Alternative numerical systems

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Abstract

The article is devoted to the construction of numerical systems, alternative to the system of real numbers and applicable in curvilinear space-time. Examples of such systems are given. Within the framework of a stationary numerical system, it is admissible to sum the diverging series like the Dirichlet series for the Riemann zeta function without resorting to its analytic continuation in the plane of the complex argument. In the framework of a non-stationary numerical system, a description of the Hubble effect is obtained, taking into account the corrections that correspond to the apparently accelerated recession of galaxies without invoking the hypothesis of dark energy.

1. Introduction

There are different points of view on the nature of numbers – let's note the extreme ones. One, which originated in ancient Greece in the 5th century BC considered numbers as a gift from the gods, sent to people for knowledge and use. Socrates said that mathematics is not a product of the mind, but a memory. Another point of view, which insisted on the earthly nature of numbers as a tool invented by people to systematize the results of various activities, according to it, numbers are an invention of an inquiring mind. As an example in favor of this point of view, we note here the invention of zero as a full-fledged number in India in the 7th century.

At present, despite the well-known attempts to return to ancient Greek views described by Bell (1991) [1], numbers, in particular real numbers, which in the context of this article we call a traditional numerical system, are considered as a tool for describing events in the world around us. This is so familiar that there is no need to describe its properties. If desired, its description can be found in any university textbook on mathematics.

The need to talk about alternative numerical systems arises when the capabilities of traditional numerical systems are insufficient for solving some mathematical problems. It is well known that complex numbers appeared when it was required to find expressions for the roots of algebraic equations. Complex numbers are said to extend the system of real numbers. The common property of these two numerical systems is the flat metric¹, given in one case on the real line, and the other on the complex plane. This, in particular, allows the real line to be smoothly embedded in the complex plane.

Some problems make one think about the introduction of numerical systems, alternative to real numbers for other reasons. Consider, for example, an infinite series

$$1 + 2 + 3 + 4 + \dots + n + \dots = -\frac{1}{12} (?) \quad (1)$$

representing the Dirichlet series for the Riemann zeta function (Janke et al (1960) [2]) for $s = 1$

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \quad (2)$$

It is easy to see that using a system of real numbers to determine its sum leads to an erroneous value (∞), while using the analytic continuation of the zeta function to the entire complex plane gives $\zeta(-1) = -1/12$. This contradiction is usually explained by the fact that the divergent series such as series (1), do not have a sum in the traditional sense, applicable to the converging series (Hardy (1949) [3]), and the result (1) is obtained by methods of analysis, by using special summation formulas or renormalization methods. However, as shown by Zayko (2016)[4], the sum of series (1) can be obtained without going beyond the framework of arithmetic if the calculations are performed in a numerical system with a non-Euclidean metric²

$$ds^2 = \left(1 + \frac{x}{x_c}\right) c^2 dt^2 - \left(1 + \frac{x}{x_c}\right)^{-1} dx^2 \quad (3)$$

Let us make some clarifications. Metric (3) represents the metric of $1 + 1$ dimensional space-time continuum; s, t – interval and time, x – coordinate expressed by the numbers of the entered numerical system, x_c – some numerical parameter characterizing the metric of this system. When $x_c \rightarrow \infty$ we get the usual Euclidean numerical system.

The structure of the article is as follows. Section 2 is devoted to the derivation of metric (3) and a discussion of its properties. Section 3

¹ It is about a spatial metric.

² About the accuracy of calculations – see Ref. [5]

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considers non-stationary numerical systems and their application to problems of cosmology. Section 4 is devoted to a discussion of the results. The Conclusion briefly summarizes the results of the work.

2. Calculation of the sums of divergent series

The sums of divergent series, in particular of series (1), in the traditional numerical system are found using the Euler-Maclaurin formula [3]. Calculation of the sum of the Dirichlet series of the zeta function $\zeta(-1)$ was performed in the work [4] using an alternative numeric system with the stationary metric (3).

To obtain metric (3), consider the partial sums of the series (1)

$$S_m = \sum_{n=1}^m n = \frac{m(m+1)}{2} = S_0 + \frac{m}{2} + \frac{m^2}{2} \quad (4)$$

$$S_0 = 0$$

which coincide with the formula for the distance traveled by the time $t = m$ a material point moving along a straight line with constant acceleration $w = 1$ and having a velocity $v_0 = 1/2$ in the initial position (Zayko (2016)[4], (2019) [5]). The force field providing constant acceleration (in the particle's own system) is created by an infinite plane (plate) with a constant mass density μ , perpendicular to the line of motion.

Expressions for metric (3) are obtained from solutions of the Einstein equations (Landau et al (1975) [6])

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi K}{c^2} T_{ik} \quad (5)$$

$$ds^2 = g_{ik} dx^i dx^k$$

K is the gravitational constant, g_{ik} – is a metric tensor, R_{ik} , T_{ik} – are the Ricci tensor and energy-momentum one, c – is a light speed; $R = R^i_{\ i}$, $i, k = 0, 1$. Einstein's equations are reduced to the condition imposed on the Ricci tensor $R_{ik} = 0$.

Solution (5) has the form³ $g_{00} = g_{11}^{-1} = C_1 x + C_2$, $C_{1,2}$ – are constants whose values are found from the relation $g_{00} = 1 + 2\phi/c^2$ in the region where the gravitational field of the plate is weak, $\phi(x) = 2\pi\mu Kx$ – Newtonian gravitational potential of the plate. The result is the expression (3) for the space-time interval. The calculation of $\zeta(-1)$ requires solving the relativistic equations of motion of a material point

$$\frac{d^2 x^i}{ds^2} + \Gamma^i_{kl} \frac{dx^k}{ds} \frac{dx^l}{ds} = 0 \quad (6)$$

Γ^i_{kl} – are Christoffel symbols [6], in the metric (3) with initial conditions determined by the form of partial sums (4) (m plays the role of discrete time).

Let us consider metric (3) in more detail. This one looks like the Schwarzschild metric [6], and x_c plays the role of an event horizon separating the region of motion of a material point $x > -x_c$, including spatial infinity from the sub-horizon region $x < -x_c$. From equations (6) we find expressions for the velocity v and acceleration w of a material point for $x_c < 0$ (Zayko (2016)[4])

$$v = \frac{dx}{cdt} = (1-q)\sqrt{q}, x = x^1, q = \frac{x}{|x_c|} \quad (7)$$

$$w = \frac{|x_c|}{c} \frac{dv}{dt} = \frac{1}{2} \left(\sqrt{q} + \frac{1}{\sqrt{q}} \right)$$

A point moving above the horizon reaches it in infinite time, which follows from the divergence of

the integral $\int dx/v$ at the lower limit $x = 1$. This corresponds to calculating the sum of the series (4) $S(t = \infty)$ where we have replaced m by t following the above. Following the formulation of the problem, we need to find a coordinate q on the trajectory of motion, where the speed is $v = 1/2$. This results in a value of $q_{1/2} = 1.41964$. However, in this case, the material point moves in the direction opposite to the horizon, which is not consistent with the formulation of the problem. To avoid a contradiction, we use the invariance of the relativistic equations of motion (6) with respect to time reversal: $t \rightarrow -t$. Then

$$S(-t) = S_0 + \frac{t^2}{2} - \frac{t}{2} = \sum_{n=1}^t (n-1) = S_1(t) \quad (8)$$

where we introduced the notation $S_1(t)$ for the partial sum of another series. Finally, we find that the distance traveled by the material point from $q_{1/2} = 1.41964$ to the horizon $q_0 = 1$ is equal to $q_{1/2} - q_0 = 0.41964$. Considering the above, we can calculate $S(\infty) = \zeta(-1)$

$$S_1(t) = \sum_{n=1}^t n - \sum_{n=1}^t 1 \xrightarrow{t \rightarrow \infty} S(\infty) + 0.5 = 0.41964 \quad (9)$$

$$S(\infty) = \zeta(-1) = -0.08035$$

where we used the expression for the zeta function $\zeta(0) = \sum_{n=1}^{\infty} 1 = -0.5$ [2]. The obtained value $\zeta(-1)$ differs from the exact value $-0.083333 \dots$ since the metric (3) is approximate; it corresponds to the constant acceleration w of a material particle in its own system, and not in the system of a distant observer. The relative error can be estimated using formula (7) – for our range of coordinate values, it is $\delta = 0.01539$, which leads to a relative error $\Delta = 0.03576$ in the value $\zeta(-1)$.

Of particular interest is the question of the motion of a material particle in the sub-horizon region. To solve it, strictly speaking, metric (3) cannot be used, and following the Schwarzschild analogy, it is necessary to construct another metric of the Kruskal-Szekeres type.⁴ This result can be interpreted geometrically by embedding a number line with metric (3) into a manifold of higher dimension – a plane using the formulas

$$dl^2 = \pm dx^2 \pm dy^2 = \left[\pm 1 \pm \left(\frac{dy}{dx} \right)^2 \right] dx^2 \quad (10)$$

$$\pm 1 \pm \left(\frac{dy}{dx} \right)^2 = \left(1 \pm \frac{x}{x_c} \right)^{-1}$$

where $y(x)$ – the image of the number line with metric (3). This problem was considered by Zayko (2018)[7]. Briefly, the results are as follows:

1. The embedding is possible only for half-lines bounded on one side by the singularity of the metric
2. Each half-plane of the embedding has a hyperbolic geometry:

$$I. -\infty < x < -x_c; dl^2 = dx^2 - dy^2; \quad (11)$$

$$y(x) = \pm x_c \left(\operatorname{arccosh} \sqrt{-z} + \sqrt{(1+z)z} \right)$$

$$II. -x_c < x < +\infty; dl^2 = dy^2 - dx^2;$$

$$y(x) = \pm x_c \left(\operatorname{arcsinh} \sqrt{1+z} + \sqrt{(1+z)(2+z)} \right)$$

Where $z = x/x_c$. These are illustrated in Figure 1, where branches of the embedding (11) are shown

Figure 1. Graph of the embedding results (11); $u = y(x)/x_c$, $z = x/x_c$; solid line – I, dash line – II.

³ Other components of the metric tensor $g_{ik} = 0$.

⁴ Zayko Y.N., to be published

3. Non-stationary numerical systems

Consider a one-dimensional real non-Euclidean continuum as a result of an embedding in some fictitious manifold of higher dimension - a hyperplane with the spatial metric

$$dl^2 = dx^2 \pm dy^2 \quad (12)$$

The sign "+" corresponds to the flat Euclidean geometry of the hyperplane, while the sign "-" corresponds to the non-Euclidean (hyperbolic) one. The geometry of a one-dimensional numerical continuum can be considered as the geometry of some hypersurface in the embedding space. For the sign "+" the role of this hypersurface can play a two-dimensional sphere of radius a , given by the equation

$$x^2 + y^2 = a^2 \quad (13)$$

For the sign "-", as a hypersurface, we choose the two-sheet hyperbola

$$y^2 - x^2 = a^2 \quad (14)$$

Both cases are shown in Fig. 2

Figure 2. Sphere (13) and hyperbola (14) used to represent real numbers

Using (13), the element of length (12) can be expressed as

$$dl^2 = \frac{a^2 dx^2}{a^2 - x^2} = \left(1 - \frac{x^2}{a^2}\right)^{-1} dx^2 \quad (15)$$

and the space-time metric - in the form

$$ds^2 = c^2 dt^2 - a^2(t) d\chi^2 \quad (16)$$

$$x = a \sin \chi$$

where a new spatial coordinate χ is introduced, and the radius of the hypersphere is assumed to be time-dependent. The same for (14) looks like

$$dl^2 = \left(1 + \frac{x^2}{a^2}\right)^{-1} dx^2 \quad (17)$$

$$ds^2 = c^2 dt^2 - a^2(t) d\chi^2 \quad (18)$$

$$x = a \sinh \chi$$

Following [6], we introduce a new time η using the formula $cdt = ad\eta$, which will allow us to rewrite the expression for metrics (16) and (18)

$$ds^2 = a^2(\eta) (d\eta^2 - d\chi^2) \quad (19)$$

The resulting expressions outwardly resemble the expressions for the metric of the Friedmann cosmological model [6].

To determine $a(\eta)$, we solve the Einstein equations for metric (19) in empty space. Denoting $x^0 = \eta$, $x^1 = \chi$, we find expressions for nonzero Christoffel symbols [6]

$$\Gamma_{kl}^i = \frac{1}{2} g^{im} \left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m} \right) \quad (20)$$

$$g_{ik} = \begin{pmatrix} a^2 & 0 \\ 0 & -a^2 \end{pmatrix}$$

$$\Gamma_{00}^0 = \Gamma_{11}^1 = \Gamma_{10}^1 = \Gamma_{01}^1 = a^{-1} a'$$

where the prime denotes the derivative on η . Let us calculate using (19) the Ricci tensor [6]

$$R_{ik} = \frac{\partial \Gamma_{ik}^l}{\partial x^l} - \frac{\partial \Gamma_{il}^l}{\partial x^k} + \Gamma_{ik}^l \Gamma_{lm}^m - \Gamma_{il}^m \Gamma_{km}^l \quad (21)$$

$$R_{ik} = \begin{pmatrix} R_{00} & 0 \\ 0 & R_{11} \end{pmatrix}$$

$$R_{00} = -R_{11} = -\left(\frac{a'}{a}\right)'$$

Einstein's equations in vacuum are reduced to the condition $R_{ik} = 0$ [6], which is a simple differential equation which solution looks as follows

$$a(\eta) = a_0 \exp\left(\frac{\eta}{\eta_0}\right) \quad (22)$$

where a_0 and η_0 - are constants. In the t -variable, the solution looks as follows

$$a(t) = a_0' + \frac{ct}{\eta_0} \quad (23)$$

a_0' - is a new constant. If we impose the condition $\eta(t=0) = 0$, then $a_0' = a_0$.

Thus, the radius of the hypersurface, which we are going to use for the description of the real numbers, turns out to depend on time t and, with its growth, increases indefinitely for $\eta_0 > 0$ or decreases to zero for $\eta_0 < 0$. Note that in the Friedmann problem, a similar result required a finite energy density of matter⁵.

To make these results meaningful, we need to associate them with an observed effect. The analogy with the Friedmann model of the Universe already noted above suggests that as such one can choose the observed expansion of the Universe, according to which galaxies move away from the observer at a speed V proportional to the distance l between the observer and the galaxy (Hubble law)

⁵ This is the difference between numerical (non-material) systems from material ones

$$V = hl \quad (24)$$

h – is the Hubble constant [6]. Moreover, if the traditional explanation of dependence (24) leads to the concept of a nonstationary Universe expanding as a result of the so-called Big Bang, then the explanation of dependence (24) proposed below, as well as the recently discovered deviations from it, is based on the properties of the considered numerical system.

Let's consider the expression for a length dl (15) and (17). It follows from (15) that dl grows with $x < a$, and the measurement results $\sim dl^1$ decrease, which contradicts with experiments. For (17), the situation is the opposite, which is confirmed by the formula

$$\left(\frac{dl}{dx}\right)^{-1} = \left(1 + \frac{x^2}{a^2}\right)^{1/2} \xrightarrow{x \rightarrow \infty} \frac{x}{a} \quad (25)$$

The result of the calculation is shown in Fig. 3

Figure 3. Influence of the spatial scale on the measurement results for different values of the coordinates x (abscissa). Ordinate – dl/dx – dashed line, $(dl/dx)^{-1}$ – solid line.

Thus, while measuring the distances between objects (galaxies) in a homogeneous universe using the proposed numerical system, we will observe a non-uniform distribution of galaxies at each moment of time t , at that the intergalactic distance will increase with distance from the observer.

Let us obtain an expression for the Hubble law (24). Since the expression for the metric (19) coincides with a similar expression for the open model adopted in modern cosmology, we can restrict ourselves to a brief repetition of the reasoning given, for example, in [6].

The apparent recession of galaxies manifests itself as a redshift in their emission spectra. The initial relation is the relation connecting the frequency of light ω with the parameter $a(\eta)$ [6]: $\omega a = \text{const}$, whence follows

$$\omega = \omega_0 \frac{a(\eta - \chi)}{a(\eta)} \quad (26)$$

where ω_0 – is the frequency of the light emitted by the galaxy at the moment $\eta - \chi$, and ω is the frequency of the light received by the observer at the moment η . Substituting into (26) the expression for $a(\eta)$ (22) we obtain

$$\frac{\omega}{\omega_0} = \exp\left(-\frac{\chi}{\eta_0}\right) = \exp\left(-\frac{l}{a\eta_0}\right) \quad (27)$$

where it is taken into account that the distance $l = \chi a(\eta)$. In the right-hand side of (27), we express the time η (or η in terms of the distance l using the relativistic equations of geodesics [6]

$$\frac{d^2 x^i}{ds^2} + \Gamma_{kl}^i \frac{dx^k}{ds} \frac{dx^l}{ds} = 0 \quad (28)$$

Taking into account the nonzero terms in (28), we obtain the system of equations

$$\begin{aligned} \frac{d^2 x^0}{ds^2} + \Gamma_{00}^0 \left(\frac{dx^0}{ds}\right)^2 + \Gamma_{00}^1 \left(\frac{dx^1}{ds}\right)^2 &= 0 \\ \frac{d^2 x^1}{ds^2} + 2\Gamma_{01}^1 \frac{dx^1}{ds} \frac{dx^0}{ds} &= 0; x^0 \equiv \eta, x^1 \equiv \chi \end{aligned} \quad (29)$$

The second equation in (29) can be integrated once, obtaining

$$\begin{aligned} \eta_0 \frac{d^2 \eta}{ds^2} + \left(\frac{d\eta}{ds}\right)^2 + \left(\frac{d\chi}{ds}\right)^2 &= 0 \\ \frac{d\chi}{ds} &= D e^{-2\eta/\eta_0} \end{aligned} \quad (30)$$

D – is a constant. From the Eqs. (30) we obtain the desired connection in the form of the equation

$$\begin{aligned} \frac{d^2 \tau}{d\sigma^2} + \frac{1}{(1+\tau)^3} &= 0 \\ \sigma &= \frac{l}{a_0 \eta_0}, \tau = \frac{ct}{a_0 \eta_0} \end{aligned} \quad (31)$$

Considering that according to the Doppler effect [6] $(\omega - \omega_0)/\omega_0 = -V/c$ we receive

$$\frac{V}{c} = 1 - \exp\left[-\frac{l}{a\eta_0}\right] = 1 - \exp\left[-\frac{\sigma}{1+\tau(\sigma)}\right] \quad (32)$$

where $\tau(\sigma)$ – is a solution of equation (31) with suitable boundary conditions. For small distances and times ($\sigma \ll 1$, $\tau \ll 1$) we receive from (32) Hubble law in the form (24) with the Hubble constant $h = c(a_0 \eta_0)^{-1}$ which coincides with the known value $a^{-1}a$, [6] which is adopted in the standard theory.

To find corrections to Hubble law (24), let us turn to the consideration of equation (31). This is the known Emden-Fowler equation (Bellman (1953)[8]). Although it has been sufficiently studied, for our case we can use only the known particular solution (Kamke (1977) [9]): $1+\tau=(2\sigma)^{1/2}$, which, after substitution in (32), leads to the V/c dependence (Fig. 4, solid curve), which does not agree with the redshift observations of distant galaxies as shown by Riess (1998) [10]. Other curves in Fig. 4 represent the result of the numerical solutions to equation (31) with different boundary values.

Figure 4. Dependence of the recession velocity of galaxies (32) on the distance for various solutions (31). A solid curve corresponds to the exact solution $1+\tau=(2\sigma)^{1/2}$. The rest of the curves are the result of the numerical solution (31) with different boundary values $\tau_0 = \tau$ ($\sigma = 0$), $\tau_0' = \tau$ ($\sigma = 0$). Points: $\tau_0 = 0$, $\tau_0' = 0.1$; dashed line: $\tau_0 = 0$, $\tau_0' = 2$; dash-dotted line: $\tau_0 = 0$, $\tau_0' = 1$

Based on these calculations, the following conclusion can be made. The behavior of the curves in Fig. 4 for boundary values $\tau_0' = \tau$ ($\sigma = 0$) > 1 does not confirm astronomical observations [10], according to which the universe is expanding with acceleration. These observations are satisfied only by $\tau_0' = 0.1$ (points on Fig. 4). These results correspond to geodesics inside and outside the light cone, respectively.

Let us make a few more remarks about the nature of the solutions to equation (31) based on the results of numerical calculations. Solutions obtained for $\tau_0 > 1$, are monotonic functions of distance. Solutions for $\tau_0 < 1$ characterize in non-monotonous behavior. Of these, only those that satisfy the condition $|d\tau/d\sigma|_{\sigma=0} < \alpha$, where α is a positive constant, correspond to astronomical observations, that is, the picture of the Universe expanding with acceleration. Numerical calculations made it possible to estimate the value $\alpha \approx 0.3$.

4. Discussion

The article is devoted to the study of the nature of numbers. This question has interested many scientists and philosophers for millennia and found expression in their writings that have survived to this day. For example, Plato understood numbers as an independent type of essences, irreducible to either ideas or things. He distinguished between mathematical and ideal numbers. In mathematical numbers, no one unit is different from another. Ideal numbers are uncountable (non-stackable), so one cannot imagine any sequential series of them (Losev (1993)[11]). Pythagoras considered numbers to be the basis of being ("Everything is number"). Common to them was the combination of arithmetic (numbers) and geometric (lines and planes) approaches to the study of the principles of being. This work is built on two principles:

- i. The world of numbers is inextricably linked with the material world.
- ii. Mathematics is a unified science in which individual parts cannot contradict each other (in our case, arithmetic and analysis).

The consequence of the first is the coherence of the laws governing objects of the material world and numbers and is expressed in the applicability of Einstein's equations to objects of a numerical nature. The consequence of the second, in the context of this work, is the elimination of the unnatural "dualism" when, for example, for the Riemann zeta function $\zeta(s)$ two different expressions are used - one in the form of a Dirichlet series (2) converging for $\text{Re}(s) > 1$, which limits the scope of its application, and another, which is a consequence of the analyticity of $\zeta(s)$ on the entire plane of complex s (except for $s = 1$).

The main conclusion that can be drawn based on the presented results is that whatever the nature of the numbers, they must reflect the nature of the reality which is described by them. For example, the traditional numerical system is applicable in a flat universe, in which "no one unit is distinguishable from another" (Plato). In other words, using geometric language, the length of a unit segment is constant throughout the universe. This is expressed in Euclidean geometry by the second postulate. However, in a universe characterized by a curvilinear metric, this postulate cannot be true [7].

Let us treat the abovementioned "dualism" of the representation of the Riemann zeta function from this point of view. Previously proposed ways to eliminate this "dualism" based on the so-called renormalization of divergent quantities, although they lead to correct results, today seem to be a compromise solution and cannot reveal the essence of the problem.

The numerical continuum, which was previously assumed to be immutable, which was given once and for all, turns out to be dependent on the computation for which it is used. That is, the same thing happens to it that previously was happened to Newtonian absolute space as a result of the relativistic revolution - it turned out to depend on the distribution of mass and its motion in it. On the other hand, in the language of computational theory, calculating $\zeta(-1)$ as the sum of an infinite diverging series means nothing more than solving an incomputable problem, i.e. breaking the Turing barrier.

Note that the proposed approach, along with solving some problems of analysis and arithmetic, gives rise to other problems associated with the interpretation of known facts, for example, the observed expansion of the Universe, and the Big Bang theory built on its basis. We note in this connection that the above solution to the redshift problem not only allows one to obtain the known results but also imposes a natural limitation on the galaxy recession velocity V : $V < c$. Finally, we note one curious circumstance. If we make a change of variables in expression (3) for the interval ds , then the new components of the metric tensor $g_{\mu\nu}$ will be regular functions for all values of u .

$$ds^2 = \frac{u^2}{x_c} c^2 dt^2 - 4x_c du^2 \quad (33)$$

The similarity of expression (33) with analogous expression in the article of Einstein et al (1935) [12] is striking. The replacement made allows changing x only in the region $x > -x_c$, unlike the original expression (3)⁶. This region x corresponds to two leaves of the region of the variable u , separated by a hyperplane $u = 0$ or $x = -x_c$, on which $g = 0$ and which is an analog of the "bridge" in the terminology of [12].

5. Conclusion

The article discusses numerical systems with non-Euclidean space and space-time metrics, which have some advantages over the traditional system of real numbers. Examples of such systems are given. Within the framework of a stationary numerical system, it is permissible, to sum up, divergent series like the Dirichlet series for the Riemann zeta function without resorting to its analytic continuation to the plane of the complex argument. Within the framework of a nonstationary numerical system, a description of the Hubble effect is obtained taking into account corrections corresponding to the seemingly accelerated recession of galaxies without invoking the hypothesis of dark energy.

Declaration of Conflict of Interests

The author declares that there is no conflict of interest. They have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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⁶ Note the difficulties in interpreting solutions for $x < -x_c$.