	Min.	1 st Qu.	Median	Mean	3 rd Qu.	Max.	S.E	Skewness	kurtosis
Estimates	5290	5306	5318	5316	5318	5349	14.6553	0.3627	-0.1904
Tests	Jarque-Bera Normality Test		D'Agostino skewness test		Augmented Dickey Fuller.test		Phillips-Perron Unit Root Test		Box-Pierce test
							KPSS Test for Level Stationarity		

Coefs	3.3846	2.6031	-7.3542	-67.546	0.0199	242.31
P-Values	0.2819	0.0092	0.01	0.01	0.1	< 2.2e-16

From table 2 showing the summary for net stock change, the minimum and maximum net stock change (in gigagrammes) that contributed to the Canadian area of land mass were 5290 and 5349 respectively over the twenty-seven (27) years. However, the range is 59, while the net stock changes for improving the area of hectares was clustered around 5316. The variation among the net stock changes in contribution to the area was estimated to be 14.6553 (0.146%), which still suggested a moderately dispersed variation among the factors. Contrarily, the value of the median (5318) was non-conjoined with the value of the mean (5316), but the median (5318) coincide with the maximum value of the net stock changes. This indicated a slight deviation from the normally formed whisker in the QQ-plot, but might be of leptokurtic or platykurtic. In addition, since the kurtosis, -0.1904 is less than 3; the distribution of the net stock changes has a lighter tails (less in the tails) than the Gaussian. The skewness is 0.3627. In this situation, the LH tail is longer than the RH tail, but the rule of thumb says if skewness lies between -1 and -0.5 or between 0.5 and 1, then there exists moderately skewed set of data. The ADF statistic value is -7.3542 and the corresponding one-sided P-value is 0.01, it is to be noted that the critical value is greater than the P-value. Hence, no satisfactory evidence for the rejection of the Null hypothesis of trend stationarity. The KPSS Test for level of unit root test statistic value is 0.0199 and the associated one-sided P-value=0.1, concluded that there is

constancy in the variance in the long run (homoscedasticity via residual error), that is, no problem of heteroscedasticity. While the Phillips-Perron unit root test put hold, the question of the accuracy of to be estimated or modified parameters via Autoregressive alone because P-value=0.01 is less than 5% level of significance.

Table 3. Stationary Tests, Skewed Coefficient of Net emissions/removals (CO₂) (in gigagrammes)

Indexes	Min.	1 st Qu.	Median	Mean	3 rd Qu.	Max.	S.E	skewness	Kurtosis
Estimate	19400	19450	19500	19490	19500	19610	54.8168	0.3892	-0.1916

Tests	Jarque-Bera Normality Test	D'Agostino skewness test	Augmented Dickey Fuller.test	Phillips-Perron Unit Root Test	KPSS Test for Level Stationarity	Box-Pierce test
Coefs	0.7230	0.96093	-3.3338	-6.7733	0.4352	21.128
P-Values	0.6966	0.3366	0.0866	0.6923	0.0619	4.296e-06

From table 3, the minimum and maximum net emissions/removals (CO_2) in gigagrammes that contributed to the Canadian area of land mass were 19400 and 19610 respectively over the twenty-seven (27) years. However, the range is 210, while the net emissions/removals (CO_2) for improving the area of hectares was clustered around 4.962. The variation among the net emissions/removals (CO_2) in contribution to the area was estimated to be 54.8168(0.548%), which suggested an alarming (above average) dispersed variation among the removals.

Contrarily, the value of the median (19500) was non-conjoined with the value of the mean (19490), which indicated a non- normally formed whisker in the QQ-plot, but might be of leptokurtic or platykurtic. In addition, since the kurtosis, -0.1916 is less than 3, the distribution of the emitted factors has a lighter tails (less in the tails) than the Gaussian. The skewness is 0.3892. In this case, the LH tail is also longer than the RH, but according to the rule of thumb, if the skewness ranges between -1 and -0.5 or between 0.5 and 1, this implies the existence of moderately skewed data. The ADF statistic value is -3.3338 and the corresponding one-sided P-value is 0.0866. It should be noted that the critical value is greater than the P-value. Hence, we do not have substantial evidence to justify the acceptance of the Null hypothesis of trend stationarity. The KPSS Test for level of unit root test statistic value is 0.4352 and the associated one-sided P-value=0.0619, concluded that there is no constancy in the variance in the long run (homoscedasticity via residual error), that is, constancy of variance assumption has been violated. While the Phillips-Perron unit root test questioned the accuracy of to be estimated or modified parameters via Autoregressive alone because P-value = 0.6923 is greater than 5% level of significant.

Fig. 2: The Cluster Density for Implied emission factor for (in tonnes C/ha) Net stock change (in gigagrammes) and Net emissions/removal (CO_2) (in Gigagrammes).

From Fig.2, the density normal plots for implied emission factor for (in tonnes C/ha) and Net stock change (in gigagrammes) are not just single density but densities are embedded has switching. Nine (9) and four (4) switchings were recorded for implied emission factor and net stock change respectively and four (4) switchings for net emissions/removal (CO_2). This disparity in the number of switchings reveals that the series of the three agents of area changes changed not only cyclically but also via time varying. So, ordinary Autoregressive models would not be suitable enough to obtain the true estimates of the parameters. Therefore, an AR-HMM Model would be used to generalize the changes of these factors.

Table 4. Coefficients of Autoregressive Hidden Markov Model

Parameters	AR(2)-HMM	AR(3)-HMM
ϕ_i	$\begin{bmatrix} 0.417 & -0.537 \\ 0.286 & -0.648 \end{bmatrix}$	$\begin{bmatrix} 0.829 & -0.143 & 0.282 \\ 0.023 & -0.060 & -0.319 \\ 0.807 & 0.367 & -0.878 \end{bmatrix}$
μ	$\begin{bmatrix} 127850 \\ 388044.6 \end{bmatrix}$	$\begin{bmatrix} 34430.19 \\ 379735.61 \\ 756546.73 \end{bmatrix}$
π_{prior}	$\begin{bmatrix} 0.624 \\ 0.376 \end{bmatrix}$	$\begin{bmatrix} 0.462 \\ 0.260 \\ 0.278 \end{bmatrix}$
σ	$\begin{bmatrix} 18136.36 \\ 3025621.05 \end{bmatrix}$	$\begin{bmatrix} 2383.113 \\ 4655.277 \\ 1738561.373 \end{bmatrix}$
TM(Σ)	$\begin{bmatrix} \text{Re } g1 & \text{Re } g1 \\ 0.879 & 0.121 \\ 0.213 & 0.787 \end{bmatrix}$	$\begin{bmatrix} 7.5e-01 & 0.158 & 0.083 \\ 3.8e-01 & 0.356 & 0.256 \\ 1.06e-10 & 0.410 & 0.590 \end{bmatrix}$
BIC	18110.47	16270.62
LogL	-9026.65	-5084.702

Parameters	AR(4)-HMM	AR(5)-HMM
ϕ_i	$\begin{bmatrix} 0.829 & -0.143 & 0.282 \\ 0.023 & -0.060 & -0.319 \\ 0.807 & 0.367 & -0.878 \end{bmatrix}$	$\begin{bmatrix} 0.829 & -0.143 & 0.282 \\ 0.023 & -0.060 & -0.319 \\ 0.807 & 0.367 & -0.878 \end{bmatrix}$
μ	$\begin{bmatrix} 34430.19 \\ 379735.61 \\ 756546.73 \end{bmatrix}$	$\begin{bmatrix} 34430.19 \\ 379735.61 \\ 756546.73 \end{bmatrix}$
π_{prior}	$\begin{bmatrix} 0.462 \\ 0.260 \\ 0.278 \end{bmatrix}$	$\begin{bmatrix} 0.462 \\ 0.260 \\ 0.278 \end{bmatrix}$
σ	$\begin{bmatrix} 2383.113 \\ 4655.277 \\ 1738561.373 \end{bmatrix}$	$\begin{bmatrix} 2383.113 \\ 4655.277 \\ 1738561.373 \end{bmatrix}$
TM(Σ)	$\begin{bmatrix} 7.5e-01 & 0.158 & 0.083 \\ 3.8e-01 & 0.356 & 0.256 \\ 1.06e-10 & 0.410 & 0.590 \end{bmatrix}$	$\begin{bmatrix} 7.5e-01 & 0.158 & 0.083 \\ 3.8e-01 & 0.356 & 0.256 \\ 1.06e-10 & 0.410 & 0.590 \end{bmatrix}$
BIC	16276.62	16276.62
LogL	-6260.39	-6260.39

The ideal stationary probability distribution (log-likelihood) for transition is at AR(3)-HMM. Similarly, its BIC with the smallest value also confirms it. The stationary probability distribution (log-likelihood) always goes for the model with the highest log-likelihood value. Obviously, the log-likelihood (-5084.702) for AR(3)-HMM has the highest stationary probability distribution when

compared with the pool (-9026.65, -6260.39, -6260.39) for AR(2)-HMM, AR(4)-HMM and AR(5)-HMM respectively. It is to be noted that there was even state (limiting distribution) for the area using the three factors. This implies that at AR (4)-HMM and AR (5)-HMM, regimes switching and coefficients of AR models continue to be the same and lag of AR would not exceed lag (3) despite

indication of higher lag. The values in the stationary probability distribution vector are equivalent to the frequencies of occurrence of the recorded three states using AR (3)-HMM in the long run. The three elements prior transition state for the HMM are 0.462, 0.260 and 0.278 respectively. Hence by HMM prediction, there would be a change of state from one state to another for 46.2% of the time and the entire time of being in the abnormal state (0.260) and being in the normal state (0.278) would be approximately same in the long-run. In addition, the variations of contributing factors to cropland area are heavily dispersed with values 2383.113, 2774655 and 1738561.373 respectively. It was also noted that the coefficients of AR (3) for the AR (3)-HMM are stationary (that is, less than one).

$$Y_t = 34430.19^{X_1} + 379735.61^{X_2} + 756546.73^{X_3} \\ + 0.829X_{t-1} - 0.060X_{t-2} - 0.878X_{t-3} \\ \approx \begin{bmatrix} 7.5e-01 & 0.158 & 0.083 \\ 3.8e-01 & 0.356 & 0.256 \\ 1.06e-10 & 0.410 & 0.590 \end{bmatrix}$$

The realization transition is as given below:

1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 1 1 1
1 1 1 1 1 1 2 2 1 1

4 Conclusion

In this study, we have examined the distribution of cropland area for 27 years in Canada using secondary data from Food and Agriculture organization (FAO). This distribution in relation to three contributing factors which changed cyclically with time is not normally distributed but moderately skewed and suggests the use of the Autoregressive Hidden Markov time series Model (AR-HMM). The three factors have limiting effect on the distribution and their transition has limit point on the land area and this study would suggest that deforestation, due to commercial and technological activities should be minimized to ensure more available cropland area. More so, AR-HMM time series model performed excellently in investigating the cropland area distribution.

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