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NUMERICAL STUDIES OF THE INFLUENCE OF THE PARAMETERS OF THIN-SHELL DOME STRUCTURE ON THEIR OPTIMIZATION FROM THE POSITION OF SPATIAL STABILITY

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Statement of the problem. The task was to evaluate the influence of the parameters of thin-walled dome coatings using the capabilities of modern software complexes. The method of optimization of dome covering structures with selection of criteria and parameters of the task has been improved.

Results. The article presents the results of refinement and testing of the methodology for addressing the problem of optimizing dome structures with the choice of criteria and parameters of the optimization problem using the capabilities of the Topological Optimization module of the finite-element computational complex *MidasCivil*. The objective function was considered dependent on the thickness of the dome, the modulus of elasticity of the Poisson coefficient of the material. The study employs the positions of the theory of elasticity, solid body deformation mechanics, construction mechanics, as well as mathematical modeling methods based on the use of the finite element method employing modern licensed finite-element computing complexes *MidasCivil* and the *Ing* + architectural and construction design system of the calculation module *MicroFe*.

Conclusions. Using the methods of optimal (in particular, geometric) design, the most affecting parameters of thin-walled dome coatings and their combinations were identified. This will allow us to design the most rational, economical and architectural-expressive dome structures as well as to make sound design decisions.

Keywords: dome coverings, finite element model, topological optimization, spatial stability, design optimizations, criteria and parameters of optimization problem.

Introduction. The use of domed structures has of late been gaining momentum due to the massive construction and restoration of religious buildings. The design and construction of temple structures has been an essential element of modern design and architecture over the

last 25 years. Since the early 1990s, the active construction of religious buildings of various denominations got underway. If religious architecture is to be compared with other secular types of architectural structures, it can be noted that purely architectural forms rather than economic, financial or technological considerations dominate [13].

Thus the feasibility of conducting theoretical and applied research in the field of optimizing the shape and structure of dome structures increases. Therefore in the recent years, a numerical method for calculating various structures, FEM, i.e., the finite element method, has become common. It has a range of advantages compared to other numerical methods, such as the finite difference method, etc. This is diversity in regard to design schemes, clarity of the design scheme, easy algorithmization, the capacity to automate the calculation process and to implement any boundary conditions [1, 7].

A number of studies by both Russian [2—6, 8—12] and overseas scientists [16—17, 19—21] are dedicated to the study of SSS as well as the optimization of different types of structures.

Unfortunately, not enough attention has been paid to optimizing reinforced concrete dome surfacing. In [18], the major provisions of the methodology for addressing the problem of optimizing structures of dome surfacing of a circular shape under operational loads in the structure from the viewpoint of preventing possible forms of buckling were developed and some results of numerical investigations were presented. It is obvious that the development and improvement of the methods for the optimal design of building structures as well as the use of modern specialized software systems make it simple to design rational and economical structures of a dome shape and are a relevant research area.

The objective of the study is to clarify, conduct full-scale research and test the methodology previously set forth by the authors for the optimal design of thin-walled reinforced concrete dome structures to obtain one where a decrease in material consumption could be achieved with constant values of the existing and increasing load-bearing capacities. Applying mathematical modeling methods based on the use of the finite element method in combination with modern licensed finite element computing systems *MidasCivil* and systems of end-to-end architectural and construction design *Ing* + calculation module *MicroFe* employing the results of numerical studies, it is essential to analyze the influence of different parameters and their combinations on optimization thin-walled dome surfacing.

1. Technique for optimizing the parameters of the dome surfacing. The objective of the research is a round reinforced concrete thin-walled dome surfacing of the main hall of the complex of the iconic building of the Annunciation Cathedral in Voronezh. This object was

selected as an example for optimizing the dome structure in order to obtain its version employing a smaller amount of material, which can be subsequently used in the design and construction of religious and spectacular structures in the regions, both in normal and in special construction conditions, such as heavy snow load with low temperature indicators or high humidity and temperature indicators.

In order to design a computational model, a shell hybrid finite element and a shell finite element is employed based on the displacement method taken from the FE library of the *MicroFe* design module. In this case, the following provisions were used:

1. The dome surfacing was modeled by elements of a flat shell taking into account shear deformations along the shell thickness based on the Reissner-Mindlin theory.
2. Geometric nonlinearity in the calculation is not considered due to small displacements of the structure.

Physical nonlinearity is not considered in the calculation as the entire surfacing is compressed; the effect of opening cracks, plastic deformations in concrete and reinforcement, redistribution of forces between the elements is not observed.

In order to choose the type of finite element for further use, preliminary numerical studies of the stress-strain of a reinforced concrete smooth dome surfacing with a diameter of 16 m and a shell thickness of 6 cm with a monolithic reinforced concrete ring with a cross-sectional area of 60×60 cm were conducted. In this case, the FE method of displacement and hybrid FE were used. The calculation was performed for a combination of loads: own weight and snow load of the dome surfacing with a monolithic support ring.

The analysis of the results enables us to conclude that their discrepancy is small. In the future, when performing calculations, hybrid FE will be used.

A reinforced concrete dome is optimized, which is a circle with a diameter of 16 m with an initial constant thickness of 0.06 m, pinched in a support ring or pivotally supported on a support ring made of brick with a constant cross section [18]. A diagram of the dome in question with the major geometric characteristics is shown in Fig. 1.

The greatest development for actually designed structures was obtained by problems where weight or volume is taken as an optimality criterion, subject to the conditions of strength, rigidity and stability as well as various design constraints. In this case, the optimization problem is reduced to the determination of the vector of design parameters (optimization) [18]. The optimization problem is reduced to the assignment of minimum (or maximum) constraints of the criterion using which the structure can be evaluated. This criterion depends on

the chosen parameters x_i and is called the objective function. In our case, there are several objective functions, so the optimization problem is multi-criteria.

$$f_i(x_1, x_2, \dots, x_n) \rightarrow \min. \quad (1)$$

Fig. 1. Scheme of the investigated dome with the major geometric characteristics

In this case, the optimization problem is reduced to the determination of the vector of design parameters (optimization). While forming the objective function, optimization parameters were identified and constraints were assigned to optimize the dome surfacing $x(z_i) = (x_1, x_2, \dots, x_N)$, corresponding to the minimum, e.g., the volume of the structure taken as the objective function:

$$V(x) = \min\left(\sum_{i=1}^m A_i \times x_i\right), \quad (2)$$

according to the restrictions:

— for strength:

$$\sigma_{\text{эКБ}}^{\max} \leq [\sigma]^{\pm}, \quad (3)$$

— for rigidity:

$$y_{\max} \leq [y], \quad (4)$$

— by the Euler stability of compressed elements:

$$\sigma_i = \sigma_k, (i = 1, \dots, p), \quad (5)$$

— by own frequencies:

$$\omega = [\omega], \quad (6)$$

— by the limits of change of design variables:

$$x(z_i) \geq h_0, (i = 1, \dots, n), \quad (7)$$

where A_i is the length of a one-dimensional or plan area of a two-dimensional element; x_i is the area of cross-sections of one-dimensional or thickness of two-dimensional elements; $\sigma_{\text{экв}}^{\text{max}}$ is the value of the equivalent voltage; $[\sigma]^{\pm}$ is the maximum permissible value of the intensity of tensile and compressive stresses (design resistance); $[y]$ is the maximum allowable deflection of the structure; σ_k is critical stress of buckling of rod elements; $[\omega]$ is specified frequency of natural vibrations; h_0 is the minimum allowable cross-sectional area of a one-dimensional element or thickness of a two-dimensional element.

Let us pose the problem of optimizing the dome cover, which consists in minimizing the function of the function. The dawn criterion is the weight of the dome surfacing. In the current study, the objective function will be considered dependent on the following parameters:

- variable thickness of the dome;
- modulus of elasticity;
- Poisson's ratio;
- camber of arch.

The constants are the dimensions of the support ring. The material of the dome surfacing is reinforced concrete: $E = 3.25 \cdot 10^4$ MPa, $\mu = 0.2$. The dome load is own load 0.55 T/m^2 + payload 0.15 T/m^2 + snow load at +117 m (wind load not considered).

In compliance with the task, the following constraints are assigned to optimize the dome surfacing:

— by the strength:

$$\sigma_{\text{экв}}^{\text{max}} \leq [\sigma]^{\pm}, \quad (8)$$

by the vertical displacement of the upper point of the canopy $y_{\text{нред}}$, the anticipation at which the loss of stability of the canopy takes place:

$$y_{\text{max}} \leq [y_{\text{нред}}], \quad (9)$$

— the camber of arch is assigned to the limits of changing the design variables based on the condition:

$$f = \left(\frac{1}{6} \div \frac{1}{8} \right) D. \quad (10)$$

The intensity of the complete design load should not exceed the value:

$$q \leq 0,2E_{b,dep} \left(\frac{t}{r_c} \right)^2 = \frac{E_b}{20} \left(\frac{t}{r_c} \right)^2, \quad (11)$$

where t is the thickness of the shell.

As a criterion for the loss of stability, the vertical displacement of the top of the dome at the moment of loss of stability was taken. In order to identify this value, a calculation was performed for a combination of loads with an increase in snow load until the start of the limiting state.

2. Obtained data based on research. Modern theoretical approaches to the design of structures for various purposes call for numerical modeling of the operation of structures under various operating conditions [11].

Presently, in the design of building structures, various finite element (FE) software packages (*MidasCivil*, *Sofistik*, *Lusas*, *Lira*, *Ansys*, *Nastran*, *Algor*, *Danfe*, *Mefisto*, *Femap*) are effectively employed, which allow a more correct assessment of the stress-strain (VAT) structures of the structure and to prevent zones dangerous from the standpoint of destruction and loss of stability [10, 11].

One of the major advantages of *MidasCivil* is the capacity to consider the whole range of different types of structural analysis within a single design complex. This software package includes: calculation of construction stages considering the time-dependent properties of materials, *p-delta* analysis for calculating additional moments and deflections, modeling of material destruction, modeling of prestressing and much more. *MidasCivil* enables design engineers to easily perform computational analysis and design of both simple and complex structures for various purposes, using a wide range of computational approaches based on the finite element method, as well as on the modern theory of structural analysis and visualization of the results obtained. These features contribute to efficient, versatile and efficient design of structures [6].

In [18], the influence of design variables (shell thickness, elastic modulus, and Poisson's ratio) on displacements or stresses of a structure at a given point was estimated. Correlation analysis was carried out which is presented in the form of a graph of design variables in Fig. 2.

For a comprehensive analysis of the behavior of the objective function for two variables, the response surface methodology is used. A relationship is established between several variables independent of each other and one or more response variables. Fig. 3 shows a graph of the objective function for two functions: modulus of elasticity — Poisson's ratio. The same

graphs were obtained for the functions: modulus of elasticity — shell thickness (Fig. 4) and for a combination of shell thickness – Poisson's ratio (Fig. 5).

Fig. 2. Plot of design variables (shell thickness, elastic modulus and Poisson's ratio) to move a specific point of the structure

Further, the following parameters were varied: the modulus of elasticity of the material, Poisson's ratio, and thickness of the shell. As a result, the graphs of the displacements of the dome center at various values of the design variables were obtained, which are presented in Fig. 6—7.

Fig. 3. Objective function plot for two functions: elasticity modulus — Poisson's ratio

Fig. 4. Objective function plot for two functions: elasticity modulus — thickness of the shell

Fig. 5. Objective function plot for two functions: thickness of the shell – Poisson's ratio

Fig. 8 shows the magnitude and direction of the reaction vectors in the embedment at different cambers of arch. Graph of the dependence of the vertical displacement of the upper point on the elastic modulus (a); graph of the dependence of the vertical displacement of the upper point on the thickness of the shell (b).

Fig. 9 sequentially shows the graphs of the vertical displacements of the top of the dome surfacing when the thickness of the shell changes depending on the lifting arrow where t is the thickness of the shell, f is the camber of arch. In Fig. 9 a) the camber of arch is $f = 7.5$ m; in Fig. 9 b) - $f = 8.25$ m; and in Fig. 9 c) - $f = 9.5$ m respectively.

6)

Fig. 7. Graph of the dependence of the vertical displacement of the upper point on Poisson's ratio

Fig. 8. Value and direction of the reaction vectors in the embedment at different cambers of arch

Fig. 10 shows the values of vertical displacements of the top of the dome when changing the boom, depending on the thickness of the shell for different conditions of fastening (hinged or rigid termination).

The results obtained illustrate the capabilities of the software package from the viewpoint of dome optimization enabling one, e.g., to analyze the values of vertical displacements of the dome top by means of varying the size of the lifting boom and the thickness of the dome structure shell. The obtained data make it possible to identify the most optimal parameters, which, in turn, will allow one to design dome surfacing with a rational distribution of material over its surface at a specific load.

Fig. 9. Vertical movement of the top of the dome while changing the thickness of the shell:

a) $f = 7.5$ m; b) $f = 8.25$ m; c) $f = 9.5$ m

Fig. 10. Vertical movement of the top of the canopy while changing the camber of arch:

a) $t = 0.05$ m; b) $t = 0.075$ m; c) $t = 0.1$ m

In the future, it seems possible to obtain a sketch of a finite element model based on which design of a dome surfacing can be substantiated that is attractive from the standpoint of architectural expressiveness. Given that the usual tectonics of dome surfacing, which developed in the stone architecture of antiquity, survived almost until the 21st century, in this case a new aesthetic development of the constructive form of the dome with a simultaneous decrease in the amount of material for its construction can be discussed.

The developed finite element models make it possible with a sufficient level of accuracy to address the problems of assessing the analysis of the stress-strain and to optimize the geometric parameters of the dome surfacing and the characteristics of the material.

Conclusions

1. Using the surface response method, graphs of objective functions of such parameters as: elastic modulus, Poisson's ratio and thickness of the shell were clearly presented. Through the course of the study, the design of the dome and its varying parameters were changed for a specific criterion of optimality while maintaining or improving its functionality.
2. Based on the resulting graphs, it can be concluded that there is a direct dependence of the increase in the value of vertical displacements of the top of the dome on that in its camber of arch.
3. As the wall thickness of the dome surfacing increases from $t = 0.05$ m to $t = 0.1$ m, there is a slight rise in the value of the vertical displacements of the dome top.
4. An improved technique for optimizing dome cover structures with a choice of criteria and parameters of the problem enables one to design thin-walled dome surfacing to be more rational, cost-effective and architecturally expressive while making sound engineering decisions.
5. Due to the use of the software package *MidasCivil* it becomes possible to obtain a highly efficient finite element model of a dome structure in an interactive visualized environment. An advanced graphical *CAD*-modeling environment has the capacity to import models from other systems for three-dimensional modeling of geometric objects. Hence this complex can be considered a multifunctional and convenient tool for automatically drawing finite element meshes that approximate the areas of geometric objects with the capacity to control quality making it possible to recommend it for computational analysis and design of both simple and complex structures for various purposes.

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