SIGNAL RESOLUTION POLY-GAUSSIAN ALGORITHM FOR NON-GAUSSIAN INTERFERENCE SIMULATION MODELING

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Renat F. Zaripov,

Kazan National Research Technical University named
after A.N. Tupolev-KAI, Kazan, Russia,
renat.zaripov@gmail.com

Marat M. Fatykhov,

Kazan National Research Technical University named
after A.N. Tupolev-KAI, Kazan, Russia,
marat.fatykhovs@gmail.com

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The effective use of the code division multiplexing communication systems channel resource requires the new solutions for conflict-free and high-quality reception and transmission of mixed traffic under the conditions of an exponential increase in the number and density of wireless network subscribers, in particular, those who use multiservice services. At the signal transmission channel layer this problem is solved by applying mathematical models adequate to the real signal-noise environment and algorithms for receiving noise-like signals in dynamic jamming mixtures, which provide increased noise immunity, system capacity and having acceptable computational complexity (technical feasibility of signal processing devices), which is extremely important. In practical applications, the system model includes a parametric quasi-stationary channel, a random number of quasi-determined signals and randomly fluctuating interferences, which determines the real signal-noise situation in the communication system radio interface operational area. In this article, scientific and engineering problems related to the use of software for complicated interference cases secondary signal processing, particularly algorithms based on post-correlation synthesis and analysis technology, are considered. The original simulation software modules are described, which allows to compensate inaccuracy and modeling errors that occur when traditional visual and imitative mathematical modeling tools are applied.

Information about authors:

Renat F. Zaripov, assistant, Kazan National Research Technical University named after A.N. Tupolev-KAI, Department of Nanotechnology in Electronics, Kazan, Russia

Marat M. Fatykhov, assistant, Kazan National Research Technical University named after A.N. Tupolev-KAI, Department of Nanotechnology in Electronics, Kazan, Russia

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Introduction

A number of modern investigations in the field of wireless communication networks electromagnetic environment shows that real probability distributions of desired and interfering signals in the off-hook and free channels are non-Gaussian (including poly-modal cases).

Nowadays, a number of optimal and quasioptimal algorithms for randomly fluctuating signals and interferences receiving and processing designed [1-10]. But technically “simple” self-adaptive algorithm development is challenging issue. This algorithms must support fast corrective adjustment for group signal secondary processing at the interval of one or several data frames.

The development and simulation of noise-tolerant signal processing algorithms in a complex of noise and interference with diverse probabilistic distributions adequate to the real situation in the radio channel is an urgent scientific and practical task and is directly associated with quite a lot of labor in the preparation, program description and modeling of the transmitting part of the system, radio channel, and receiving part with adequate verification of simulation results. Usually, the typical software of this level simulator developers is aimed at solving a narrow problem and its sources are restricted, which does not allow system model and the executable algorithm structure quick parameters change.

In most cases, the mathematical description of the radio channel in solving such problems is significantly simplified and does not take into account their complexity associated with variation of the signal-interference environment parameters, radio signals multipath wave propagation and fading, the “near-far” problem, the influence of mutual interference with the signal structure, the fluctuation noise, industrial and other types of interference influence, etc. [11, 12].

This paper presents a specialized program for simulation and mathematical modeling of well known, including poly-Gaussian,

signal processing algorithms in mobile communication systems, based on the MatLab language and taking into account the drawbacks inherent in both visual modeling tools and the programming language itself [13].

The available modeling tools subsequently allow multidimensional processing of random signals and interference.

Investigated model of communication system

As an example, consider a communication system at the observation time $[0, T_{\text{obs}}]$, in which we have N active user terminals. Each user has his own probability P_N of being active with duration of message $[0, T_b]$. Considering that at the observation time $[0, T_{\text{obs}}]$, base station (BS) detects random number of information bits, that are multiplexed with OVFS codes to get direct sequence spread spectrum (DSSS).

A single bit of information b_k , where $k = \overline{1, K}$ (where K is a index number of bit in the message) is processed as a code bit, passed through procedure of protective coding, and, for phase manipulation, transformed into bipolar form, i.e. $\langle 1 \rangle = \langle -1 \rangle$, a $\langle 0 \rangle = \langle 1 \rangle$.

In general, received group signal at the entrance can be described as:

$$r(t) = s_k(t) + n(t) + l_z(t), \quad (1)$$

where $s_k(t)$ are signals of number of active user terminals after amplification and carrier transfer; $n(t)$ – additive white Gaussian noise with zero mean ($m_n=0$) and randomly specified variance (σ_n^2); $l_z(t)$ – unspecified random process caused by different types of noises and interferences, such as industrial noise, internal system and chaotic pulse interference. This is non-Gaussian interference random number of signals resolution problem formulation. The generalized block diagram for the considered and simulated communication system is presented in Fig. 1.

Fig. 1. Communication system model: a) structure of a transmitter and a communication channel; b) structure of a receiver

There are few limitations and assumptions for considered system model

1. It is assumed that at the observation interval active user terminals synchronization procedure is provided. This leads to, delay estimates are not performed additionally for each signal.

2. For receiver, every type of interferences appears as poly-Gaussian interference, represented by the weighted sum of Z Gaussian composition components [14], whose parameters are known at the frame observation interval from the use of the EM-algorithm [9].

3. Communication system model is based on the determined signal receiving, and the random nature of input process fluctuations determined by signal and interference mixture poly-Gaussian probabilistic model [15].

Optimal signal resolution algorithm

In [1] an algorithm for resolving signals was proposed and an analytical calculation of likelihood ratios for one signal and interference group was performed. It is proposed, as an example, to modify the specified algorithm and the resulting expressions and analytically decompose them into separate components according to the hierarchical computational tree used in computational clusters of procedures in the modeling process.

We formalize this problem in terms of statistical hypotheses testing. At the input of the receiver there is a signal of the following form: $u = S_N + I_M + n$ (hypothesis $S_N I_M n$), or $u = n$ (hypothesis n).

Probabilistic description of hypotheses has the form:

$$S_N I_M n: W(u | S_N I_M n) = \prod_n q_{S_n} \prod_m q_{I_m} G\left\{u; \sum_n S_n + \sum_m I_m; \sigma_{I_M n}^2\right\}, \quad (2)$$

$$n: W(u | n) = G\{u; 0; \sigma_n^2\}, \quad (3)$$

where $\sigma_{I_M n}^2 = \sigma_N^2 + \sum_m \sigma_{I_m}^2$; G is Gaussian density function; q_{S_n} is a probability mix weight for N signals; q_{I_m} is probability mix weight for M interferences; N – vector string of signal numbers (for example $[N] = [1 \ 4 \ 6]$, 1st signal, 4th signal and 6th signal); M – vector string of interference numbers (for example $[M] = [1 \ 2 \ 4]$, 1st interference, 2nd interference and 4th interference); S_n and I_m are the mean of n -th signal and m -th interference correspondingly.

In this case likelihood ratio has form:

$$l_{S_N I_M n} = \prod_n q_{S_n} \prod_m q_{I_m} \frac{w(u | \{S_1, S_2, \dots, S_n\}, \{I_1, I_2, \dots, I_M\}, n)}{w(u | n)} \quad (4)$$

Eq. (4) shows that the likelihood ratio calculation unit has high computational redundancy in the computational cluster, therefore, the likelihood ratio finding algorithm requires optimization in the computational part.

Accordingly, the final expression for the likelihood ratio and its components in accordance with [1] is:

$$l_{S_N I_M n} = l_{0 S_N I_M n} + Z_{S_N I_M n}(u) + 0,5 E_{S_N I_M n}(u) - 0,5 E_{S_N I_M n}. \quad (5)$$

Here and after

$$l_{0 S_N I_M n} = \ln \left(\prod_n q_{S_n} \prod_m q_{I_m} \sqrt{\frac{\sigma_n^2}{\sigma_n^2 + \sum_m \sigma_{I_m}^2}} \right);$$

$$Z_{S_N I_M n}(u) = u \frac{\sum_n S_n + \sum_m I_m}{\sigma_n^2 + \sum_m \sigma_{I_m}^2} E_{S_N I_M n} = \frac{\left(\sum_n S_n + \sum_m I_m \right)^2}{\sigma_n^2 + \sum_m \sigma_{I_m}^2};$$

$$E_{S_N I_M n}(u) = u^2 \frac{\sum_m \sigma_{I_m}^2}{\sigma_n^2 + \sum_m \sigma_{I_m}^2}, \text{ where } Z_{S_N I_M n}(u) \text{ is correla-}$$

tion of input and N signals, normalized on power of noise and M interferences; $E_{S_N I_M n}$ is mutual energy of M interferences, normalized on $\sigma_{I_M n}^2$; $E_{S_N I_M n}(u)$ is a cross-correlation of input energy and energy of noise and M interferences.

The resulting formula describes the case of a one sample of the input implementation one-dimensional distribution. In case of $k = \overline{1, K}$ samples, we have:

$$l = l_{0 S_N I_M n} + \sum Z_{S_N I_M n}(\bar{u}) + 0,5 \sum E_{S_N I_M n}(\bar{u}) - 0,5 \sum E_{S_N I_M n}, \quad (6)$$

where $\bar{u} = \{u_1, u_2, \dots, u_k\}$.

Rewrite expression (6) in a simplified form for calculation:

$$l = l_0 + \sum_{k=1}^K u_k \underbrace{S l_{\sigma_k}}_{\text{Vector}} + 0,5 \sum_{k=1}^K u_k^2 \underbrace{\sigma_{I_M n}}_{\text{Scalar}} - 0,5 \sum_{k=1}^K u_k \underbrace{S l_{\sigma_k}^2}_{\text{Vector}}, \quad (7)$$

and see that l_0 , $S l_{\sigma_k}$, $\sigma_{I_M n}$ and $0,5 \sum_{k=1}^K S l_{\sigma_k}^2$ changes only when

changing the value of a parameter, for example, after after processing the input implementation by the EM clustering algorithm [16, 17]:

$$l_{0 S_N I_M n} = \ln \left(q_{S_N I_M n} \sqrt{\frac{\sigma_n^2}{\sigma_{I_M n}^2}} \right), \quad S l_{\sigma_k} = \frac{S_{N_k} I_M}{\sigma_{I_M n}}, \quad \sigma_{I_M n} = \frac{\sigma_{I_M}}{\sigma_{I_M n} \sigma_n^2},$$

$$S l_{\sigma_k}^2 = \frac{(S_{N_k} I_M)^2}{\sigma_{I_M n}^2}; \quad q_{S_N I_M n} = \prod_n q_{S_n} \prod_m q_{I_m};$$

$$S_{N_k} I_M = \sum_n S_{n_k} + \sum_m I_m; \quad q_{I_M} = \sum_m \sigma_{I_m}^2.$$

Software simulation complex for mobile communication systems signal processing algorithms modeling

For simulation of the optimal and quasi-optimal signal resolution algorithms, a software-simulation complex has been developed [13], in which all program elements are made in the form of specialized functional modules. MatLab platform and language were used as a programming language and development environment. MatLab platform runs on 2 engines.

The first is responsible for mathematical calculations written in C++ using the Runtime library from Microsoft Corporation, the second is responsible for outputting all graphical objects written in Java using the Oracle library from Oracle Corporation. Since MatLab is based on two platforms that most developers use, software written in this environment can be transferred to other object-oriented languages. To increase the speed of calculations, a proprietary database management system was developed.

The developed software system allows to significantly save the time spent by developers on creating an environment of simulation modeling adequate for real signal-interference situations and to evaluate the effectiveness of the synthesized signal and interference processing algorithms. Signal processing algorithms modeling time saving is achieved by:

1. Using a simulation automation method.
2. Use of a program modules code special structure.
3. Optimization of memory usage by a database in a cellular form. This makes it possible not to switch to 64-bit architecture and not to acquire specialized equipment for performing design work. Due to the use of a small amount of memory, the access time is reduced; this leads to a significant increase in the speed of calculations in simulation.
4. Combinations of discrete-continuous values with discrete ones. The essence of this approach is to process data containing information about the time instants of each element of the array. This is necessary when modeling quasi-real communication channels.
5. Elimination of the remainder of the division. This problem is widespread in time-dependent process modeling. Most software solutions have libraries that cut off the remainder after the division operation. But, in particular, MatLab does not perform this procedure and returns a value with the remainder. For example, in MatLab, the expression $1/1099 = 1.0000e-99$ will not always be true.
6. Using the developed software module for quasi-real communication channels modeling. This program module, through the use of mixtures of different probability distribution densities, makes it possible to simulate quasi-real noise and interference, fast and slow fading of channels, chaotic impulse interferences, simulations of Doppler signal shifts, reduction and increase in wavelength, which cause intersymbol interference and other destabilizing factors.

Results of simulation modeling

In the Fig. 2., the results of minimizing the mean square error (MMSE) and poly-Gaussian (PG-MMSE) synthesized algorithm asymptotic efficiency and the bit error probability under non-Gaussian interference numerical modeling are presented. Modeling conditions: average number of users: 10; OVSF 127 codes used; sufficient statistics - over 1000 frames with a length of 128 bits; equality of received signals energies (when using power control). The parameters of arbitrary non-Gaussian interference were randomly selected from the following ranges: number of probability components: 3-8, components weighting coefficients: 0.15-0.8 (under the normalization condition), standard deviation: 0.5-4.5, mathematical expectation: 0-5.

Fig. 2. Bit error probability as a function of SNR

The presented results of mathematical modeling shows a real gain in the noise immunity of the poly-Gaussian algorithm with respect to the well-known MMSE algorithm by almost a decade with equal signal/noise or signal/(interference+noise) ratios.

Conclusions

The modeled original algorithm for resolving signals in non-Gaussian interference is a quasi-optimal signal processing algorithm in which the decision on the signal-noise combination is made according to the maximum likelihood criterion. The algorithm shows the high performance for receivers in channels with non-Gaussian interference.

The disadvantage of the poly-Gaussian algorithm is high computational complexity, more precisely, with an increase in the number of users and interference components, the number of hypotheses increases exponentially. It should be noted that the complexity of the calculations is noticeable against the background of an increasing number of communication systems users. This requires the controlled avoidance of optimality for quasi-optimal procedures development.

However, an undoubted advantage is a limited set of standard operations of no more than 2nd order. Given the modification of the algorithm, an increase in efficiency is also observed in the calculation part. In general, this gives technical simplification when implementing the algorithm on modern FPGAs.

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МОДЕЛИРОВАНИЕ ПОЛИГАУССОВОГО АЛГОРИТМА РАЗРЕШЕНИЯ СИГНАЛОВ В НЕГАУССОВСКИХ ПОМЕХАХ

Зарипов Ренат Фаридович, Казанский национальный исследовательский технический университет им. А.Н. Туполева – КАИ, Казань, Россия, renat.zaripov@gmail.com

Фатыхов Марат Монирович, Казанский национальный исследовательский технический университет им. А.Н. Туполева – КАИ, Казань, Россия, marat.fatykhovs@gmail.com

Аннотация

Рассмотрены научно-инженерные задачи, касающиеся применения программных комплексов для моделирования алгоритмов вторичной обработки сигналов в сложных помеховых комплексах, в частности, алгоритмов, основанных на технологии посткорреляционного синтеза и анализа. Изложены оригинальные модули ПО для имитационного моделирования, позволяющие скомпенсировать погрешности и ошибки моделирования, возникающие при применении традиционных средств визуального и имитационного математического моделирования.

Ключевые слова: среда имитационного моделирования, негауссовские каналы связи, алгоритм разрешения сигналов, Matlab.

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Информация об авторах:

Зарипов Ренат Фаридович, ассистент кафедры НТвЭ, Казанский национальный исследовательский технический университет им. А.Н. Туполева – КАИ, Казань, Россия

Фатыхов Марат Монирович, ассистент кафедры НТвЭ, Казанский национальный исследовательский технический университет им. А.Н. Туполева – КАИ, Казань, Россия