The role of thermal radiation on the boundary layer past a stationary flat plate with constant surface boundary condition

Basant K Jha¹, Gabriel Samaila^{*2}

¹Department of Mathematics, Ahmadu Bello University, Zaria, Nigeria

²Department of Mathematics, Air Force Institute of Technology, Kaduna, Nigeria

Keywords

Thermal radiation, boundary layer, constant surface boundary condition and flat plate.

Abstract

This study is devoted to investigating the importance of thermal radiation on the boundary layer over a horizontal surface considering classical constant surface boundary condition. The mathematical model consists of coupled two-dimensional partial differential equations which are transformed to the set of ordinary differential equations via the similarity transformation. The final set of dimensionless equations is solved numerically using Runge Kutta Fehlberg (RKF45) method in Maple software. The significant effect of the thermal radiation is examined using four fluids namely; water, Sulphur oxide, air and mercury whose respective Prandtl numbers are 7, 2, 0.72 and 0.044. The influence of other prominent parameters affecting the flow formation and temperature profile is demonstrated using tables and graphs. The results indicated that the thermal boundary layer thickness could be increase by reducing the Prandtl number. The results also showed that increasing the thermal radiation parameter has a positive impact on the boundary layer thickness. The heat transfer rate could be improved by increasing thermal radiation or decreased by increasing the values of the Prandtl number. Regarding the temperature gradient, an observable increasing is seen far from the flat plate with the growing of thermal radiation whereas the opposite trend is true near the plate surface.

1. Introduction

Recently, analysis of boundary layer flow has witnessed global attention because of its various applications in the areas such as separation and attachment in aerodynamics, golf ball aerodynamics, mixing enhancement, heat transfer enhancement etc. Early investigations such as [1]–[6] explored the behaviour of fluid flow on the boundary layer past a vertical/horizontal wall. Furthermore, recent work such as [7]–[10] presented the influence of the active parameters on boundary layer flow considering various geometries. Similarity transformation is an important instrument used to transform PDE to ODE. It is mostly used in boundary layer analysis due to the coupled and highly nonlinear nature of the boundary layer model. Aziz (2009) [11] present a similarity solution for boundary layer flow with emphasis on the convective boundary condition. Makinde and Olanrewaju (2010) [12] also adopted a similarity solution to study boundary layer flow with the buoyancy effect. Some literature on similarity solution can be found in [13]–[17].

In the last decade, the roles of thermal radiation has attracted the attention of several workers due to its application in industries. Thermal radiation is applied in the design of industrial equipment such as gas turbines, nuclear power plants and missiles. Raju *et al.* (2013) [18] explored the implication of the heat source and thermal radiation on MHD natural convection over a vertical wall. Sharma *et al.* (2012) [19] and Thakur *et al.* (2014) [20] presented the influence of Dufour and Soret on MHD mixed convection flow over a vertical plate. Hayat *et al.* (2013) [21] also presented the importance of thermal radiation on MHD 3-D flow.

An oscillatory flow of natural convection with thermal radiation over a vertical wall was studied by El-Hakim (2000) [22]. Makinde (2008) [23] also presented boundary layer flow with mass transfer and thermal radiation past a continuously moving plate. Jha *et al.* (2016) [24] presented the importance of suction/injection on MHD natural convection flow with thermal radiation. Hossain *et al.* [25] addressed the radiation effect on mixed convection over a vertical wall. Raptis

(1998) [26] published work on free convection flow through a porous medium with radiation. Makinde (2005) [27] expounded on the effects of mass transfer and thermal radiation on natural convection past a continuously moving plate. Ibrahim [28] presented a combined influence of absorption radiation on unsteady MHD free convection flow over a vertically continuous moving wall. Suneetha *et al.* (2008) [29] investigated the effect of thermal radiation on MHD natural convection flow over a vertical wall considering the variable concentration and temperature.

To the best of the authors' knowledge, the effect of thermal radiation on the boundary layer flow over a horizontal plate using constant surface boundary condition has been neglected by several researchers. In our contribution to the existed literature on boundary layer, we demonstrated the significant role of the thermal radiation over a horizontal surface using constant surface boundary condition. As it customarily in boundary layer analysis, ∞ is replaced by 10 in all our calculation.

2. Mathematical Formulation

Consider hydrodynamics and the boundary layer flow with thermal radiation effect past a horizontal plate. Let U_∞ be the uniform velocity of the fluid moving over the plate's surface at temperature T_∞ . The x -axis is considered to be along with the plate while the y -axis normal to the plate. Following Aziz (2009) [11], the governing can be formulated as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left[\frac{\partial^2 T}{\partial y^2} - \frac{1}{K} \frac{\partial q_r}{\partial y} \right] \quad (3)$$

*Corresponding Author: gabbbooooo@yahoo.com
 G. Samaila ORCID <https://orcid.org/0000-0002-7463-9427>

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Where u and v are respectively velocity components along and normal to the plate and K denotes the thermal conductivity. The quantity q_r in Eqn. (3) denotes the radiative heat flux normal to the plate and it can be simplified through Rosseland diffusion approximation as.

$$q_r = -\frac{4\sigma}{3k^*} \frac{\partial T^2}{\partial y} \quad (4)$$

The boundary conditions for the velocity are as follows;

$$U(x, 0) = v(x, 0) = 0$$

$$u(x, \infty) = U_\infty \quad (5)$$

The plate surface is considered to be heating through convection by the hot fluid at the temperature T_f with heat flux coefficient T_f . Thus, the temperature boundary condition can be written as $T(x, y) = T_0$

$$T(x, \infty) = T_\infty \quad (6)$$

We define the similarity variable η , stream function $f(\eta)$ and the dimensionless parameters defined as

$$\eta = y \sqrt{\frac{U_\infty}{\nu x}}, \quad u = U_\infty f'(\eta),$$

$$v = \frac{1}{2} \sqrt{\frac{U_\infty \nu}{x}} (\eta f'(\eta) - f) \quad (7)$$

$$R = \frac{4\sigma(T_f - T_\infty)^3}{k^* K}, \quad C_T = \frac{T_\infty}{T_f - T_\infty},$$

$$Pr = \frac{\mu C_T}{K}, \quad \theta = \frac{(T - T_\infty)}{(T_f - T_\infty)} \quad (8)$$

The reduced dimensionless equations and boundary conditions can be written as

$$2f(\eta)'''(\eta) + f(\eta)f(\eta)''(\eta) = 0$$

$$\theta'(\eta) \left[1 + \frac{4R}{3} (\theta + C_T)^2 \right] + 4R[C_T + \theta]^2 [\theta'(0)]^2 +$$

$$\frac{1}{2} Pr \theta'(\eta) f(\eta) = 0 \quad (9)$$

$$f(0) = f'(0) = 0, \quad f'(\infty) = 1$$

$$\theta(0) = 1, \quad \theta(\infty) = 0 \quad (10)$$

Where prime represent derivative with respect to η and all the parameters in the above equations are defined in the nomenclature section.

Figure 1: Physical model

3. Results and Discussion

In this section, the influence of embedded parameters such as temperature difference (C_T), Prandtl number (Pr) and the thermal radiation (R) on the plate temperature and the flow formation are fully discussed in details. The numerical solution to the governing equation is obtained through the RKF45 method in maple software. It is found that assigning the value of $\infty \geq 10$ gives excellent results compared to $\infty < 10$ thus, $\infty = 10$ is used throughout the simulation. The study considered four working fluids namely; water ($Pr = 7$), Sulphur oxide ($Pr = 2$), air ($Pr = 0.72$) and mercury ($Pr = 0.044$). Figure 2-5 portray the importance of varying R on the plate temperature for the four fluids under examination. It is observed from the figures that the temperature profiles for the four fluids could be enhanced by increasing the thermal radiation effect. Regarding the temperature gradient, an observable increasing is seen far from the flat plate with the growing of thermal radiation whereas the opposite trend is true near the plate surface as depicted in Figure 6. The role of C_T on the temperature distribution is demonstrated in Figure 7. The figure revealed that increasing the temperature difference between the ambient and free stream leads to an increase in the temperature distribution. The implication of Pr is portrayed in Figure 8 where Pr augment has the potential to reduce the thickness of the boundary layer. Generally, the momentum and thermal diffusion or the velocity and thermal boundary layer coincide when $Pr = 1$. However, for liquid fluid $Pr > 1$, the viscous/momentum diffusivity dominate the fluid flow and in turn, the velocity boundary layer becomes thicker than the thermal boundary layer whereas contrary behaviour is true for $Pr < 1$. This can be seen in Figure 7 where the temperature profile of water reached its lowest values at approximately half ($\frac{1}{2}$) of the sulphur oxide, one-third ($\frac{1}{3}$) of air and one-fifth ($\frac{1}{5}$) of mercury.

Table 1 represents the impact of R on the Nusselt number for various fluid under examination with $C_T = 0.2$. It is worthy to remark from the table that the Nusselt number drops as the values of R increase whereas increases with growing values of Pr . Therefore, the heat transfer rate could be improved by increasing R or decreased by increasing the values of Pr .

Figure 2: Variation of R on the temperature profile.

Figure 3: Variation of R on the temperature profile. Figure 6: Variation of R on the temperature gradient profile. Figure 4: Variation of R on the temperature profile Figure 7: Variation of C_T on the temperature profile Figure 5: Variation of R on the temperature profile. Figure 8: Variation of Pr on the thermal boundary layer.

Table 1. Numerical values for $-\theta'(0)$ for $C_T = 0.2$

R	$Pr = 0.044$	$Pr = 0.72$	$Pr = 2.0$	$Pr = 7.0$
0.0	0.12166	0.29564	0.42231	0.64592
0.5	0.07355	0.17572	0.25160	0.38525
1.0	0.05848	0.13650	0.19598	0.30047
1.5	0.05105	0.11605	0.16708	0.25651
2.0	0.04664	0.10309	0.14884	0.22884
2.5	0.04372	0.09396	0.13603	0.20944
3.0	0.04166	0.08708	0.12640	0.19488
3.5	0.04012	0.08164	0.11881	0.18343
4.0	0.03893	0.07719	0.11262	0.17412

4. Conclusion

The importance of thermal radiation on the boundary layer flow over a stationary flat plate is examined. The numerical solution to the sets of the differential equation is obtained using the RKF45 method. The significant effect of the thermal radiation and other active parameters are examined using four fluids namely; water, Sulphur oxide, air and mercury. The results showed that;

- (i). The thickness of the boundary layer decreases with growing values of Pr .
- (ii). Augmenting R has a positive impact on the temperature distribution for the four fluids under examination.
- (iii). The temperature difference augment has an increasing impact on the plate temperature for the four fluids.
- (iv). The heat transfer rate could be improved by increasing R or decreased by increasing the values of Pr .

Nomenclature

R	Thermal radiation (dimensionless)
q_r	Radiative heat flux
Pr	Prandtl number (dimensionless)
θ	Temperature (dimensionless)
T	Temperature
Re	Reynold number
ν	Kinematic viscosity
k^*	mean absorption
C_T	Temperature difference (dimensionless)
u, v	velocity components
σ	Stefan Boltzmann constant

Declaration of Conflict of Interests

The authors declare that there is no conflict of interest. They have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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