Civil Engineering Beyond Limits

www.acapublishing.com

RESEARCH ARTICLE

Full and Partial Interaction Analysis of Single Span Double Skin Composite Beams with Frictional Force

Orhan DOĞAN

Kırıkkale University, Engineering Faculty, Civil Engineering Department, 71451 Kırıkkale-TURKEY

Abstract

Double skin composite (DSC) construction consists of a layer of a plain concrete, sandwiched between two layers of relatively thin steel plate, connected to the concrete by welded stud shear connectors. This construction acts in a similar way to doubly reinforced concrete elements but the flexibility of connection between the steel plates and concrete gives rise to interface slip and additional overall element deflection. This results in a strong and efficient structure with certain potential advantages over conventional forms of construction. This paper presents a theoretical analysis of the behavior of simply supported single span DSC beams, assuming both full and partial interaction. The partial interaction analysis takes into account the flexibility of connection on both tension and compression faces. The partial interaction analysis is extended to cover the influence of frictional forces between the concrete and external steel plates, at the supports and load points. The theoretical solutions based on partial interaction theory, assuming realistic material and shear connector properties and incorporating the influence of interface frictional forces between the concrete and external steel plates, at the supports and load points, are compared with the results of tests on DSC beams. It is concluded that the proposed method shows good correlation with real behavior and may be reliably used for the analysis of simply supported single span DSC beams.

Keywords: Double skin composite construction, Sandwich beams, Partial interaction, Full interaction, shear connectors, slip, Frictional force.

1.Introduction

Steel-concrete composite systems generally consist of steel plate, concrete and reinforcement. Shear connectors are usually utilized to develop the composite action between steel and concrete. In steel-concrete composite members, the natural bonding, friction, and mechanical interlocking actions of shear connectors have a significant influence on the degree of interaction [1, 2].

It is known that the degree of interaction between steel and concrete influences the shear flow and strain distribution. Also, it has an impact on the structural performance such as strength, stiffness and failure mode. The degree of interaction in steel-concrete composite systems can be evaluated as full-interaction, partial-interaction, and no-interaction. [1, 2]. The assumption of full-interaction may result in an overestimation of the structural performance while the assumption of no-interaction may cause an underestimation of the structural performance. Therefore, the partial-interaction assumption and analysis with a degree of interaction becomes more practical and seems to be essential for a precise prediction of behavior. Actually, the steel-concrete composite members generally show partial-interaction due to the deformation of shear connectors and slip at the interface under the applied loads [3-16].

In many situations, slip and its influence on the structural behavior of steel-concrete composite systems may be small enough to be neglected in the analysis (i.e., full interaction). However, in some cases it may be feasible to use either fewer connectors than are required for complete shear connection or connectors which possess a relatively

low stiffness. In such situations the influence of slip may not be negligible and result in reduced stiffness of the system (i.e., partial interaction). In general, the stiffness of the connectors has a significant influence on both the slip and deformations of a composite beam. The stiffness of the shear connectors may be determined experimentally from so called push-shear tests.

An important aspect of the design of double skin composite (DSC) beams is the design of the so called shear connectors, which transfer both shear and normal forces between the concrete infill and external steel plates. The shear connection is sometimes defined as complete when the bending strength cannot be increased by the provision of additional connectors. However, all connectors possess finite stiffness and therefore slip must occur between the concrete and steel plates if the shearing forces are to develop. Slip results in a strain discontinuity at the steel-concrete interface, with a corresponding reduction in flexural stiffness.

Analysis of the influence of slip in composite beams, assuming both linear and non-linear material and shear connector behavior [17, 18, 19-23], has generally been based upon an approach attributed to Newmark et al [19]. The equilibrium and compatibility equations for an element of the beam are reduced to a single second order differential equation in terms of either the resultant axial force in the concrete or the interface slip. Solutions for the axial force or interface slip are substituted back into the basic equilibrium and compatibility equations, which can then be solved to give displacements and strains throughout the beam.

*Corresponding Author: odogan@kku.edu.tr
(O:Doğan)

Newmark et al [19] presented the results of tests and analysis for evaluating the load deflection behavior of simply supported, partially interactive, composite concrete and steel T-beams. The theoretical analysis was based upon the assumption that a continuous imperfect connection existed between the two elements. A second order differential equation expressing the relationship between the longitudinal forces, transmitted through the shear connection from the concrete slab to the steel beam, and the applied bending moment, was derived and solved for the case of a beam loaded with a concentrated load.

Newmark et al's approach has been developed by Yam and Chapman [18, 20, 21] to incorporate non-linear material and shear connector behavior. The resulting non-linear differential equations were solved iteratively and the influence of slip on the ultimate flexural strength of composite beams was studied.

Johnson et al [22, 23] have presented modified versions of Newmark et al's theory in which the differential equations are formulated in terms of interface slip. The equations were used as the basis of an extensive theoretical study of the loss of interaction in short-span composite beams and slabs.

An alternative approach to the analysis of composite beams with partial interaction has been presented by Roberts [24], in which the basic equilibrium and compatibility equations are formulated in terms of the displacements of the layers. The resulting differential equations are then solved simultaneously by expressing the displacement derivatives in finite difference form. The development of this approach to incorporate non-linear material and shear connector behavior has been described by Al-Amery and Roberts [25]. The resulting non-linear differential equations are expressed in finite difference form and solved iteratively.

Experimental and theoretical studies of the behavior of DSC beams with partial interaction have been reported by Oduyemi and Wright [26], Wright et al [27-28], and Narayanan et al [29]. Wright and Oduyemi [30] presented closed form solutions for the partial interaction analysis of simply supported DSC beams. The analysis takes into account the flexibility of the connection on both the tension and compression faces, and incorporates the influence of concrete cracking and non-linear connector behavior using a step-wise linearization technique. Two coupled differential equations for the axial forces in the tension and compression plates were formulated and closed form solutions found for various load combinations. The solutions were compared with the results of tests on several DSC beams and good agreement between the theory and experiment was found.

The objective of this study is to present a theoretical analysis of the behavior of DSC beams, assuming both full and partial interactions between steel plates and concrete infill. The partial interaction analysis introduced by Wright and Oduyemi [30] is extended to incorporate the influence of frictional forces between the concrete and external steel plates, at the supports and load points. Several errors in the original presentation have also been identified and corrected. Theoretical solutions are compared with the experimental results on DSC beams reported by Edwards [31-34] and are found to be in reasonably close agreement.

2. Theoretical Analysis

2.1 Full Interaction Theory

Full interaction analysis of DSC beams is based on the following assumptions: both steel and concrete are linearly elastic materials, concrete subjected to tensile strain is cracked and ineffective in resisting load, the shear connection between the concrete and steel is sufficiently stiff to ensure that slip is negligible, and plane sections remain plane.

2.1.1 General solution of governing differential equations

The assumed linear strain distribution over the depth of a DSC section subjected to bending is shown in Fig. 1a, together with the associated resisting forces in the concrete and steel plates. Fig. 1b shows the assumed positive conventions for displacements u and v in the x and y directions, moments M , shear forces V and curvature k .

Figure 1a. Internal forces and strain distribution over the depth of a DSC section for full interaction.

Figure 1b. The assumed positive assumptions for displacement u and v in x and y directions.

Referring to Fig. 1a, the compressive forces in the uncracked portion of the concrete core F_{cu} and in the steel compression plate F_{sc} , and the tensile force in the steel tension plate F_{st} , are given by

$$F_{cu} = k \left(\frac{d_{cu}}{2} \right) E_c A_{cu} \quad (1a)$$

$$F_{sc} = k \left(d_{cu} + \frac{t_{sc}}{2} \right) E_{sc} A_{sc} \quad (1b)$$

$$F_{st} = k \left(d_c - d_{cu} + \frac{t_{st}}{2} \right) E_{st} A_{st} \quad (1c)$$

where k is curvature, A_{cu} is cross-section area of uncracked portion of concrete core ($b \cdot d_{cu}$), b is width of beam section, d_{cu} is uncracked depth of concrete section, A_{sc} is cross-section area of steel tension plate ($b \cdot t_{sc}$), t_{sc} is thickness of steel plate in compression, A_{st} is cross-section area of steel compression plate ($b \cdot t_{st}$), t_{st} is thickness of steel plate in tension, E_c is Young's modulus of concrete core, E_{sc} is Young's modulus of steel compression plate and E_{st} is Young's modulus of steel tension plate.

The depth of the neutral axis d_{cu} can be determined from the condition that the resultant axial force is zero. Hence

$$F_{sc} + F_{cu} = F_{st} \quad (2)$$

Substituting Eqs. (1a-c) into Eq. (2) and solving for d_{cu} gives

$$d_{cu} = - \frac{B^*}{2A^*} \mp \frac{\sqrt{(B^*)^2 - 4A^*C^*}}{2A^*} \quad (3a)$$

where

$$A^* = E_c b \quad (3b)$$

$$B^* = 2(E_{sc}A_{sc} + E_{st}A_{st}) \quad (3c)$$

$$C^* = E_{sc}A_{sc}t_{sc} - (2d_c + t_{st})E_{st}A_{st} \quad (3d)$$

Taking moments about the line of action of F_{cu} gives the resultant moment M as

$$M = M_{sc} + M_{st} + M_c + F_{sc}y_{sc} + F_{st}y_{st} \quad (4a)$$

$$y_{sc} = \frac{d_{cu} + t_{sc}}{2} \quad (4b)$$

$$y_{st} = d_c - \frac{d_{cu}}{2} + \frac{t_{st}}{2} \quad (4c)$$

in which M_{sc} is bending moment in the steel compression plate, M_{st} is bending moment in the steel tension plate, M_c is bending moment in the uncracked concrete core, y_{sc} is moment lever arm of F_{sc} and y_{st} is moment lever arm of F_{st} .

Assuming that all the layers deform with equal curvature, Eqs. (4a-c) can be rewritten in the form

$$M = k(E_{sc}I_{sc} + E_{cu}I_{cu} + E_{st}I_{st}) + F_{sc}y_{sc} + F_{st}y_{st} \quad (5)$$

where the elemental second moment of cross-section area is denoted about local centroidal axes and I_{sc} is second moment of cross-section area of steel compression plate I_{cu} is second moment of cross-section area of uncracked portion of concrete core, I_{st} is second moment of cross-section area of steel tension plate.

Eq. (5) can be rewritten in terms of a so called composite stiffness factor α , which depends upon the section geometry and material properties. Substituting for F_{sc} and F_{st} from Eqs. (1a-c) and rearranging gives

$$M = k \sum EI(1 + \alpha) \quad (6a)$$

$$\alpha = \frac{E_{sc}A_{sc}}{\sum EI} y_{sc} \left(d_{cu} + \frac{t_{sc}}{2} \right) + \frac{E_{st}A_{st}}{\sum EI} y_{st} \left(d_c - d_{cu} + \frac{t_{st}}{2} \right) \quad (6b)$$

$$\sum EI = E_{sc}I_{sc} + E_{cu}I_{cu} + E_{st}I_{st} \quad (6c)$$

$$y_{sc} = \frac{t_{sc} + d_{cu}}{2} \quad (6d)$$

$$y_{st} = \frac{t_{st} + 2d_c - d_{cu}}{2} \quad (6e)$$

Combining Eqs. (1b), (1c) and (6a-e) yields

$$F_{sc} = \rho_1 M \quad (7a)$$

$$F_{st} = \rho_2 M \quad (7b)$$

where

$$\rho_1 = \frac{E_{sc}A_{sc}}{\sum EI(1 + \alpha)} \left(d_{cu} + \frac{t_{sc}}{2} \right) \quad (7c)$$

$$\rho_2 = \frac{E_{st}A_{st}}{\sum EI(1 + \alpha)} \left(d_c - d_{cu} + \frac{t_{st}}{2} \right) \quad (7d)$$

The interface shear forces per unit length q_{sc} and q_{st} are equal to the rate of change of the axial forces in the steel plates (see Fig. 2a).

Figure 2a. Interface shearing forces.

Figure 2b. Support, loading and bending moment diagram.

Hence

$$q_{sc} = - \frac{dF_{sc}}{dx} \quad (8a)$$

$$q_{st} = - \frac{dF_{st}}{dx} \quad (8b)$$

Approximating the curvature k by d^2v/dx^2 in which v denotes the transverse displacement of the beam in the y direction, Eq. (6a) can be written in the form

$$\frac{d^2v_s}{dx^2} = - \frac{M}{\sum EI(1 + \alpha)} \quad (9)$$

Eq. (9) is the governing differential equation for bending of a fully composite DSC beam. Integrating Eq. (9) twice, the general solution for the transverse displacement of the beam in the y direction can be written in the form

$$v_s(x) = - \frac{1}{\sum EI(1 + \alpha)} \int \left\{ \int M dx \right\} dx + C_1 x + C_2 \quad (10)$$

in which C_1 and C_2 are constants of integration which can be determined from particular boundary conditions.

2.1.2 Particular solution of governing differential equations

A particular solution of the governing differential equations is now obtained for a simply supported beam subjected to symmetrical two point loading, as shown in Fig. 2b. The bending moment diagram is symmetrical about the centreline and only the left half of the beam need be considered. However, the bending moment diagram is discontinuous at the load points and therefore solutions have to be

obtained for parts A and B of the beam. The bending moments in parts A and B of the beam are

$$M_A = P \frac{x}{2} \quad (11a)$$

$$M_B = P \frac{u}{2} \quad (11b)$$

In part A of the beam, the axial forces in the compression and tension plates (see Eqs. (7a-d)) are therefore

$$F_{scA} = \rho_1 \left(\frac{P}{2} \right) x \quad (12a)$$

$$F_{stA} = \rho_2 \left(\frac{P}{2} \right) x \quad (12b)$$

Hence, the interface shear forces per unit length (Eq. (8a) and Eq. (8b)) are given by

$$q_{scA} = -\rho_1 \left(\frac{P}{2} \right) \quad (13a)$$

$$q_{stA} = -\rho_2 \left(\frac{P}{2} \right) \quad (13b)$$

In part B of the beam, F_{scB} and F_{stB} are given by

$$F_{scB} = \rho_1 \left(\frac{P}{2} \right) u \quad (14a)$$

$$F_{stB} = \rho_2 \left(\frac{P}{2} \right) u \quad (14b)$$

Hence

$$q_{scB} = 0 \quad (15a)$$

$$q_{stB} = 0 \quad (15b)$$

Substituting the appropriate expression for the bending moment into Eq. (9) and integrating twice gives

$$v_{sA}(x) = -\frac{P}{2 \sum EI (1+\alpha)} \frac{x^3}{6} + C_1 x + C_2 \quad (16a)$$

$$v_{sB}(x) = -\frac{P u}{2 \sum EI (1+\alpha)} \frac{x^2}{2} + C_3 x + C_4 \quad (16b)$$

The four constants of integration can be determined from four boundary conditions, which are

$$\text{at } x = 0 \quad v_A = 0$$

$$\text{at } x = \frac{L}{2} \quad \frac{dv_B}{dx} = 0$$

$$\text{at } x = u \quad v_A = v_B \quad \text{and} \quad \frac{dv_A}{dx} = \frac{dv_B}{dx}$$

Hence

$$C_1 = \frac{P u (L - u)}{4 \sum EI (1 + \alpha)} \quad (16c)$$

$$C_2 = 0 \quad (16d)$$

$$C_3 = \frac{P u L}{4 \sum EI (1 + \alpha)} \quad (16e)$$

$$C_4 = -\frac{P u^3}{12 \sum EI (1 + \alpha)} \quad (16f)$$

The particular solution of the governing differential equations for deflection along the left half of the beam is therefore

$$v_{sA}(x) = \frac{1}{\sum EI (1 + \alpha)} \left[-\frac{P}{12} x^3 - \left(\frac{P u^2}{4} - \frac{P u L}{4} \right) x \right] \quad (17a)$$

$$v_{sB}(x) = \frac{1}{\sum EI (1 + \alpha)} \left[-\frac{P u}{4} x^2 + \frac{P u L}{4} x - \frac{P u^3}{12} \right] \quad (17b)$$

2.2 Partial Interaction Theory

In this section, the partial interaction analysis introduced by Wright and Oduyemi [5] is extended to incorporate the influence of frictional forces between the concrete and external steel plates, at the supports and load points. Theories of partial interaction are based on the following simplifying assumptions: (a) both steel and concrete are linearly elastic materials, (b) deflections are small, (c) shear deformations within each material are negligible, (d) the shear connection between the concrete and steel plates is continuous along the beam i.e. the discrete stud connectors act as a continuous (smeared) connection, (e) the shear stiffness of the connection is linear, (f) the distribution of strain throughout the depth of each individual layer is linear, (g) at every section of the beam, each layer is bent to the same radius of curvature i.e. each layer deflects by the same amount and no buckling or separation of layers occurs, (h) the concrete subjected to tensile strain is cracked and ineffective in resisting load and (i) the depth of the neutral axis is constant and related to the beam geometry and material properties.

Assumptions (a) and (b) are normally associated with elastic beam analysis. Assumption (a) is valid for the steel plates over most of the load range, and since the neutral axis is generally high in the section, the error in assuming that the small area of concrete in compression behaves linearly, is not thought to be significant.

So long as the ratio of bending to shear forces in the section is relatively high, assumption (c) will be justified. For a beam subjected to a reasonably uniform distribution of loading, this will normally be the case.

Assumptions (d) and (e) are necessary requirements for the analysis. Discrete connections can be allowed for using numerical techniques [24, 25]. However, in practice, detailing requirements will be such as to ensure a reasonably close spacing of connectors, which will approximate to a smeared connection. Although the actual load slip behaviour of studs is highly non-linear [26], the shear connection is assumed linear for simplicity.

Assumptions (f) and (g) are valid for the small deflections which occur during the major part of the load range. However, there is some possibility of compression plate buckling which should be avoided by ensuring a sufficiently close spacing of connectors [26].

Assumption (h) is often used in design situations and is particularly valid in this analysis. It has been observed [26] that the stud connectors on the tension face may act as crack inducers, giving rise to discrete cracks in the tension region of the concrete from a very early stage of loading.

Assumption (i) is required to simplify the form of the equations. The neutral axis depth d_{cu} , previously defined by Eqs. (3a-d), depends on the cross-section geometry and material properties, and is assumed constant throughout the length of the beam. However, the depth of the neutral axis is influenced by slip between the concrete and steel plates, and is not therefore strictly a constant.

2.2.1 General solution of governing differential equations

Based on the assumptions given in the previous section, the differential equations governing the behaviour of DSC beams with partial interaction are now derived. The assumed strain distribution over the depth of a DSC section subject to bending is shown in Fig. 3a, together with the associated resisting forces in the concrete and steel plates. Due to slip between the concrete and steel plates, strain

discontinuities ε_1 and ε_2 occur at the compression and tension plate interfaces.

Figure 3a. Internal forces and strain distribution for partial interaction.

Compression and tension plate slips are related to the stud shear forces and shear stiffness by the equations

$$S_{sc} = \frac{Q_{sc}}{K_{sc}} \quad (18a)$$

$$S_{st} = \frac{Q_{st}}{K_{st}} \quad (18b)$$

where

$$Q_{sc} = \frac{q_{sc} p_{sc}}{n_{sc}} \quad (18c)$$

$$Q_{st} = \frac{q_{st} p_{st}}{n_{st}} \quad (18d)$$

where s_{sc} is slip between concrete core and steel compression plate, s_{st} is slip between concrete core and steel tension plate, Q_{sc} is shear force in stud connector welded to steel compression plate, Q_{st} is shear force in stud connector welded to steel tension plate, K_{sc} is shear stiffness of stud connector welded to steel compression plate, K_{st} is shear stiffness of stud connector welded to steel tension plate, q_{sc} is shear force per unit length of steel compression plate, q_{st} is shear force per unit length of steel tension plate, p_{sc} is longitudinal spacing of stud connectors welded to steel compression plate, p_{st} is longitudinal spacing of stud connectors welded to steel tension plate, n_{sc} is number of stud connectors across the width of the compression plate and n_{st} is number of stud connectors across the width of the tension plate.

The interface shearing forces per unit length q_{sc} and q_{st} are equal to the rate of change of the axial forces in the steel plates (see Fig. 2a). Hence

$$q_{sc} = - \frac{dF_{sc}}{dx} \quad (19a)$$

$$q_{st} = - \frac{dF_{st}}{dx} \quad (19b)$$

Eqs. (19a-b) can be substituted into Eqs. (18a-d) to give

$$S_{sc} = - \frac{p_{sc}}{n_{sc} K_{sc}} \frac{dF_{sc}}{dx} \quad (20a)$$

$$S_{st} = - \frac{p_{st}}{n_{st} K_{st}} \frac{dF_{st}}{dx} \quad (20b)$$

The rate of change of slip ds/dx is equal to the strain difference ε at the interface of layers. Hence

$$\frac{ds_{sc}}{dx} = - \frac{p_{sc}}{n_{sc} K_{sc}} \frac{d^2 F_{sc}}{dx^2} = \varepsilon_1 \quad (21a)$$

$$\frac{ds_{st}}{dx} = - \frac{p_{st}}{n_{st} K_{st}} \frac{d^2 F_{st}}{dx^2} = \varepsilon_2 \quad (21b)$$

The strain differences ε are assumed to be caused by the axial forces F and bending moments M acting at the centroid of each of the layers, as shown in Fig. 3a. If tensile strains are assumed positive, the strain differences and strains are related to the forces by the following equations derived, from simple bending theory.

$$\varepsilon_1 = \varepsilon_{sc1} - \varepsilon_{c1} \quad (22a)$$

$$\varepsilon_2 = \varepsilon_{c2} - \varepsilon_{st2} \quad (22b)$$

where

$$\varepsilon_{sc1} = + \frac{M_{sc}}{E_{sc} I_{sc}} \frac{t_{sc}}{2} - \frac{F_{sc}}{E_{sc} A_{sc}} \quad (22c)$$

$$\varepsilon_{c1} = - \frac{M_{cu}}{E_c I_{cu}} \frac{d_{cu}}{2} - \frac{F_{cu}}{E_c A_{cu}} \quad (22d)$$

$$\varepsilon_{c2} = + \frac{M_{cu}}{E_c I_{cu}} \left(d_c - \frac{d_{cu}}{2} \right) - \frac{F_{cu}}{E_c A_{cu}} \quad (22e)$$

$$\varepsilon_{st2} = - \frac{M_{st}}{E_{st} I_{st}} \frac{t_{st}}{2} + \frac{F_{st}}{E_{st} A_{st}} \quad (22f)$$

Combining Eqs. (21a-b) and (22a-f) gives

$$\frac{d^2 F_{sc}}{dx^2} = \frac{n_{sc} K_{sc}}{p_{sc}} \cdot \varepsilon_1 \quad (23a)$$

$$\frac{d^2 F_{st}}{dx^2} = \frac{n_{st} K_{st}}{p_{st}} \cdot \varepsilon_2 \quad (23b)$$

Equating the axial compressive and tensile forces acting within the section gives

$$F_{sc} + F_{cu} = F_{st} \quad (24)$$

Taking moments about the line of action of F_{cu} gives the resultant moment M as

$$M = M_{sc} + M_{st} + M_{cu} + F_{sc} y_{sc} + F_{st} y_{st} \quad (25)$$

where y_{sc} and y_{st} are as defined by Eqs. (4b) and (4c).

Using assumption (g) that all the layers deform with equal curvature implies that

$$- \frac{d^2 v_p}{dx^2} = \frac{M_{sc}}{E_{sc} I_{sc}} = \frac{M_{st}}{E_{st} I_{st}} = \frac{M_{cu}}{E_c I_{cu}} = \frac{M - F_{sc} y_{sc} - F_{st} y_{st}}{\sum EI} \quad (26)$$

where v_p denotes the displacement of a partially composite beam. Combining Eqs. (22a-f), (23a-b) v_p and (26) yields

$$\frac{d^2 F_{sc}}{dx^2} - a_1 F_{sc} + a_2 F_{st} = -a_0 M \quad (27a)$$

$$\frac{d^2 F_{st}}{dx^2} - b_1 F_{st} + b_2 F_{sc} = -b_0 M \quad (27b)$$

where

$$a_1 = \frac{n_{sc} K_{sc}}{p_{sc}} \left(\frac{1}{E_{sc} A_{sc}} + \frac{1}{E_c A_{cu}} + \frac{y_{sc}^2}{\sum EI} \right) \quad (27c)$$

$$a_2 = \frac{n_{sc} K_{sc}}{p_{sc}} \left(\frac{1}{E_c A_{cu}} - \frac{y_{st} y_{sc}}{\sum EI} \right) \quad (27d)$$

$$a_0 = \frac{n_{sc} K_{sc}}{p_{sc}} \left(\frac{y_{sc}}{\sum EI} \right) \quad (27e)$$

$$b_1 = \frac{n_{st} K_{st}}{p_{st}} \left(\frac{1}{E_{st} A_{st}} + \frac{1}{E_c A_{cu}} + \frac{y_{st}^2}{\sum EI} \right) \quad (27f)$$

$$b_2 = \frac{n_{st} K_{st}}{p_{st}} \left(\frac{1}{E_c A_{cu}} - \frac{y_{st} y_{sc}}{\sum EI} \right) \quad (27g)$$

$$b_0 = \frac{n_{st} K_{st}}{p_{st}} \left(\frac{y_{st}}{\sum EI} \right) \quad (27h)$$

Eqs. (27a) and (27b) are the two simultaneous differential equations for the axial forces in the steel compression and tension plates of a DSC beam. Eqs. (27a) and (27b) can be uncoupled and expressed in the form

$$[D^4 - (a_1 + b_1) D^2 + (a_1 b_1 - a_2 b_2)] F_{sc} = [-a_0 D^2 + (a_0 b_1 + a_2 b_0)] M \quad (28)$$

$$F_{st} = - \left(\frac{D^2 F_{sc} - a_1 F_{sc} + a_0 M}{a_2} \right) \quad (29)$$

in which D denotes the differential operator d/dx. Eq. (28) is a fourth order differential equation for the axial force in the steel compression plate. After solving this equation, the force in the steel tension plate can be obtained by substitution in Eq. (29). The general solutions of Eqs. (28) and (29) are

$$F_{sc} = A_1 \cos \sqrt{m_1} x + A_2 \sin \sqrt{m_1} x + A_3 \cos \sqrt{m_2} x + A_4 \sin \sqrt{m_2} x + g_1 M + g_2 D^2 M \quad (30a)$$

$$F_{sc} = A_1 g_3 \cos \sqrt{m_1} x + A_2 g_3 \sin \sqrt{m_1} x + A_3 g_4 \cos \sqrt{m_2} x + A_4 g_4 \sin \sqrt{m_2} x + g_5 M + g_6 D^2 M \quad (30b)$$

where

$$m_1 = \frac{(a_1 + b_1)}{2} + \left(\frac{(a_1 + a_2)^2}{4} - (a_1 b_1 - a_2 b_2) \right)^{1/2} \quad (30c)$$

$$m_2 = \frac{(a_1 + b_1)}{2} - \left(\frac{(a_1 + a_2)^2}{4} - (a_1 b_1 - a_2 b_2) \right)^{1/2} \quad (30d)$$

$$g_1 = \frac{1}{m_1 m_2} (a_0 b_1 - a_2 b_0) \quad (30e)$$

$$g_2 = \frac{1}{m_1 m_2} \left[\frac{m_1 + m_2}{m_1 m_2} (a_0 b_1 - a_2 b_0) - a_0 \right] \quad (30f)$$

$$g_3 = \frac{a_1 - m_1}{a_2} \quad (30g)$$

$$g_4 = \frac{a_1 - m_2}{a_2} \quad (30h)$$

$$g_5 = \frac{a_1 g_1 - a_0}{a_2} \quad (30i)$$

$$g_6 = \frac{a_1 g_2 - g_1}{a_2} \quad (30j)$$

Eqs. (30a) and (30b) represent the axial forces in the steel compression and tension plates. The constants of integration A_1 to A_4 can be determined from the boundary conditions, for various load cases. Eqs. (26) and (27a-h) can be combined and rearranged in the form

$$\frac{d^2 v_p}{dx^2} = - \frac{M}{\sum EI} \left[1 - \frac{(a_2 b_0 + a_0 b_1)}{(a_1 b_1 - a_2 b_2)} y_{sc} - \frac{(a_1 b_0 + a_0 b_2)}{(a_1 b_1 - a_2 b_2)} y_{st} \right] +$$

$$\lambda_1 \frac{d^2 F_{sc}}{dx^2} + \lambda_2 \frac{d^2 F_{st}}{dx^2} \quad (31a)$$

where

$$\lambda_1 = \frac{(b_2 y_{st} + b_1 y_{sc})}{(a_1 b_1 - a_2 b_2)} \frac{1}{\sum EI} \quad (31b)$$

$$\lambda_2 = \frac{(a_1 y_{st} + a_2 y_{sc})}{(a_1 b_1 - a_2 b_2)} \frac{1}{\sum EI} \quad (31c)$$

The first term of Eq. (31a) involving the bending moment gives the deflection of the beam with full interaction v. The second and third terms represent the additional curvature caused by loss of interaction between the layers. If there is no slip, the second and third terms in Eq. (31a) are zero and the equation can be written in the form

$$\frac{d^2 v}{dx^2} = - \frac{M}{\sum EI (1 + \alpha)} \quad (32a)$$

where

$$\alpha = \frac{E_{sc} A_{sc}}{\sum EI} y_{sc} \left(d_{cu} + \frac{t_{sc}}{2} \right) + \frac{E_{st} A_{st}}{\sum EI} y_{st} \left(d_c - d_{cu} + \frac{t_{st}}{2} \right) \quad (32b)$$

$$\frac{1}{1 + \alpha} = 1 - \frac{(a_2 b_0 + a_0 b_1)}{(a_1 b_1 - a_2 b_2)} y_{sc} - \frac{(a_1 b_0 + a_0 b_2)}{(a_1 b_1 - a_2 b_2)} y_{st} \quad (32b)$$

Eq. (31a) can be integrated directly to give the following general equation for the deflection of a beam with partial interaction v_p .

$$v_p(x) = v_s(x) + \lambda_1 F_{sc} + \lambda_2 F_{st} \quad (33)$$

where $v_s(x)$ is given by Eq. (10).

2.2.2 Particular solution - no frictional forces

Fig. 2b shows a DSC beam subjected to symmetrical, two-point loading. The applied bending moment diagram is symmetrical about the centreline and only half of the beam need be considered. However, there is a discontinuity in the bending moment at the load point and Eqs. (30a) and (30b) need to be determined for the portions A and B of the beam. Hence

$$F_{sCA} = A_1 \cos \sqrt{m_1} x + A_2 \sin \sqrt{m_1} x + A_3 \cos \sqrt{m_2} x + A_4 \sin \sqrt{m_2} x + g_1 M_A + g_2 D^2 M_A \quad (34a)$$

$$F_{stA} = A_1 g_3 \cos \sqrt{m_1} x + A_2 g_3 \sin \sqrt{m_1} x + A_3 g_4 \cos \sqrt{m_2} x + A_4 g_4 \sin \sqrt{m_2} x + g_5 M_A + g_6 D^2 M_A \quad (34b)$$

$$F_{sCB} = A_5 \cos \sqrt{m_1} x + A_6 \sin \sqrt{m_1} x + A_7 \cosh \sqrt{m_2} x + A_8 \sinh \sqrt{m_2} x + g_1 M_B + g_2 D^2 M_B \quad (34c)$$

$$F_{stB} = A_5 g_3 \cos \sqrt{m_1} x + A_6 g_3 \sin \sqrt{m_1} x + A_7 g_4 \cos \sqrt{m_2} x + A_8 g_4 \sin \sqrt{m_2} x + g_5 M_B + g_6 D^2 M_B \quad (34d)$$

The general equation of deflection (Eq. 33) can also be written for each section of the beam in the following forms:

$$v_{pA}(x) = - \frac{P}{2 \sum EI (1 + \alpha)} \frac{x^3}{6} + \lambda_1 \frac{F_{sCA}}{\sum EI} + \lambda_2 \frac{F_{stA}}{\sum EI} + C_1 x + C_2 \quad (35a)$$

$$v_{pB}(x) = - \frac{P}{2 \sum EI (1 + \alpha)} \frac{x^2}{2} + \lambda_1 \frac{F_{sCB}}{\sum EI} + \lambda_2 \frac{F_{stB}}{\sum EI} + C_3 x + C_4 \quad (35b)$$

The constants of integration A_1 to A_8 in Eqs. (34) and (35) have now to be determined from specified boundary conditions. In this section,

friction and adhesion between the concrete and external steel plates, particularly at the supports and load points, is neglected. Therefore, the appropriate boundary conditions are

$$\text{at } x = 0 \quad F_{sCA} = 0 \quad \text{and} \quad F_{stA} = 0$$

$$\text{at } x = \frac{L}{2} \quad DF_{scB} = 0 \quad \text{and} \quad DF_{stB} = 0$$

$$\begin{aligned} \text{at } x = u \quad F_{sCA} = F_{scB} \quad \text{and} \quad F_{stA} = F_{stB} ; \quad DF_{scA} \\ = DF_{scB} \quad \text{and} \quad DF_{stA} = DF_{stB} \end{aligned}$$

The final forms of the constants A_1 to A_8 are

$$A_1 = 0 \quad (36a)$$

$$A_2 = f_1 \frac{P}{2} \left(\cos \sqrt{m_1} u - \sin \sqrt{m_1} u \tan \sqrt{m_1} \frac{L}{2} \right) \quad (36b)$$

$$A_3 = 0 \quad (36c)$$

$$A_4 = f_2 \frac{P}{2} \left(\cos \sqrt{m_2} u - \sin \sqrt{m_2} u \tan \sqrt{m_2} \frac{L}{2} \right) \quad (36d)$$

$$A_5 = f_1 \frac{P}{2} \sin \sqrt{m_1} u \quad (36e)$$

$$A_6 = -f_1 \frac{P}{2} \sin \sqrt{m_1} u \tan \sqrt{m_1} \frac{L}{2} \quad (36f)$$

$$A_7 = f_2 \frac{P}{2} \sin \sqrt{m_2} u \quad (36g)$$

$$A_8 = -f_2 \frac{P}{2} \sin \sqrt{m_2} u \tan \sqrt{m_2} \frac{L}{2} \quad (36h)$$

where

$$f_1 = \frac{1}{\sqrt{m_1}} \frac{(g_5 - g_1 g_4)}{(g_4 - g_3)} \quad (36i)$$

$$f_2 = \frac{1}{\sqrt{m_2}} \frac{(g_5 - g_1 g_3)}{(g_3 - g_4)} \quad (36j)$$

The four constants of integration C_1 to C_4 in Eqs. (35a) and (35b) have also to be determined from four boundary conditions, which are

$$\text{at } x = 0 \quad v_A = 0$$

$$\text{at } x = \frac{L}{2} \quad Dv_B = 0$$

$$\text{at } x = u \quad v_A = v_B \quad \text{and} \quad Dv_A = Dv_B$$

The final forms of the constants C_1 to C_4 are

$$C_1 = \frac{P u (L - u)}{4 \sum EI (1 + \alpha)} \quad (37a)$$

$$C_2 = 0 \quad (37b)$$

$$C_3 = \frac{P u L}{4 \sum EI (1 + \varepsilon)} \quad (37c)$$

$$C_4 = -\frac{P u^3}{12 \sum EI (1 + \alpha)} \quad (37d)$$

2.2.3 Particular solution - friction at supports and load points

In this case, it is assumed that frictional forces of equal magnitude F_f exist between the steel plates and the concrete core, at the supports and the load points, which induce axial forces in the steel plates (see Fig. 3b).

Figure 3b. Support, loading and frictional forces F_f at the support and load points.

Again the eight constants of integration A_1 to A_8 have to be determined from eight boundary conditions, which are

$$\text{at } x = 0 \quad F_{sCA} = 0 \quad \text{and} \quad F_{stA} = F_f$$

$$\text{at } x = \frac{L}{2} \quad DF_{scB} = 0 \quad \text{and} \quad DF_{stB} = 0$$

$$\text{at } x = u \quad F_{sCA} + F_f = F_{scB} \quad \text{and} \quad F_{stA} = F_{stB} ;$$

$$DF_{scA} = DF_{scB} \quad \text{and} \quad DF_{stA} = DF_{stB}$$

The final forms of the constants A_1 to A_8 are

$$A_1 = g \frac{P/2}{(g_3 - g_4)} \quad (38a)$$

$$A_2 = -A_1 / \tan \sqrt{m_1} u + B_1 / \tan \sqrt{m_1} u + B_2 - f_4 F_f / \sin \sqrt{m_1} u \quad (38b)$$

$$A_3 = -g \frac{P/2}{(g_3 - g_4)} \quad (38c)$$

$$A_4 = -A_3 / \tan \sqrt{m_2} u + B_3 / \tan \sqrt{m_2} u + B_4 + f_3 F_f / \sin \sqrt{m_2} u \quad (38d)$$

$$A_5 = A_1 + f_1 \frac{P}{2} \sin \sqrt{m_1} u + f_4 F_f \cos \sqrt{m_1} u \quad (38e)$$

$$A_6 = -B_1 \tan \sqrt{m_1} \frac{L}{2} \quad (38f)$$

$$A_7 = A_3 + f_2 \frac{P}{2} \sin \sqrt{m_2} u - f_3 F_f \cos \sqrt{m_2} u \quad (38g)$$

$$A_8 = -B_3 \tan \sqrt{m_2} \frac{L}{2} \quad (38h)$$

where

$$f_1 = \frac{1}{\sqrt{m_1}} \frac{(g_5 - g_1 g_4)}{(g_4 - g_3)} \quad (38i)$$

$$f_2 = \frac{1}{\sqrt{m_2}} \frac{(g_5 - g_1 g_3)}{(g_3 - g_4)} \quad (38j)$$

$$f_3 = \frac{g_3}{(g_4 - g_3)} \quad (38k)$$

$$f_4 = \frac{g_4}{(g_4 - g_3)} \quad (38m)$$

The four constants of integration in Eqs. (35a) and (35b) can be determined from the boundary conditions

$$\text{at } x = 0 \quad v_A = 0$$

$$\text{at } x = \frac{L}{2} \quad Dv_B = 0$$

$$\text{at } x = u \quad v_A = v_B \quad \text{and} \quad Dv_A = Dv_B$$

The final forms of the constants C_1 to C_4 are

$$C_1 = \frac{P u (L - u)}{4 \sum EI (1 + \varepsilon)} + \frac{\lambda_3}{\sum EI} [q_{sca}(u) - q_{scB}(u)] \quad (39a)$$

$$C_2 = -\frac{\lambda_4 F_f}{\sum EI} \quad (39b)$$

$$C_3 = \frac{P u L}{4 \sum EI (1 + \alpha)} \quad (39c)$$

$$C_4 = -\frac{P u^3}{12 \sum EI (1 + \alpha)} + (\lambda_3 - \lambda_4) F_f + \lambda_3 u [q_{sca}(u) - q_{scB}(u)] \quad (39d)$$

3. Comparison of Results

The behavior of DSC beams is extremely complex and therefore various assumptions are used in full and partial interaction analysis in order to simplify the system as mentioned before. Particular solutions have been obtained for a simply supported and symmetrically loaded DSC beam, as shown in Fig. 2b, which are now used to investigate the influence of various parameters, in particular the stiffness of the shear connection and frictional forces between the steel plates and concrete infill. For the beam shown in Fig. 2b the applied bending moment diagram is symmetrical about the centerline and therefore only half of the beam need be considered.

When comparing the results of full and partial interaction theories the parameter variations involving material properties were simplified. For comparison of theoretical solutions with the experimental test results, the system geometry and material properties assumed were the same as reported by Edwards [31] and are presented in the following sections.

3.1 Comparison of full and partial interaction theories

In this section full and partial interaction theories are compared, firstly neglecting friction between the layers at the supports and load points and secondly taking these forces into consideration; boundary conditions are studied separately. Axial forces in the steel plates, shear flows (shear force per unit length), slip between the layers and transverse displacements are compared in each case.

For the comparison of full and partial interaction theories the assumed beam geometry was the same as tested by Edwards [31], having a span $L=1500$ mm, a breadth $b=200$ mm, a concrete core depth $d_c=150$ mm, top and bottom steel plate thicknesses of 8 mm, stud spacing on both plates 180 mm and the distance of the applied load from the support $u=575$ mm. The Young's moduli of the steel and concrete were assumed to be $210\,000$ N/mm² and $25\,000$ N/mm² respectively.

(i) Neglecting frictional forces

For the first assumption with no frictional forces between the layers, Figs. 4a and 4b show the variation of axial forces in the tension and compression steel plate and shear flows for various values of the connection stiffness K . The axial forces increase with an increase of shear connection stiffness. When the assumed stiffness of the connection is increased, the results based on partial interaction theory tend to those based on full interaction theory.

Figure 4a. Comparison of axial forces in tension steel plate Fst and compression steel plate Fsc for full interaction theory and partial interaction theory with no frictional forces.

Figure 4b. Comparison of shear flow between the layers for full interaction theory and partial interaction theory with no frictional forces.

Fig. 4c shows the slip between the steel plates and concrete infill. The slip is zero at midspan and increases towards the end of the beam. The slip decreases with an increase of the shear connection stiffness and tends to zero as the shear connection stiffness tends to infinity.

Figure 4c. Slip s between the layers for different shear connection stiffness K and for full interaction theory.

Figure 4d. Comparison of beam deflection v for full interaction theory and partial interaction theory.

The deflection curve of the beam, shown in Fig. 4d, decreases with an increase of the shear connection stiffness, and tends to the full interaction curve when the shear connection stiffness tends to infinity. The second and third terms on the right hand side of Eq. (31a) represent the additional curvature caused by loss of interaction between the layers. When these terms are zero, the first term on the right hand side of Eq. (31a) is numerically equal to the right hand side of Eq. (9). Eq. (31a) can therefore be rewritten as

$$\frac{d^2 v_p}{dx^2} = -\frac{M}{\Sigma EI (1+\alpha)} + \lambda_3 \frac{d^2 F_{sc}}{dx^2} + \lambda_4 \frac{d^2 F_{st}}{dx^2} \quad (40)$$

(ii) Including frictional forces

For the second assumption with frictional forces at the supports and load points, Figs. 5a and 5b show that the axial forces in the tension and compression plates for $K=10$ kN/mm and a range of frictional coefficients g .

Figure 5a. Comparison of axial forces in tension and compression plates for full and partial interaction theory.

Figure 5b. Comparison of shear flow between the layers for full interaction theory and partial interaction theory.

Fig. 5c shows the variation of slip with the variation of the frictional forces between the layers. The slip decreases with an increase of the frictional forces when the shear connection stiffness is constant. The frictional force at the support makes the tension plate end slip smaller than the maximum slip along the tension plate, while the compression plate end slip is maximum at the end of the beam.

Figure 5c. Comparison of slip s between the layers for different shear connection stiffness K -partial interaction theory with frictional forces.

Fig. 5d shows the variation of the beam deflection curve with a variation of the frictional forces between the layers. The deflection curve of the beam decreases with an increase of frictional forces.

Figure 5d. Comparison of the beam deflection curve with a variation of the frictional forces between the layers with frictional forces.

Figs. 6a, 6b, 6c and 6d show the variations of the steel plate axial forces, shear flows, slip and deflections with variation of shear connection stiffness, when the frictional coefficient is assumed constant and equal to 0.2. With an increase of shear connection stiffness the axial forces and shear flows increase while the slip and deflection decrease.

Figure 6a. Comparison of the steel plate axial forces with frictional forces.

Figure 6b. Comparison of the shear flow between the layers with frictional forces.

Figure 6c. Comparison of the slip between the layers with frictional forces.

Figure 6d. Comparison of the deflection for full and partial interaction.

3.2 Comparison of partial interaction theory with experimental results

To validate the partial interaction theory, theoretical results for slip and deflection have been compared with experimental results for beams, B7 and B8, tested by Edwards [31]. The beams had a span $L=1500$ mm, a breadth $b=200$ mm, a concrete core depth $d_c=150$ mm, top and bottom steel plate thicknesses of 8 mm and the distance of the applied load from the support $u=575$. The stud spacing on the tension and compression plates varied for the two tests. The assumed material properties were as determined by Edwards [31] (steel plate Young's modulus $210\,000$ N/mm², concrete Young's modulus $30\,000$ N/mm² for beam B7 and $27\,130$ N/mm² for beam B8). The stiffness of the shear connection was varied from 30 to 40 kN/mm. The frictional coefficient at the supports and load points was assumed to be either zero or 0.5.

Figs. 7a and 7b show the comparison of compression and tension plate slip for partial interaction theory with the results of tests on beam B7. For a frictional coefficient of 0.5 theoretical and experimental results show reasonably close agreement.

Figure 7a. Comparison of the slip between the compression plate and concrete infill with frictional forces for partial interaction theory and experimental results of beam B7.

Figure 7b. Comparison of the slip between the tension plate and concrete infill with frictional forces for partial interaction theory and experimental results of beam B7.

Fig. 7c compares theoretical and experimental deflections for beam B7, which also show reasonably close agreement.

Figure 7c. Comparison of the deflection for partial interaction and experimental results of beam B7.

Figs. 8a and 8b show the comparison of tension and compression plate slip for partial interaction theory with the results of tests on beam B8. For a frictional coefficient of 0.5 theoretical and experimental results show reasonably close agreement. Fig. 8c compares theoretical and experimental deflections for beam B8, which also show reasonably good agreement.

Figure 8a. Comparison of the slip between the tension plate and concrete infill with frictional forces for partial interaction theory and experimental results of beam B8.

Figure 8b. Comparison of the slip between the tension plate and concrete infill with frictional forces for partial interaction theory and experimental results of beam B8.

Figure 8c. Comparison of the deflection for partial interaction and experimental results of beam B8.

4. Discussion and Conclusions

The governing differential equations for full and partial interaction analysis of double skin composite beams have been presented and developed. Errors in the previously presented theory for partial interaction analysis have been corrected and the influence of frictional forces between the concrete and external steel plates, at the supports and load points, incorporated in the analysis.

Theoretical results based on partial interaction theory converge to those based on full interaction theory, as the stiffness of the shear connection is increased. Also, theoretical results based on partial interaction theory, assuming realistic material and shear connector properties and incorporating the influence of interface frictional forces, show satisfactory correlation with test results.

This study has presented the formulation of differential equations suitable for the solutions of fully and partially interactive double skin composite beam elements. Closed form solutions for double load

arrangement on simple span beams have been presented to establish the applicability of partial connection in DSC beam elements.

Comparisons have been made between experimental data and theoretical predications of behavior using the method of full and partial interaction analysis. The second assumption of partial interaction analysis with frictional force gives better results than the others in comparison to the results of tests. The predicted frictional coefficient at the interface of steel-concrete has a significant effect on the definition of the behavior of DSC beam and must be calculated.

Consequently, agreement of partial interaction theory between the full and partial interaction theories appears good. The agreement between the solutions of partial interaction theory with frictional force and the results of tests on DSC beams reported by Edwards [31] are found to be reasonably close, although further experimental work is required to establish the applicability of partial connection in DSC beam elements.

References

- [1] Veljkovic, M.. Behavior and resistance of composite slab. Ph.D. thesis. Sweden: Lulea University; (1996).
- [2] Oehlers, DJ., Seracino, R., Yeo, MF.. Fatigue behavior of composite steel and concrete beams with stud shear connections. *Progress in Structural Engineering Materials* (2000) 2: pp.187–95.
- [3] Johnson, R.P.. Beams, slabs, columns, and frames for buildings. Composite structures of steel and concrete, Blackwell Scientific Publications (1994), vol. 1.
- [4] Dogan, O.. Fatigue of welded stud shear connectors in double skin composite construction. Ph.D. Thesis. UK: University of Wales College of Cardiff (1997).
- [5] Roberts, TM.. Dogan O. Fatigue of welded stud shear connectors in steel-concrete-steel sandwich beams. *Journal of Constructional Steel Research* (1998) 45(3): pp.301–320.
- [6] Oehlers, DJ., Bradford, MA.. Elementary behavior of composite steel & concrete structural members. Butterworth-Heinemann (1999).
- [7] Jeong, Youn-Ju, Kim Hyeong-Yeol, Kim Sang-Hyo. Partial-interaction analysis with push-out tests. *Journal of Constructional Steel Research* (2005) 61(9): pp.1318–1331.
- [8] Ranzi, G., Gara, F., Ansourian, P.. General method of analysis for composite beams with longitudinal and transverse partial interaction. *Computers & Structures* (2006) 84(31–32): pp.2373–2384.
- [9] Gara, F., Ranzi, G., Leoni, G.. Time analysis of composite beams with partial interaction using available modelling techniques: A comparative study. *Journal of Constructional Steel Research* (2006) 62(9): pp.917–930.
- [10] Queiroza, FD., Vellascob, PCGS., Nethercota, DA.. Finite element modelling of composite beams with full and partial shear connection. *Journal of Constructional Steel* (2007) 63(4): pp.505–521.
- [11] Ranzi, G., Bradford, MA.. Direct stiffness analysis of a composite beam-column element with partial interaction. *Computers & Structures* (2007) 85(15–16): pp.1206–1214.
- [12] Ranzi, G., Bradford, MA.. Composite beams with both longitudinal and transverse partial interaction subjected to elevated temperatures. *Engineering Structures* (2007) 29(10): pp.2737–2750.
- [13] Jeong, Youn-Ju.. Simplified model to predict partial-interactive structural performance of steel-concrete composite slabs. *Journal of Constructional Steel Research* (2008) 64(2): pp.238–246.

- [14] Ranzi, G.. Locking problems in the partial interaction analysis of multi-layered composite beams. *Engineering Structures* (2008) 30(10): pp.2900-2911.
- [15] Girhammar, UA., Pan, DH., Gustafsson, A.. Exact dynamic analysis of composite beams with partial interaction. *International Journal of Mechanical Sciences* (2009) 51(8): pp.565-582.
- [16] Sousa, Jr. JBM., Oliveira, CEM., da Silva AR.. Displacement-based nonlinear finite element analysis of composite beam columns with partial interaction. *Journal of Constructional Steel Research* (2010) 66(6): pp.772-779.
- [17] Knowles, PR.. *Composite Steel and Concrete Construction*. Butterworths, London (1973).
- [18] Yam, LCP.. *Design of Composite Steel-Concrete Structures*. Surrey University (1981).
- [19] Newmark, NM., Siess, CP., Viest, IM.. Tests and analysis of composite beams with incomplete interaction. *Proc. Soc. Exper. Stress. Anal* (1951) 9(1): pp.75-92.
- [20] Yam, LCP., Chapman, JC.. The inelastic behaviour of simply supported composite beams of steel and concrete. *Proc. Inst. Civ. Engrs* (1968) 4: pp.651-683.
- [21] Yam, LCP., Chapman, J.C.. The inelastic behavior of continuous composite beams of steel and concrete. *Proc. Inst. Civ. Engrs* (1972) 53: pp.487-501.
- [22] Johnson, RP.. *Composite Structures of Steel and Concrete. Beams, Columns, Frames, and Applications in Building*. Crosby Lockwood Staples, London (1975) Vol.1.
- [23] Johnson, RP.. Loss of interaction in short-span composite beams and plates. *Journal of Constructional Steel Research* (1981) 1(2): pp.11-16.
- [24] Roberts, TM.. Finite difference analysis of composite beams with partial interaction. *Computers and Structures* (1985) 3(21): pp.469-473.
- [25] Al-Amery RIM., Roberts, TM.. Non-linear finite difference analysis of composite beams with partial interaction. *Computers and Structures* (1990) 35 (1): pp.81-87.
- [26] Oduyemi, TOS., Wright, HD.. An experimental investigation into the behavior of double skin sandwich beams. *Journal of Constructional Steel Research* (1989) 14: pp.197-220.
- [27] Wright, HD., Oduyemi, TOS, Evans, HR.. The experimental behavior of double skin composite elements. *Journal of Constructional Steel Research* (1991) 19: 97-110.
- [28] Wright, HD., Oduyemi, TOS., Evans, HR.. The design of double skin composite elements. *Journal of Constructional Steel Research* (1991) 19: pp. 111-132.
- [29] Narayanan, R., Wright, HD., Francis, RW., Evans, HR.. Double-skin composite construction for submerged tube tunnels. *Steel Construction Today* (1982) 1: pp.185-189.
- [30] Wright, HD., Oduyemi, TOS.. Partial interaction analysis of double skin composite beams. *Journal of Constructional Steel Research* (1991) 19: 253-283.
- [31] Edwards, DN.. Experimental and theoretical studies of double skin composite construction for immersed tube tunnels. PhD Thesis. UK: University of Wales College of Cardiff (1990).
- [32] Dogan, O., Roberts T.M., Fatigue performance and stiffness variation of stud connectors in steel-concrete-steel sandwich systems, *Journal of Constructional Steel Research* (2012) Vol. 70, pp. 86-92.
- [33] Dogan, O., Roberts, T.M.. Comparing experimental deformations of steel-concrete-steel sandwich beams with full and partial interaction theories, *International Journal of the Physical Sciences* (2010) Vol. 5(10), pp. 1544-1557.
- [34] Dogan, O., Roberts, T.M., Comparison of experimental internal forces with full and partial interaction theories in steel-concrete-steel sandwich beams, *International Journal of the Physical Sciences*, (2010) Vol. 5(15), pp. 2322-2334.